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Modelling of a Witricity System Using GSSA Method

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Abstract

With the rapid development of mobile appliances, wireless power transfer technique has been a hot issue for researchers. A resonant coupled power system called witricity with high efficiency and the middle range transfer distance is presented by MIT. The main circuit of the witricity system acts as a resonant converter operating in high frequency. The converter is a complex time variant and non-linear systems. So, it is difficult to obtain its accurate mathematical models. In this paper, the generalized state-space averaging method is applied to model this converter. With appropriate values for the circuit parameters, numerical results are compared with which in time domain model. The results show that theoretical analysis are well agree with the simulation, and by proposed method the computational time is remarkably reduced.

Keywords: *wireless power, witricity, generalized-state-space-averaging*

1. Introduction

Without cords and plugs, electronic and electrical products has been more friendly and safer than traditional method in special occasion such as coal mine, under water and implanted devices [1, 2, 3, 4, 5]. The wireless power transfer system based on inductively coupled power transfer method has the bottleneck of short transfer distance. If the distance between energy transmitter and receiver is more than tens millimetres, the system efficiencies will drop sharply. However, the theory of magnetic resonance coupling proves a method of wireless power transfer(WPT) in longer distances. Andre [6] presented his work about this method which called witricity (wireless electricity). A 100 W bulb was lit 1m apart from the source by magnetic resonance coupling method. Currently, considerable efforts have been made to the development of this power transfer method. It has been shown that the distance of wireless power transfer is significantly increased by placing intermediate resonators between transmitter and receiver. Though above researches have great contributes to the transfer distance and efficiency [7, 8, 9, 10, 11, 12], the studies about mathematical model of the witricity system are seldom. The circuit of witricity requires operating with a high frequency, normally more than 1 MHz, to obtain high transfer efficiency in long distance. Usually, a resonant converter with PWM switches is employed to drive the witricity system. The power converter model is fast changing with the time in nature because of the switching behaviour, so this resonant converter is a time varying, nonlinear and complex system. Thus, it is difficult to acquire the exact mathematical model of this converter.

There are two methods available to model a circuit. One is based on the circuit topology, which can be simulated by some soft packages like SIMULIK or PSIM. The shortcoming of this method is that it requires vast resources of computer and a long simulation time. The other method is to derive its mathematical model directly through mathematical method. The advantage is that any parameter can be change to observe characteristics of proposed system in different condition. Based on this idea, the state space averaging method is developed for the analysis and design of PWM converters [13]. Because this method considers only the DC components of variables, it is not valid for modelling of resonant converters in which the AC components are the main contributions. In addition, this method is based on an assumption that the variables should be much slow in time domain than switching frequencies. However, in a resonant converter, the switching

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frequencies are near to its nature resonant frequencies. After that, a modified state space averaging approach is proposed, which is applied to model the quasi resonant converters with high accuracy. But it till cannot describe the resonant converter in detail. Sanders [14] proposed a generalized state space averaging (GSSA) method to model resonant converters. Compared to other resonant converter modelling techniques, this GSSA method represents variables both in a slow time-varying DC variables and fast oscillatory AC variables. This method is particularly suitable to full resonant converters. Carlos[15] has successfully employed this method in modelling a full-bridge rectifier. Besides, GSSA method was applied to the harmonics estimation of PWM converters [16, 17, 18, 19]. In the application of resonant converters, a contactless power supply system is modelled using GSSA method with a completer 9th order system [20]. And then, Xin[21] employed this method to derive the mathematical model of an inductively coupled power transfer system, and a robust controller has designed based on this model. In addition, a mathematical model to simulate transcutaneous energy transmission systems by GSSA method is presented[22], and through selective modal analysis method, the system was keeping up to the first-order. However, the mathematical model of the witricity system based on GSSA or other modelling method is seldom to be found up to date.

In this paper, the GSSA method is employed to the modelling of a witricity system and the steps are presented in detail. The comparison of the GSSA method and circuit model simulation in time domain by PSIM is analysis, which shows that GSSA has a sufficient precision to present the dynamic system of a resonant converter. This paper is organized as follows. In section 2 the GSSA method is reviewed. In section 3, the mathematical model is derived by GSSA method and numerical analysis is presented at last. The conclusions are given in section 4.

2. Overview of GSSA Method

The GSSA method is based on the concept that any waveform $x(t)$ on a time interval $[t, t+T]$ can be expressed in Fourier series as,

$$x(t) = \frac{1}{2}A_0 + \sum_{n=1}^{\infty}(a_n cos(\omega nt) + b_n sin(\omega nt)), \tag{1}$$

where $t \in [0, T]$, $\omega = 2\pi/T$, and T is the period of a sliding window. It is mention to note that the variable $x(t)$ is not necessarily periodic and can be imagined so by summing the window repeats over all time domain. Thus, a Fourier series exists and its coefficients can be determined from,

$$a_n(t) = \frac{2}{T}\int_t^{t+T} x(t)cos(n\omega t)dt, \tag{2}$$

and

$$b_n(t) = \frac{2}{T}\int_t^{t+T} x(t)sin(n\omega t)dt. \tag{3}$$

It is clear that the average value $A_0/2$ and the amplitude $\sqrt{a_n^2 + b_n^2}$ of each component change with time. In other word, the coefficient a_n and b_n can reflect the envelope of the original signal $x(t)$. For resonant converters, variables usually have both quasi-sinusoidal and direct components, and if an appropriate sliding window is selected, the fundamental term of the variable in each sliding window will represented of the original signal $x(t)$ well. In general, the period T is chosen near the nature resonant period of the converter under studied. The value of n represents the accuracy level of the signals and if n approaches infinity, the approximation error theoretically approaches zeros.

3. Mathematical Modelling of the Witricity System

The witricity system employs a high frequency resonant converter to generate the excitation of the power transfer system. In this part, the operational principle of witricity will be review first, and then the circuit model is discussed. At last the mathematical model is derived by GSSA.

3.1. Operational Principles of Witricity System

Witricity is based on the near-field, strongly coupled magnetic resonance, and the fundamental principle is that resonant objects exchange energy efficiently, while non-resonant objects interact weekly. Figure1 shows the schematic of a witricity system using two magnetically coupled resonators, which includes the source coil, the device coil, the energy source, and the load [].

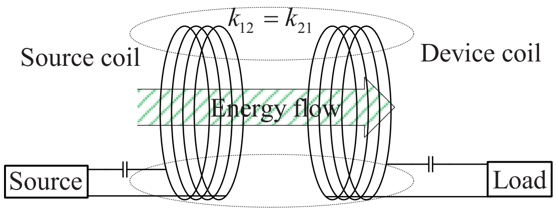

Figure 1. Basic components of witricity system

Observed from figure 1, energy resonates between the source coils and the device coils through the electromagnetic fields, though there is a big gap between the source coils and the device coils which generates a low coupling coefficient between this two coils. This physical phenomenon can be explained using the coupled mode theory []. As seen from Figure 1, it can assume that this two coils are two resonators, and $a_s(t)$ and $a_d(t)$ represent the amplitude of these resonators. In which, $a_s(t)$ is the source resonator and $a_d(t)$ is the device resonator respectively. The two resonators obey the following equations [].

$$\frac{da_s(t)}{dt} = (i\omega_s - \Gamma_s)a_s(t) + ik_{sd}a_d(t) \tag{4}$$

$$\frac{da_d(t)}{dt} = (i\omega_d - \Gamma_d)a_d(t) + ik_{ds}a_s(t) \tag{5}$$

Where, ω_s and ω_d are the individual angular frequency of source coils and device coils, respectively; $k = k_{12} = k_{21}$ is the coupling coefficient between the source and device; Γ_s and Γ_d are the individual intrinsic decay rates for the source and device, respectively. Define the coupling factor as

$$CF = \frac{\omega k}{2\sqrt{\Gamma_s \Gamma_d}}. \tag{6}$$

When this meets the following conditions, $CF \gg 1$ and $\omega_s = \omega_d$, the witricity system can transfer power efficiently from source coils and device coils[].

3.2. Circuit Model

In this paper, the source device employs a half-bridge to excite the resonant converter, and the serial-parallel compensation net is used which is shown in figure 2.

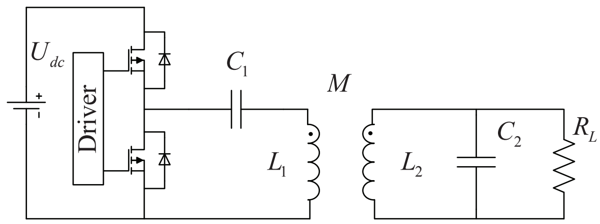

Figure 2. Schematic of proposed half-bridge converter

In figure2, L_1 and L_2 denote the primary coil self-inductances and the secondary coil self-inductances, respectively; C_1 and C_2 represent compensated capacitors on both sides; M is the mutual inductance between the primary coil and secondary coil and R_L is the equivalent AC resistor of loads. By means of mutual inductance theory, the equivalent circuit of this system is deduced and shown in Figure3.

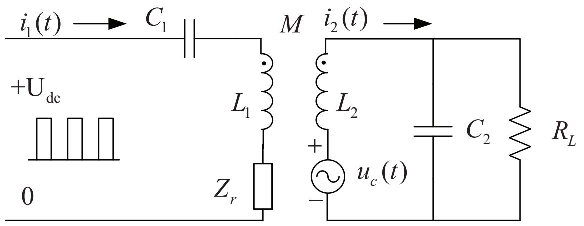

Figure 3. Mutual inductance equivalent circuit of proposed converter

In figure3, Z_r is the reflected impedance from the secondary side and $u_c(t)$ is the induced voltage of secondary side. Z_r can be expressed by equation (7).

$$Z_r = \frac{\omega^2 M^2}{Z_2} \tag{7}$$

In which, Z_2 is the paralleled impedance of secondary side, which is expressed in equation (8).

$$Z_2 = j\omega L_s + \frac{R_L}{1 + j\omega C_2 R_L} \tag{8}$$

Substituting (8) into (7) the reflected resistance and reactance from the secondary coil to the primary is, respectively,

$$\text{Re}Z_r = \frac{\omega^2 M^2 R_L}{R_L^2(\omega^2 C_2 L_2 - 1)^2 + \omega^2 L_2^2} \tag{9}$$

and

$$\text{Im}Z_r = \frac{-\omega^3 M^2 [C_2 R_L^2(\omega^2 C_2 L_2 - 1) + L_2]}{R_L^2(\omega^2 C_2 L_2 - 1)^2 + \omega^2 L_2^2} \tag{10}$$

Then, the equivalent impedance looking from the input side of the half-bridge inverter is,

$$Z_{\text{eq}} = j\omega L_1 + \frac{1}{j\omega C_1} + Z_r \tag{11}$$

3.3. Mathematical Models Using GSSA

According to Kirchhoffs circuit laws, the equivalent circuit equations are as follows,

$$\begin{cases} s(t)U_{dc} = L_1 \dfrac{di_1(t)}{dt} + u_{c1}(t) + i_1 Z_r \\ i_1(t) = C_1 \dfrac{du_{c1}(t)}{dt} \\ j\omega M i_1(t) = -L_2 \dfrac{di_2(t)}{dt} + u_2(t) \\ i_2(t) = -C_2 \dfrac{du_2(t)}{dt} - \dfrac{u_{c2}(t)}{R} \end{cases}, \tag{12}$$

where $i_1(t)$ and $i_2(t)$ represent the resonant current of primary side and secondary side respectively, and $u_1(t)$ denotes the voltage of primary capacitor while $u_2(t)$ is the voltage of the secondary capacitor. Besides, $s(t)$ is the switch function, which can be expressed as,

$$s(t) = \begin{cases} 1nT \le t < (2n+1)T/2 \\ 0(2n+1)T/2 \le t < (n+1)T \end{cases}. \tag{13}$$

Because the circuit model of the witricity system is a resonant converter which can filter out harmonic components, the dynamic system associated with a zero and first order Fourier coefficient expressions can provide the enough accuracy. So, by using GSSA method, the state variables can be expressed by the zero and first order components and the function is as follows,

$$\begin{cases} \langle i_1 \rangle_1 = x_1 + jx_2 \\ \langle u_{c1} \rangle_1 = x_3 + jx_4 \\ \langle i_2 \rangle_1 = x_5 + jx_6 \\ \langle u_{c2} \rangle_1 = x_7 + jx_8 \\ \langle i_1 \rangle_0 = x_9 \\ \langle u_{c1} \rangle_0 = x_{10} \\ \langle i_2 \rangle_0 = x_{11} \\ \langle u_{c2} \rangle_0 = x_{12} \end{cases} . \tag{14}$$

Then, from equation 11, the differential equations in state-space have the following matrix form,

$$\frac{d\vec{x}}{dt} = A\vec{x} + U_{dc}. \tag{15}$$

Where, $\vec{x}$ representing the zero-order and first-order state variables, is defined as a column vector,

$$\vec{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & ... & x_9 & x_{10} & x_{11} & x_{12} \end{bmatrix}^T . \tag{16}$$

In addition, the matrix A can be divided into four which is expressed as,

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \tag{17}$$

where A_1 and A_2 are shown as follows,

$$A_1 = \begin{bmatrix} -\frac{Z_r}{L1} & -\frac{1}{L_1} & 0 & 0 \\ \frac{1}{C_1} & 0 & 0 & 0 \\ -\frac{j\omega M}{L_2} & 0 & 0 & \frac{1}{L_2} \\ 0 & 0 & \frac{1}{C_2} & -\frac{1}{R_L C_2} \end{bmatrix} \tag{18}$$

and

$$A_2 = \begin{bmatrix} -\frac{\mathrm{Re}(Z_r)}{L_1} & \frac{\omega + \mathrm{Im}Z_r}{L_1} & -\frac{1}{L_1} & 0 & 0 & 0 & 0 & 0 \\ -\omega - \frac{\mathrm{Im}Z_r}{L_1} & -\frac{\mathrm{Re}Z_r}{L_1} & 0 & -\frac{1}{L_1} & 0 & 0 & 0 & 0 \\ \frac{1}{C_1} & 0 & 0 & \omega & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{C_1} & -\omega & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\omega M}{L_2} & 0 & 0 & 0 & \omega & \frac{1}{L_2} & 0 \\ -\frac{\omega M}{L_2} & 0 & 0 & 0 & -\omega & 0 & 0 & \frac{1}{L_2} \\ 0 & 0 & 0 & 0 & -\frac{1}{C_2} & 0 & -\frac{1}{R_L C_2} & \omega \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{C_2} & -\omega & -\frac{1}{R_L C_2} \end{bmatrix} \tag{19}$$

In addition, the coefficient b is expressed as,

$$b = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{2}{\pi L_1} & ... & 0 & 0 \end{bmatrix}. \tag{20}$$

To obtain the numerical solution, the continuous linear model should be transfer to the discrete mode first. Through the equation (15) and (16), the model is coded in MATLAB, and the results are compared with the simulation results in time domain by PSIM software package.

$$x(k+1) = \Phi x(k) + \Gamma u(k) \tag{21}$$

$$[\Phi, \Gamma] = f(A, B, T_s) \tag{22}$$

where, $k = 0, 1, 2...n.$

The parameter values is shown in table 1, and the simulation results are shown in figure 4 and figure 5. Figure 4 shows the waveforms of resonant current of primary side, in which the red line represents the solution from GSSA while the blue lines is the results from the PISM package.

Table 1. The Performance of the witricity system

Variable	Result	Unit
L_1	18	μH
C_1	800	pF
L_2	18	μH
C_2	800	pF
M	5	μH
R_L	12	Ω

Comparing curves in figure 4, the resonant current curves based on GSSA method is very similar to the simulation result from PSIM package. And figure 5 shows the errors of these two methods. From which we can see that the errors is always under 0.2 A. thus, it is clearly that the first approximation is sufficiently precise.

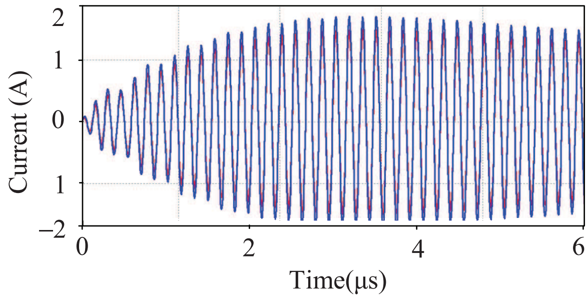

Figure 4. Resonant current of primary side using GSSA and simulation method

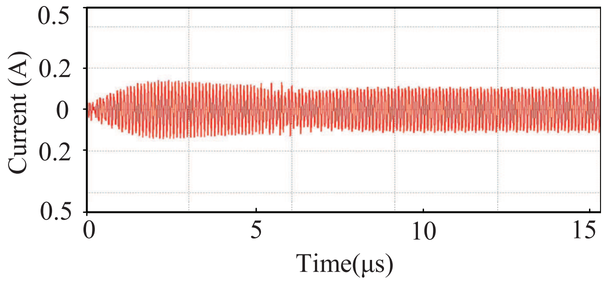

Figure 5. Errors between the GSSA method and the simulation method

4. Conclusion

In this paper, a tutorial material of a method for modelling the witricity system is presented. The GSSA method has been reviewed first. Then the circuit model of the witricity is introduced. Based on which, the mathematical model is derived using GSSA method, and the steps are presented in detail. The results of GSSA are compared with the simulation from PSIM package in time domain, which shows that there is little error between each other. The numerical analyses show that a first order approximation of GSSA is enough to present the precision of a witricity system, and there is no need to include higher-order terms in the analysis. The models derived from the GSSA method are suitable for dynamic analysis of the witricity system, and its computational time has been considerably reduced compared to other time domain simulation method.

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