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Two Novel Decoding Algorithms for Turbo Codes Based on Taylor Series in 3GPP LTE System

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Abstract

This paper proposes two novel methods to simply Logarithmic Maximum a posteriori (Log-MAP) algorithm for turbo codes in the Third Generation Partnership Project Long Term Evolution (3GPP LTE). Firstly, we exploit a new function to replace the logarithmic term in the Jacobian logarithmic function based on Taylor series, which has the best approximated accuracy compared with the existing methods. With this method, we get algorithm I. Secondly, to further simplify the complexity, we propose a new piece-wise ladder function to approximate the logarithmic term according to algorithm I. In this way, we obtain algorithm II. Simulation results show that the performance of the algorithm I is most close to the optimal algorithm. Algorithm II owns the complexity which is similar to the MAX-Log-MAP algorithm, meanwhile it can offer 0.37-0.4db performance gains than MAX-Log-MAP algorithm.

***Keywords**: 3GPP LTE, turbo codes, Log-MAP, Taylor series*

1. Introduction

The Third Generation Partnership Project Long Term Evolution (3GPP LTE) [1], supported by the most telecommunication operators from the whole world, is investigated in order to ensure the competitiveness of Universal Mobile Telecommunications System (UMTS) for the next 10 years and beyond. LTE is a high-data-rate, low-latency and packet-optimized radio access technology [2]. Turbo code [3], capable of achieving close-to-Shannon capacity and amenable to hardware-efficient implementation, has been adopted by many wireless communication standards, including HSDPA [4] and LTE [5].

The 3GPP working group adopts the 1/3 code rate turbo codes to obtain the high data rate in consideration of their powerful error correcting capability [1]. In addition, LTE employs the turbo code with a new contention-free internal interleaver based on quadratic permutation polynomial (QPP), which requires small parameter storage, provides the excellent performance [6, 7]. The encoding and decoding structure of 3GPP LTE turbo codes is simply shown in Figure 1 [4], where x_k and L_k represent the systematic bits and the Log-likelihood ratio (LLR), respectively.

The symbol-by-symbol Log-MAP algorithm is optimal for iterative decoding in white Gaussian noise [8, 9]. However, reading data from a big table is a time consuming process and logarithm is not easy to implement in hardware. Its sub-optimal variants, the Max-Log-MAP algorithm [10], reduces the computational complexity greatly. It is reported that the Max-Log-MAP has a performance degradation about 0.4dB [10, 11], which will bring almost 10% capacity loss in the system [11]. To improve the performance of Max-Log-MAP algorithm, many efforts have been devoted in literatures including [12-16].

In this paper, we propose two novel decoding algorithms for 3GPP LTE turbo codes, which can yield good BER performance with lower complexity. The proposed algorithms can offer the best approximated performance to Log-MAP and it need not compute the logarithmic term.

This paper is organized as follows. In Section 2, there is an introduction of existing turbo decoding algorithm. Section 3 describes the proposed algorithms. Simulation results are shown in Section 4 and we present design architecture in Section 5. The conclusion is given in Section 6.

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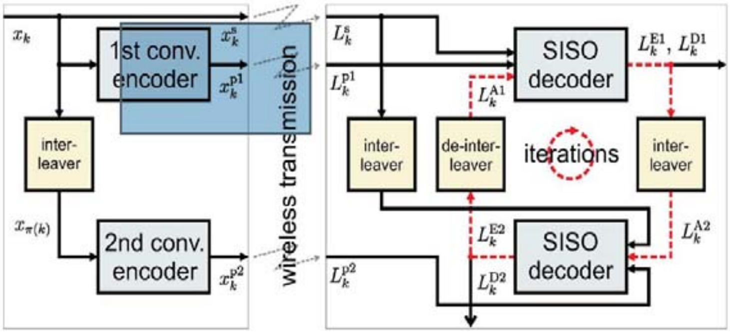

Figure 1. The Encoding and Decoding Structure of 3GPP LTE Turbo Codes

2. Existing Turbo Decoding Algorithms

We briefly review serval classic turbo decoding algorithms. Detailed explanations are given in [13].

2.1. The Optimal Algorithm

Log-MAP algorithm is the optimal algorithm for turbo code. The goal of the Log-MAP algorithm is to compute log-likelihood ratio (LLR) [13]:

$$L(u_k)=\ln[\sum_{(s_{k-1},s_k),u_k=+1} e^{\alpha_k^*(s_{k-1})+\beta_{k+1}^*(s_k)+\gamma_k^*(s_{k-1},s_k)}]-\ln[\sum_{(s_{k-1},s_k),u_k=-1} e^{\alpha_k^*(s_{k-1})+\beta_{k+1}^*(s_k)+\gamma_k^*(s_{k-1},s_k)}] \quad (1)$$

Where u_k is the information bits, s_k and s_{k-1} denote the state at k th and $k-1$th time instant. To compute the Equation (1), we need to recursively calculate forward and backward metrics, denoted as $\alpha_k(s_k)$ and $\beta_k(s_k)$.

Define the following function:

$$\max{}^*(x,y)=\ln(e^x+e^y)=\max(x,y)+\ln(1+e^{-|x-y|}) \quad (2)$$

Where $\ln(1+e^{-|x-y|})$ is a correction, which makes Log-MAP optimal.

According to the Equation (2), the forward and backward metrics can be computed as:

$$\begin{cases}\alpha_k^*(S_k)=\ln(\alpha_k(S_k))=\max^*_{S_{k-1}\in\sigma_{k-1}}(\gamma(S_{k-1},S_k)+\alpha_{k-1}^*(S_{k-1}))\\ \beta_k^*(S_k)=\ln\beta_k(S_k)=\max^*_{S_{k+1}\in\sigma_{k+1}}(\gamma(S_k,S_{k+1})+\beta_{k+1}^*(S_{k+1}))\end{cases} \quad (3)$$

Where σ_{k-1} and σ_k are collection of all states at the moment $k-1$and k respectively, and γ is the branch metrics.

Finally, we rewrite Equation (1) as:

$$\begin{aligned}L(u_k)=&\max[\beta_k^*(S_k)+\gamma(S_{k-1},S_k)+\alpha_{k-1}^*(S_{k-1})]\\ &-\max[\beta_k^*(S_k)+\gamma(S_k,S_{k-1})+\alpha_{k-1}^*(S_{k-1})]\ .\end{aligned} \quad (4)$$

2.2. The Suboptimal Algorithms

The suboptimal algorithms main contain Max-Log-MAP [10], linear Log-MAP [12], the improved Max-Log-MAP [13], non-linear Log-MAP [14] and the constant Log-MAP [15, 16] and they are obtained by the following expressions to replace $\ln(1+e^{-|x-y|})$ in turn.

$$\ln(1+e^{-|x-y|}) \approx 0 \quad (5)$$

$$\ln(1+e^{-|x-y|}) \approx \max(0, \ln 2 - \frac{1}{4}|x-y|) \quad (6)$$

$$\ln(1+e^{-|x-y|}) \approx \max(0, \ln 2 - \frac{1}{2}|x-y|) \quad (7)$$

$$\ln(1+e^{-|x-y|}) \approx \frac{\ln 2}{2^{|x-y|}} \quad (8)$$

$$\ln(1+e^{-|x-y|}) \approx \begin{cases} 0.375, |x-y| < 2 \\ 0, |x-y| > 2 \end{cases} \quad (9)$$

$$\ln(1+e^{-|x-y|}) \approx \begin{cases} 0.5, |x-y| < 1.5 \\ 0, |x-y| > 1.5 \end{cases} \quad (10)$$

3. The Proposed Algorithms

As is known to all, $\ln(1+e^{-|x-y|})$ in Log-MAP brings lots of undesirable problems. Firstly, saving the results of $\ln(1+e^{-|x-y|})$ in a lookup table would involve a quantization error caused by truncation of the input of the lookup table. Secondly, lookup tables are required for a wide range of operating signal-to-noise ratios (SNRs), which increases the hardware cost [13]. Last but not the only one, reading data from logarithm tables is a time consuming process. So we find a new function to replace it. Derivation process is as follows.

Let:

$$f(t) = \ln\frac{1+t}{1-t}. \quad (11)$$

Computing its derivative, we can get the following expressions:

$$f^{'}(t) = \frac{2}{1-t^2} \quad (12)$$

According to the Taylor series:

$$\frac{1}{1\text{-}t} = 1+t+t^2+t^3+...+t^n, -1 < t < 1, \quad (13)$$

Equation (12) is modified into Equation (14).

$$f^{'}(t) = 2(1+t^2+t^4+...+t^{2n}), -1 < t < 1 \quad (14)$$

We obtain the following expression through computing the integral of Equation (14).

$$f(t) = 2(t + \frac{1}{3}t^3 + \frac{1}{5}t^5 + ...), -1 < t < 1 \quad (15)$$

Omitting the high-order terms, we rewrite Equation (15) as Equation (16)

$$f(t) \approx 2t, -1 < t < 1 \quad (16)$$

Let:

$$\ln(1 + e^{-m}) = \ln\frac{1+t}{1-t}, \quad (17)$$

Then:

$$t = \frac{0.5}{0.5 + e^m}. \quad (18)$$

Combining Equation (16) with Equation (18), we get the following approximation.

$$\ln(1 + e^{-m}) \approx \frac{1}{0.5 + e^m} \quad (19)$$

Inspired by observing the curve of the exact correction term, we propose the following correction function.

$$\ln(1 + e^{-m}) \approx \frac{1.025}{0.5 + e^m} \quad (20)$$

To simply the computational complexity, we further propose a ladder function approximation.

$$\ln(1 + e^{-m}) \approx \frac{1.025}{0.5 + e^{\lfloor m \rfloor}} \quad (21)$$

Where $\lfloor m \rfloor$ is the largest integer that is smaller or equal to m. As shown in Figure 2, these two correction terms are more accurate than equations (5), (6), (7), (8), (9) and (10).

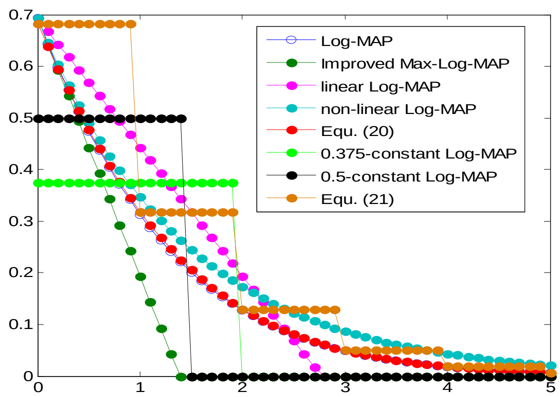

Figure 2. The Comparison of the Approximations

According to P. Robertson's study, just selecting the t between 0 and 5 could obtain the ideal approximation [10], so we can get:

$$\ln(1+e^{-m}) \approx \begin{cases} \dfrac{1.025}{0.5+e^{m}}, m < 5 \\ 0, m > 5 \end{cases} \quad (22)$$

$$\ln(1+e^{-m}) \approx \begin{cases} \dfrac{1.025}{0.5+e^{\lfloor m \rfloor}}, m < 5 \\ 0, m > 5 \end{cases} . \quad (23)$$

Through Equation (22) and (23) to compute the logarithm in Equation (2), we get two novel algorithms, and we donate as algorithm I and algorithm II respectively. In practice, we could choose different algorithms according to our requirement. For the systems which need higher reliability such as satellite communications, we can employ algorithm I decode. For the system which need higher validity system such as power line communications, we could choose algorithm II.

4. Simulation Results

4.1. Performance Comparison

Figure 3 show the simulated performance under AWGN channel for the proposed algorithms and others, including Log-MAP [9], Max-Log-MAP [10], linear Log-MAP [12], the improved Max-Log-MAP [13], non-linear Log-MAP [14] and the constant Log-MAP [15, 16]. The bit error rate (BER) performance is simulated in a rate-1/3, 8-states turbo coded system with the generator [7, 5]. The frame size is N=1024 and the maximum number of iterations for decoding was set to 6.

Figure 4 show the performance for algorithm I, algorithm II, Log-MAP and Max-Log-MAP algorithm. Figure 4 has similar simulated environment with figure 5, while its frame size is N=512.

As shown in Figure 3, algorithm I offers almost the same performance as Log-MAP algorithm. The extra coding gain is about 0.4db compared to the Max-Log-MAP algorithm. 0.15db to the linear Log-MAP, 0.12db to the improved Max-Log-MAP, 0.1db to the constant Log-MAP, 0.08db to the non-linear Log-MAP. It is also slightly superior to the algorithm II.

As can be seen in Figure 4, algorithm I and algorithm II can offer similar performance, which is almost equal to the Log-MAP algorithm. and they have nearly 0.37dB gain over Max-Log-MAP algorithm.

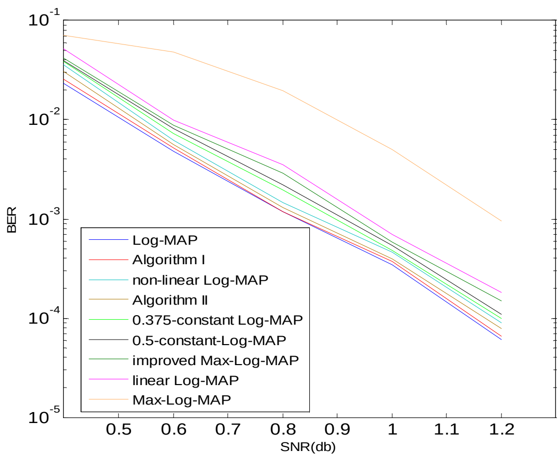

Figure 3. BER Performances for Algorithm I, Algorithm II and the Existing Turbo Decoding Algorithms

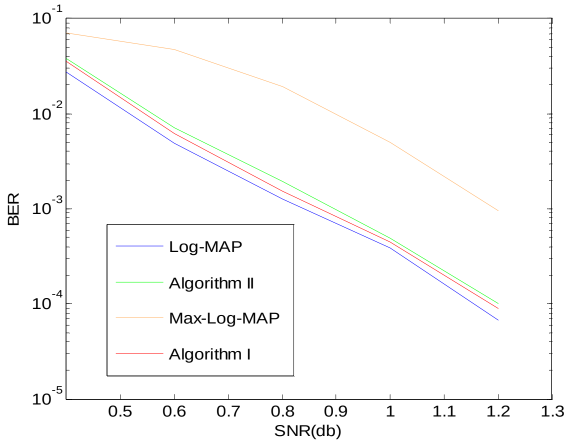

Figure 4. BER Performances for Algorithm I, Algorithm II, Log-MAP and Max-Log-MAP

4.2. Complexity Analysis

In order to calculate the complexity of algorithm II, we gain the probability of calculating $\alpha_l(s_l)$, $\beta_l(s_l)$ and LLR when t locates in different range by statistics. We select SNR=1db to make statistics, then we get the Table 1.

Table 1. The probability of calculating $\alpha_l(s_l)$, $\beta_l(s_l)$ and LLR, SNR=1db

	(0,1)	(1,2)	(2,3)	(3,4)	(4,5)	(5, ∞)
$\alpha_l(s_l)$	0.0540	0.0532	0.0527	0.0547	0.0542	0.7312
$\beta_l(s_l)$	0.0565	0.0555	0.0511	0.0543	0.0601	0.7226
LLR	0.0708	0.0550	0.0601	0.0610	0.0610	0.6921

According to Table I, we obtain $p_1, p_2, p_3, p_4, p_5, p_6$.

$$\begin{cases} p_1 = 0.0604 \\ p_2 = 0.0545 \\ p_3 = 0.0546 \\ p_4 = 0.0567 \\ p_5 = 0.0584 \\ p_6 = 0.7153 \end{cases} \tag{24}$$

Using these data, we can calculate the computation invoking the Equation (23) once. The number of addition is:

$$(2.5\times 2^M - 1)\times(1-p_6) \approx 0.7118\times 2^M - 0.2847 \tag{25}$$

The number of comparison is:

$$2 + (1 - p_6)\left[1 + \sum_{i=1}^{3} p_i(1 + \sum_{i=2}^{3} p_i) + \sum_{i=4}^{5} p_i\right] \approx 2.371 \tag{26}$$

At last, we obtain the Table 2 by referring to [10].

Table 2. The Complexity of Turbo Decoding Algorithms

Algorithm	Comparisons	Additions	Multiplications	Look-ups
Log-MAP	$5\times 2^{M}-2$	$15\times 2^{M}+9$	8	$5\times 2^{M}-2$
Max-Log-MAP	$5\times 2^{M}-2$	$10\times 2^{M}+11$	8	0
Algorithm II	$11.855\times 2^{M}-4.742$	$11.7118\times 2^{M}+0.7153$	8	0

As we all know, computing the “comparison” is almost no time consuming.for the computer. From above table we can see that the complexity of algorithm II is almost equal to the Max-Log-MAP algorithm.

5. Design architecture

We describe the detail design architecture of algorithm II in this section. The block diagram in Figure 5 shows the node metric calculation units [12]. In this figure, s_j refers to the state j at time k, while $s_j^{'}$ and $s_j^{''}$ refer to those pervious states at time k-1, which enter state at time k. The delay elements shown by “D” in this figure are employed in order to provide the node metric values at time k-1 [13]. In our method, we compare $|x-y|$ with 5 using comparer1 first, than we compare $|x-y|$ with integer 2, 4, 3, 1 in turns by the method of dichotomization. With this method, we can further improve the computational efficiency.

The detailed architecture of each block is shown in Figure 6 and $\beta_k(s)$ is determined using the same structure in backward recursion. This figure proves that the implementation of algorithm II is much simpler than Log-MAP that requires multiple lookup tables for a wide range of SNRs. In this way, algorithm II reduces the implementation cost.

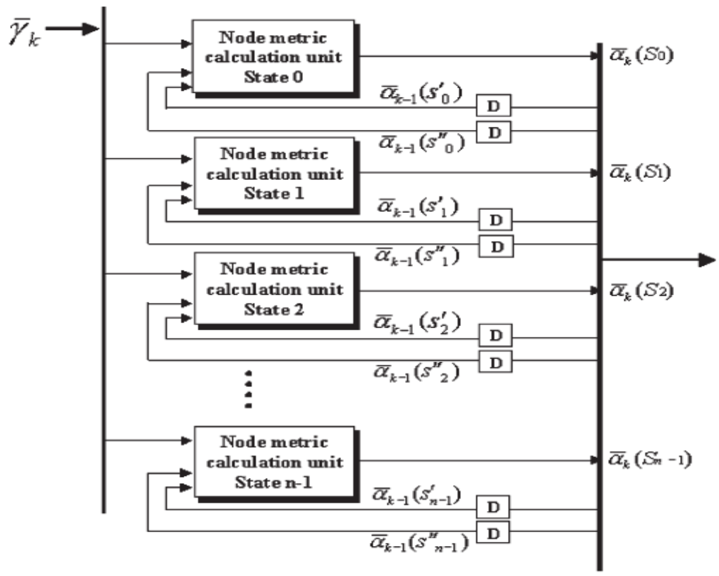

Figure 5. Node Metric Calculation Unit for n Different States

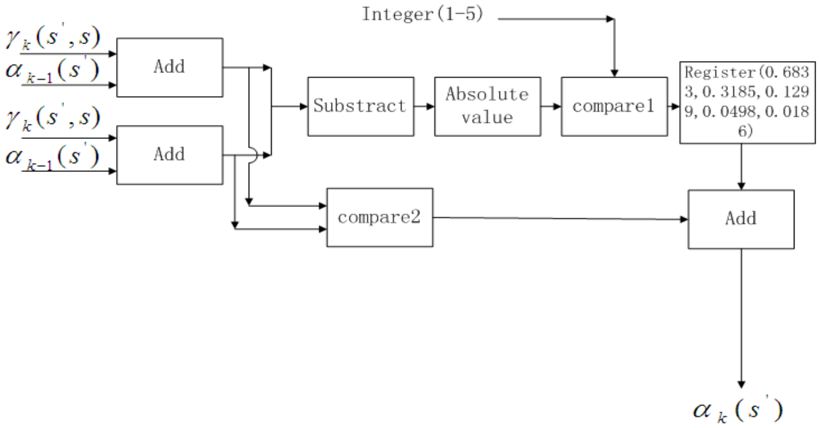

Figure 6. Detailed Architecture of Algorithm II

6. Conclusion

In this paper, we propose two novel decoding algorithms for turbo code in 3GPP LTE system, which result in almost equivalent performance to the optimum algorithm and avoid high complexity. Firstly, we exploit a new approximated correction terms for the Log-MAP algorithm, and it can offer the best approximated accuracy in contrast to the existing algorithms. Then, a novel wise-piece ladder function is proposed to replace the logarithmic term. The simulations show that the novel decoding schemes are superior to Max-Log-MAP algorithm in performance with slightly increased complexity. In addition, the proposed algorithms are very flexible, we could choose different algorithms according to different systems.

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