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Partial-state Finite-time Stabilization Control of Chaos in PMSM with Uncertain Parameters

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Abstract

In this paper, a novel partial-state finite-time control scheme is presented for stabilizing chaos in permanent magnet synchronous motor with uncertain parameters. First, a new concept of partial-state finite-time stability is introduced. Based on the cascade-connected system theory, a controller is then designed in detail to improve the performance of the system. The stability of the proposed scheme is verified according to Lyapunov stability. This method is demonstrated to be highly robust again system parametric variations. Finally, numerical simulation results are presented to illustrate the effectiveness of the proposed method.

Keywords: *permanent magnet synchronous motor, finite-time stability, chaos control, cascade-connected system theory*

1. Introduction

Since the late 1980s, research has confirmed that chaos is a real phenomenon in all motor drive systems, such as induction motors, DC motors, and switched reluctance motors [1]. Chaotic behavior in permanent magnet DC motor open drive systems was first addressed by Hemati [2]. Li has found that chaos was also existed in permanent magnet synchronous motor (PMSM) [3]. With its changing operating parameters, PMSM exhibits complex behavior that reduces system performance, including limit cycles and chaos oscillation. Thus, this behavior should be suppressed or even eliminated. To address this problem, Li proposed generic mathematical models and conducted in-depth theoretical analysis, which are the foundations for controlling chaos.

Despite the numerous methods used to control chaos, few can be directly applied to PMSM. The main methods include decoupling control [4], feedback control [5, 6], back-stepping control [7], passivity control [8], sliding mode control [9], adaptive control [10, 11], and fuzzy control [12, 13]. However, decoupling control, feedback, back-stepping control, and passivity control, all of which depend on the mathematical model of the system, cannot guarantee dynamic performance because of uncertain system parameters. PMSM requires a parameter adaptive mechanism for the adaptive control technique. This requirement increases the cost and complexity of the system and reduces its response capacity. Sliding mode control requires uncertain terms to meet specific match conditions and exhibits inherent chattering. Fuzzy control is usually based on Takagi–Sugeno fuzzy models of the system. Li et al. [13] proposed the fuzzy feedback control scheme, which exhibits slow responsiveness. Li et al. [12] then proposed optimal fuzzy guaranteed cost control, which exhibits better responsiveness but has a structure that is too complex for application.

The abovementioned control methods can only ensure the asymptotic exponential stability of the system and not its time optimality (i.e., the shortest adjustment time). Finite-time control achieves system stability in finite time and contains fractional power, giving this method higher robustness than the above methods [14]. With the advantages of fast response, high tracking precision and strong robustness for uncertain parameters, this method has been successfully applied to control all kinds of systems, such as robotic manipulators [15], spacecraft systems [16], AC servo systems [17] and chaotic systems [18, 19]. Wei et al. [20] applies this method to control a chaotic permanent magnet synchronous motor (PMSM) system.

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However, the speed state equation has only one external controllable variable (i.e., load torque), which is generally not arbitrarily controllable. Thus, it is difficult to implement in practice and is not considered an uncertain parameter for the methods in [8, 20]. To solve these problems, we propose a novel partial-state finite-time chaotic controller (PSFTCC) that accounts for parameter uncertainties in PMSMs. With this controller, the system exhibits not only rapid response but also robustness under uncertain parameters.

This paper is organized as follows. Section 2 introduces the basic concepts and lemmas of the cascade-connected system theory (CCST) and the finite-time stability theory (FTST). Section 3 presents the chaos model of PMSM. Section 4 describes in detail the PSFTCC design and verifies the stability of the controller according to Lyapunov stability. Section 5 presents the simulation results to illustrate the effectiveness of the method. Finally, Section 6 concludes.

2. Basic Concepts and Lemmas

Important concepts and lemmas necessary for controller design are given below.

Consider a cascade-connected system described by:

$$\begin{cases} \dot{x} = f(x,z) \\ \dot{z} = g(z) \end{cases}, \tag{1}$$

Where $x \in R^n$ and $z \in R^m$ are system states, $f(x,z)$ and $g(z)$ are C^1 vectors, and $f(0,0)=0$ and $g(0)=0$.

Definition 1 [21]**.** Consider the dynamic system $\dot{x} = f(x)$, where $x \in R^n$ is the system state. This system is finite-time stable if it has a constant $T>0$ (which may depend on the initial state) that meets the following conditions: ① $\lim_{t \to T}\|x(t)\| = 0$ and ② $\|x(t)\| \equiv 0$, if $t \geq T$.

Definition 2. Consider system (1), if there exists control input $u=(u_1,u_2,...,u_m)$, $m<n$, such that partial states of the system (1) are finite-time stable and other states are globally asymptotically stable, then system (1) is called partial-state finite-time stable.

Lemma 1 [22]**.** If system (1) meets the following conditions, ① $\dot{x} = f(x,0)$ and $\dot{z} = g(z)$ are globally asymptotically stable at $x=0$ and $z=0$, respectively, and ② all states of system (1) are bounded, then system (1) is globally asymptotically stable at equilibrium (x, z) = (0, 0).

Lemma 2 [21]**.** If a continuous positive definite function $V(t)$ satisfies the following differential inequality, $\dot{V}(t) \leq -mV^{\xi}(t)$ for $\forall t \geq t_0$, and $V(t_0) \geq 0$, where $m>0$ and $0<\xi<1$ are constants, then $V(t)$ satisfies the inequality ① $V^{1-\xi}(t) \leq V^{1-\xi}(t_0) - m(1-\xi)(t-t_0)$ for $t \in [t_0, t_m]$ and ② $V(t) \equiv 0$ for $t \geq t_1$ at any initial time t_0. t_1 is given as $t_1 = t_0 + \dfrac{V^{1-\xi}}{m(1-\xi)}$, which is the response time of the system.

Lemma 3. For positive real numbers a, b, and c, if $c \in (0,1)$, then $a^c + b^c \geq (a+b)^c$.

3. Controller Design for PMSM Chaotic System

In this section, it is given the chaos model of the PMSM drive system and at the same time the controller is designed in detail. Then, The stability of the proposed control scheme is verified via Lyapunov stable theory.

3.1. Dymanic Model and Chaotic Characteristics of PMSM Chaotic System

The transformed model of PMSM with the smooth air gap can be expressed as follows [3]:

$$\begin{cases} \dot{i}_d = -i_d + wi_q + v_d \\ \dot{i}_q = -i_q - wi_d + \gamma w + v_q \\ \dot{w} = \sigma(i_q - w) - T_L \end{cases}, \quad (2)$$

Where v_d, v_q, i_d, and i_q are the transformed stator voltage components and current components in the d-q frame, w and T_L are the transformed angle speed and external load torque respectively, and γ and σ are the motor parameters.

Considering the case that, after an operation of the system, the external inputs are set to zero, namely, $v_d = v_q = T_L = 0$, system (2) becomes an autonomous system:

$$\begin{cases} \dot{i}_d = -i_d + wi_q \\ \dot{i}_q = -i_q - wi_d + \gamma w \\ \dot{w} = \sigma(i_q - w) \end{cases}, \quad (3)$$

The modern nonlinear theory such as bifurcation and chaos has been used to study the nonlinear characteristics of PMSM drive system in [3]. It has found that, with the operating parametersγ and σ falling into a certain area, PMSM will exhibit complex dynamic behavior, such as periodic, quasi periodic and chaotic behaviors. In order to make an overall inspection of dynamic behavior of the PMSM, the bifurcation diagram of the angle speed w with increasing of the parameter γ is illustrated in Figure 1(a). We can see that the system shows abundant and complex dynamical behaviors with increasing parameter γ. The typical chaotic attractor is shown in Figure 1(b) with $v_d = v_q = T_L = 0$, $\gamma = 25$, and $\sigma = 5.46$.

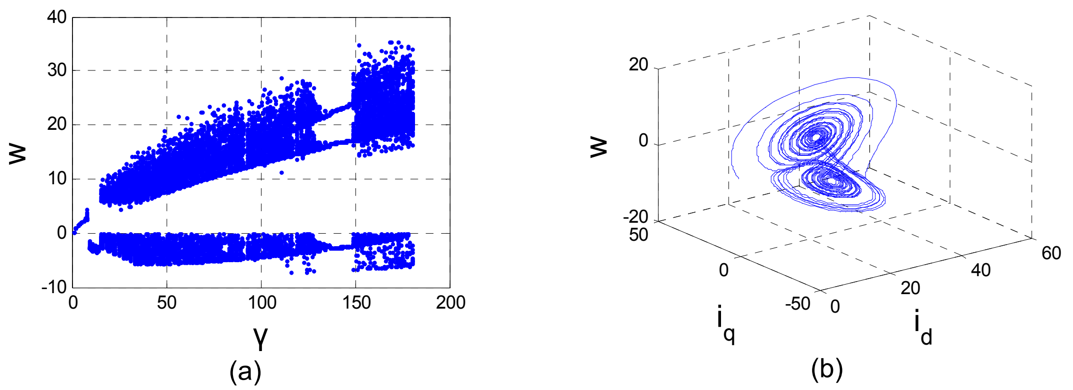

Figure 1. Bifurcation Diagram and the Characterizations of Chaos in PMSM (a) Bifurcation diagram of state variable w with the parameter γ (b) typical chaotic attractor.

With uncertain parameters, the dynamic model of the system can be described as follows:

$$\begin{cases} \dot{i}_d = -i_d + wi_q \\ \dot{i}_q = -i_q - wi_d + (\gamma + \Delta_\gamma)w \\ \dot{w} = (\sigma + \Delta_\sigma)(i_q - w) \end{cases}, \quad (4)$$

Where Δ_γ and Δ_σ represent the uncertainty of γ and σ respectively and are both bounded.

Following an actual operation, this article assumes that the fluctuation range of system parameters is 30%, that is, $\|\Delta_\gamma\| \le \delta_1 \le 0.3\gamma$, $\|\Delta_\sigma\| \le \delta_2 \le 0.3\sigma$.

3.2. Controller Design

System (3) indicates three equilibrium points: $S_0(0,0,0)$ and $S_{2,3}(\gamma-1, \pm\sqrt{\gamma-1}, \pm\sqrt{\gamma-1})$. Given that $\gamma = 25$, S_0 is locally stable, and S_1 and S_2 are both locally unstable [3] . Assuming that one equilibrium point of system (3) is $S(i_{dd}, i_{qd}, w_d)$, then:

$$\begin{cases} \dot{i}_{dd} = -i_{dd} + w_d i_{qd} = 0 \\ \dot{i}_{qd} = -i_{qd} - w_d i_{dd} + \gamma w_d = 0 \\ \dot{w}_d = \sigma(i_{qd} - w_d) = 0 \end{cases} \cdot \tag{5}$$

To quickly stabilize to equilibrium point $S(i_{dd}, i_{qd}, w_d)$, u_1 and u_2 are used to control the system (4). Under the control efforts u_1 and u_2, the controlled system can be represented as:

$$\begin{cases} \dot{i}_d = -i_d + w i_q + u_1 \\ \dot{i}_q = -i_q - w i_d + (\gamma + \Delta_\gamma) w + u_2 \\ \dot{w} = (\sigma + \Delta_\sigma)(i_q - w) \end{cases} \cdot \tag{6}$$

Let $e_1 = i_d - i_{dd}$, $e_2 = i_q - i_{qd}$ and $e_3 = w - w_d$, we can obtain the dynamic error equations of the system:

$$\begin{cases} \dot{e}_1 = -e_1 + e_2 e_3 + e_2 w_d + e_3 i_{qd} + u_1 \\ \dot{e}_2 = -e_2 - e_1 e_3 - e_1 w_d - e_3 i_{dd} + \gamma e_3 + \Delta_\gamma (e_3 + w_d) + u_2 \\ \dot{e}_3 = (\sigma + \Delta_\sigma)(e_2 - e_3) \end{cases} \cdot \tag{7}$$

The control objective is to stabilize the system (6) at the equilibrium point $S(i_{dd}, i_{qd}, w_d)$, that is, we design the controller to stabilize the error system (7) at *e*=0. So we will focus on the controller designing for system (7).

System analysis is required prior to controller design. If the states e_1 and e_2 are close to zero after time t_m, namely, $e_1 \equiv 0$ and $e_2 \equiv 0$ for $t > t_m$, then system (6) can be given as:

$$\begin{cases} \dot{e}_1 = -b e_1 + e_2 e_3 + e_2 w_d + e_3 i_{qd} + u_1 \\ \dot{e}_2 = -e_2 - e_1 e_3 - e_1 w_d - e_3 i_{dd} - c e_3 + u_2 \\ \dot{e}_3 = -a e_3 \end{cases} \tag{8}$$

System (8) is a typical cascaded-connected system. The third equation of system (8) indicates that error e_3 is globally asymptotically stable at $e_3 = 0$. Thus, if the errors e_1 and e_2 are stabilized at zero, the system becomes globally stable. Next, we design the controller to stabilize e_1 and e_2 at (0, 0).

Based on CCST and FTST, the controller is designed as follows:

Theorem 1. Consider dynamic error system (8). If the controller is designed as:

$$\begin{cases} u_1 = -e_3 i_{qd} - k_1 e_1^\alpha \\ u_2 = e_3 i_{dd} - \gamma e_3 - L|w| sign(e_2) - k_2 e_2^\alpha \end{cases} \tag{9}$$

Then, system (8) exhibits partial-state finite-time stability at equilibrium point O (0, 0, 0), where k_1 and k_2 are the coefficients of the terminal attractors that are positive real numbers, $\alpha = \frac{q}{p}$ (where $p > q$ and both terms are positive odd integers), and $L \geq \delta_1 = 0.3\gamma$.

Proof. Controlling u_1 and u_2 transforms the first and second equations of Equation (8) into:

$$\begin{cases} \dot{e}_1 = -e_1 + e_2 e_3 + e_2 w_d - k_1 e_1^{\alpha} \\ \dot{e}_2 = -e_2 - e_1 e_3 - e_1 w_d + \Delta_{\gamma} w - L|w| sign(e_2) - k_2 e_2^{\alpha} \end{cases} \tag{9}$$

If the candidate Lyapunov function is defined as $V_1 = \frac{1}{2}(e_1^2 + e_2^2)$, then the time derivative of V_1 along the trajectory of (9) is:

$$\begin{aligned} \dot{V}_1 &= e_1 se_1 + e_2 se_2 \\ &= e_1(-e_1 + e_2 e_3 + e_2 w_d - k_1 e_1^{\alpha}) + e_2(-e_2 - e_1 e_3 - e_1 w_d + \Delta_{\gamma} w - L|w| sign(e_2) - k_2 e_2^{\alpha}) \\ &= -e_1^2 - e_2^2 - k_1 e_1^{\alpha+1} - k_2 e_2^{\alpha+1} - (L|we_2| - \Delta_{\gamma} we_2) \\ &\leq -k_1 e_1^{\alpha+1} - k_2 e_2^{\alpha+1} \\ &= -k_1 (\tfrac{1}{2})^{-0.5(\alpha+1)} (\tfrac{1}{2} e_1^2)^{0.5(\alpha+1)} - k_2 (\tfrac{1}{2})^{-0.5(\alpha+1)} (\tfrac{1}{2} e_2^2)^{0.5(\alpha+1)} \\ &\leq -m((\tfrac{1}{2} e_1^2)^{0.5(\alpha+1)} + (\tfrac{1}{2} e_2^2)^{0.5(\alpha+1)}) \leq -mV_1^{\xi}, \end{aligned}$$

Where $m = \min((\frac{1}{2})^{-0.5(\alpha+1)} k_1, (\frac{1}{2})^{-0.5(\alpha+1)} k_2) > 0$ and $\xi = \frac{1}{2}(\alpha + 1)$. If $0 < \alpha < 1$, then $0 < \xi < 1$. Lemma 2 indicates that the states e_1 and e_2 come close to zero within finite time t_m, that is, subsystem (9) is finite-time stable.

After t_m, $e_1 \equiv 0$ and $e_2 \equiv 0$. Substituting $e_1 = 0$ and $e_2 = 0$ into the third equation of system (9) yields:

$$\dot{e}_3 = -(\sigma + \Delta_{\sigma}) e_3 \tag{10}$$

Thus, subsystem (10) is globally asymptotically stable.

Based on the above equations, condition ① is met for Lemma 1. Moreover, the PMSM chaotic system is bounded; that is, condition ② is also met for Lemma 1. Thus, system (8) is globally asymptotically stable. e_1 and e_2 are finite-time stable, and e_3 is globally asymptotically stable. Definition 2 indicates that system (8) is partial-state finite-time stable at O (0, 0, 0).

4. Simulation results

We use SIMULINK of MATLAB to verify the feasibility of the proposed PSFTCC for a PMSM chaotic system. In the simulation, the fourth-order Runge–Kutta method is used to solve the systems with time step size of 0.001. The parametric values of PMSM are the same as those in Section 3. Without loss of generality, we select $S_0(0,0,0)$ as the desired equilibrium point. The control method takes effect after t=15 s.

The simulation shows the result of the proposed method with the uncertain parameters disregarded (Figure 2). The selective parameters in theory are $k_1 = k_2 = k = 50$, $\alpha = \frac{7}{9}$, and $L = 10$ ($L \geq 0.3\gamma = 6$). With parameter uncertainties assumed to be the same as those in Section 3, the results of the proposed control law are given in Figure 3 ($k_1 = k_2 = k = 1$) and Figure 4 ($k_1 = k_2 = k = 10$).

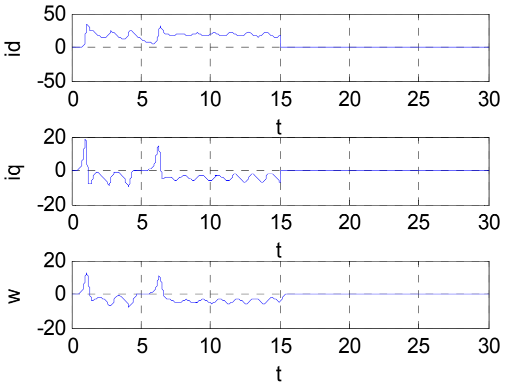

Figure 2. The States Response of the Proposed Scheme without Uncertain Parameters

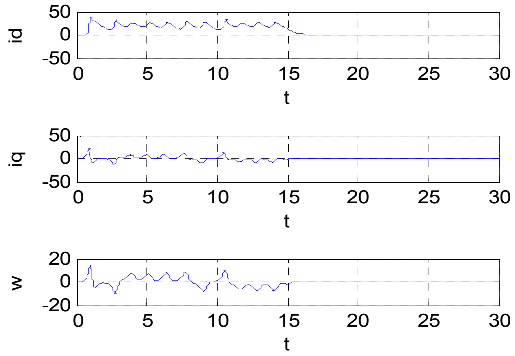

Figure 3. The States Response of the Proposed Scheme with Control Parameters k=1 Considering Uncertain Parameters

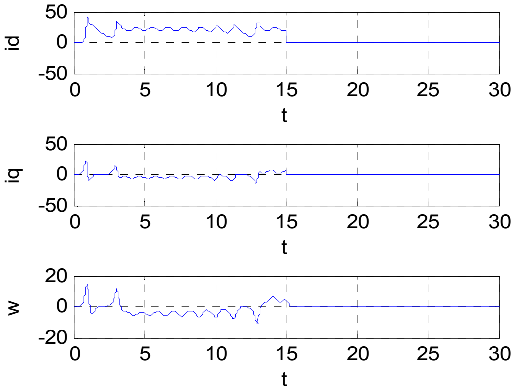

Figure 4. The States Response of the Proposed Scheme with Control Parameters k=10 Considering Uncertain Parameters

Figure 4 shows that system states can quickly stabilize to the equilibrium point $S_0(0,0,0)$. When uncertain parameters are considered, increasing the control parameters can enhance system response.

5. Conclusion

In this paper, a novel partial-state finite-time control scheme is proposed for PMSM chaotic system in the presence of parameter uncertainties. This method applies the finite-time stability theory to cascade-connected systems to improve their performance. Simulation results verify that the proposed controller exhibits quick responsiveness and strong robustness. Adding the control voltage to the state equation of the system maintains stability. Moreover, the structure of this controller is easy to design and implement. Future research should investigate the implementation of the proposed control scheme using an experimental setup. The scheme can also be extended to synchronize PMSM chaotic systems with uncertain parameters.

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