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Nonlinear Robust Control Approach Based on Integrity

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Abstract

A kind of Nonlinear Robust Control (NRC) approach based on integrity for multivariable systems is presented. It uses model estimatorwhich provides the approximatemodel information to compensate the non-modeled dynamics, system uncertainties, and external disturbances of a system. Firstly, the existence of NRC with integrity is examined. Then, stable regions of each NRC's parameters are calculated, and some parameters are obtained by placing suitable closed-loop poles, for meeting the design specifications of the whole control system. The proposed method is applied to twoillustrative examplesfrom literature. Results demonstrate that NRC is feasible and robust for complicated multivariable systems.

Keywords: *nonlinear robust control (NRC), model estimator, integrity*

1. Introduction

Most industrial processes are Multi-Input Multi-Output (MIMO) in essence. Due to interactions between loops, simply extending the design method for single-input single-output (SISO) control system to MIMO system generally causes the performancedeterioration, and even system instability. Nowadays, the methods for tuning MIMO system are categorized into two main groups: multivariable control and decentralized control.

Generally speaking, the performance of a multivariable control system is superior to the decentralized one. However, the latter iseasierto design and realize [1]. It has less parameters to be tuned, and is convenient to deal with when some loops break down. Because of these, decentralized control has been widely used in industrial processes. Especially robust decentralized control has a promising future in industrial processes.

Some researchers have paid more attention to tuneparameters of decentralized PI/PID controller in recent years, such as Detuningmethod [2],Sequential-closemethod [3], Mono-variable method [4], Effective open-loop process (EOP) method [5], and Iterative method [6], etc. (1) Detuning method [2]: Each PI/PID controller is designed independently based on SISO method, and then the controller parameters are detuned to compensate the couplings between loops when all loops are closed. The greatest advantage of this method is simple, but the performance and stability specifications are not clear during the detuning, so it is some kind of trial and error. (2) Sequential-close method [3]: Close each loop using the SISO method in certain sequence. The method is conceptually simple, but the resulted performance greatly depends on the closing sequence. When some loops break down, the stability of the rest cannot be guaranteed automatically. (3) Mono-variable method [4]: Each controller is designed by SISO method under certain constraints on stability and performance specifications. The entire performance may be unsatisfied because of neglecting of detailed information of controllers in other loops. (4) Effective open-loop process (EOP) method [5]: Regulate the *i*-th controllers according to *i*-th EOP when all the other loops are closed. The whole performance can be guaranteed since the EOP design has considered the information of all other loops, but the calculation of EOP becomes progressively complicated with the increase of system dimension and model order, so does the approximation model error. So the method is suitable only for low-dimension and low-order system. (5) Iterative method [6]: It is similar to sequential-close

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method, but tuning process is carried out iteratively until all controller parameters converge. The effort on tuning is enormous, but the relations between tuning process and resulted performance are weak.

The common characteristics of above methods are decomposing or transforming multivariable system with *n* inputs variables and *n* outputs variables into *n* single-loop systems. They all have some kind of conservativeness due to the interaction between channels, so overall performance could be possibly optimized further. The EOP method is considered an entire performance tool. However, model reduction error makes EOP method hard to be extendedformultivariable and high-order system.

In order to improve robustness performance for system uncertainties, as well as to guarantee the high integrity of controlsystem, this paper proposes a kind ofNonliearRobust Control (NRC) approach. Based on integrity theorem and parameter tuning methods, the control parameters are obtained directly and easily.No model reduction is needed, so no model error is introduced.

This paper is organized as follows. Section 2 outlines the problem and requirements.Section 3 introduces NRC approach and stable region calculation method for 1-order and 2-order SISO system. In Section 4,the integrity theorem and reference stable region for MIMO coupling system are presented.The entire design procedure of NRC approach is proposed in section5.Some simulation results are given in section 6, following with analysis and summary. The conclusion is drawn in the last section.

2. Problem Description

In this paper, we assume that the process matrix has diagonal dominance and the input-output variables are paired in the diagonal way. If not, a new input/output pairing should be considered [6], or a compensator should be introduced [7] to minimize interaction of the system.

We consider a stable *n* by *n*controlled process, which is described by a nominal transfer function matrix $G_p(s)$ as:

$$G_p(s)=\begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1}(s) & g_{n2}(s) & \cdots & g_{nn}(s) \end{bmatrix} \tag{1}$$

Where $g_{ij}(s)$ represents the transfer function from the *j*-th input to the *i*-th output.

If we ignore the coupling among the subsystems, the problem of designing a decentralized controller turns into *n* independent controllers. Then the process is to be controlled in a negative feedback configuration by the decentralized controller $G_c(s)$:

$$G_c(s)=\begin{bmatrix} G_{c1}(s) & 0 & \cdots & 0 \\ 0 & G_{c2}(s) & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \cdots & 0 & G_{cn}(s) \end{bmatrix} \tag{2}$$

Then, the final decentralized PI/PID control system is as follows:

$$\begin{cases} Y(s)=G_p(s)U(s) \\ U(s)=\varepsilon\ G_c(s)(R(s)-Y(s)) \end{cases} \tag{3}$$

Where $Y(s)=[y_1(s),y_2(s),\cdots,y_n(s)]^T$ is an output vector, $U(s)=[u_1(s),u_2(s),\cdots,u_n(s)]^T$ is an input vector, and $R(s)=[r_1(s),r_2(s),\cdots,r_n(s)]^T$ is a set-point vector. Matrix ε indicates the loop failure of sensors or actuators, $\varepsilon = diag\{\varepsilon_1,\cdots,\varepsilon_n\}, \varepsilon_i \in \{0,1\}, i = 1,\cdots,n$. If the *i*-th subsystem is working properly, then $\varepsilon_i = 1$, else $\varepsilon_i = 0$.

The control diagram is shown in Figure 1.

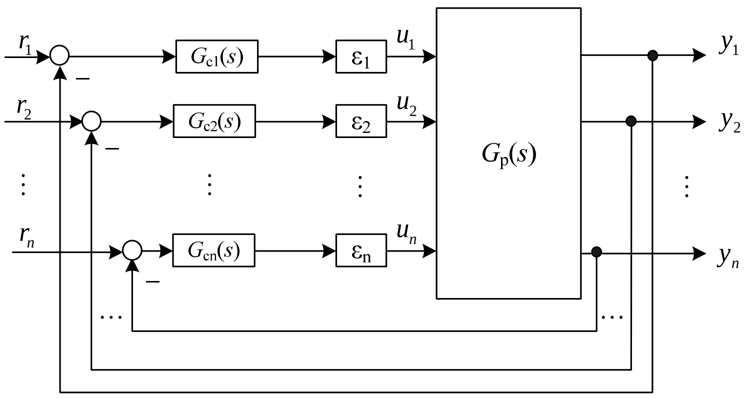

Figure 1. MIMO Control System Considering Integrity

3. Nonlinear Robust Control (NRC) Approach Design

3.1. SISO NRC

The r-order SISO NRC using a kind of model estimator is designed as [8]:

$$u = -\sum_{i=0}^{r-1} h_i z_{i+1} - \hat{d} \tag{4}$$

Where the model estimator is:

$$\begin{cases} \hat{d} = \xi + kz_r \\ \dot{\xi} = -k\xi - k^2 z_r - ku \end{cases} \tag{5}$$

Where $\hat{d}$ denotes model information evenincluding un-modeled dynamics, uncertainties in system parameters, and disturbances; $h_i, i = 0,\cdots,r-1$ are suitable positive constants, which ensure polynomial $h(s) = h_0 + h_1 s + \cdots h_{r-1}s^{r-1} + s^r$ is Hurwiwz and satisfy desired dynamics of closed-loop system; *k* is a key controller parameter, which determine stability directly; $z_i, i = 0,\cdots r$ are system states, and $z_i = y^{(i-1)}, i = 1,\cdots r$.

Taking the Laplace transformation of (4) and (5), it is straightforward to calculate the transfer function of NRC:

$$G_c(s) = \frac{1}{s}\sum_{i=0}^{r} K_i s^i \tag{6}$$

Where $K_0 = kh_0$, $K_i = kh_i + h_{i-1}, i = 1,\cdots r-1$, $K_r = k + h_{r-1}$.

It can be clearly seen that the structure of (6) is similar to common PID controller.

3.2. Stable Region

For 1-order and 2-order SISO system, the stable regions of NRCapproach are easy to calculate.

3.2.1. 1-order Plant

We consider a general 1-order plant:

$$\dot{y} = -ay + bu \tag{7}$$

Where y is the system output, u is the control signal, and constants a and b are both unknown.
The equation can be rewritten as follows:

$$\dot{y} = d + u \tag{8}$$

Where $d = -ay + (b-1)u$.
The 1-order NRC is designed as follows [8]:

$$\begin{cases} u = -h_0 y - \hat{d} \\ \hat{d} = \xi + ky \\ \dot{\xi} = -k\xi - k^2 y - ku \end{cases} \tag{9}$$

The transfer function of (9) is similar to PI controller as:

$$G_c(s) = (k + h_0) + \frac{kh_0}{s} \tag{10}$$

Substituting (9) into (8), we obtain:

$$\dot{y} = -h_0 y + \tilde{d} \tag{11}$$

Where $\tilde{d} = d - \hat{d}$.
Let us now computer the time derivative of $\hat{d}$,

$$\begin{aligned} \dot{\hat{d}} &= \dot{\xi} + k\dot{y} \\ &= -k\xi - k^2 y - ku + k(d+u) \\ &= k(d - \xi - ky) \\ &= k\tilde{d} \end{aligned} \tag{12}$$

And the expression of $\tilde{d}$ is:

$$\begin{aligned} \tilde{d} &= d - \hat{d} \\ &= -ay + (b-1)(-h_0 y - \hat{d}) - \hat{d} \\ &= (h_0 - bh_0 - a)y - b\hat{d} \end{aligned} \tag{13}$$

Taking into account (11) and (12), the dynamics of $\dot{\tilde{d}}$ can be obtained by computing the time derivative of (13).

$$\begin{aligned} \dot{\tilde{d}} &= (h_0 - bh_0 - a)\dot{y} - b\dot{\hat{d}} \\ &= (h_0 - bh_0 - a)(-h_0 y + \tilde{d}) - bk\tilde{d} \\ &= (ah_0 + bh_0^2 - h_0^2)y + (h_0 - bh_0 - a - bk)\tilde{d} \end{aligned} \tag{14}$$

Defining new state variable vector $\xi = [\xi_1, \xi_2]^T = [y, \tilde{d}]^T$, the close-loop system can be described as follows:

$$\begin{cases} \dot{\xi}_1 = -h_0\xi_1 + \xi_2 \\ \dot{\xi}_2 = (ah_0 + bh_0^2 - h_0^2)\xi_1 + (h_0 - bh_0 - a - bk)\xi_2 \end{cases} \tag{15}$$

That is:

$$\dot{\xi} = A_z\xi \tag{16}$$

Where $A_z = \begin{bmatrix} -h_0 & 1 \\ ah_0 + bh_0^2 - h_0^2 & h_0 - bh_0 - a - bk \end{bmatrix}$.

System (16) is asymptotically stable only if A_Z is Hurwize. Then we have:

$$\begin{cases} a + bh_0 + bk > 0 \\ bkh_0 > 0 \end{cases} \tag{17}$$

Because $h_0 > 0$ and $b \neq 0$, the stable condition becomes:

$$\begin{cases} \mu > 0 \\ \mu > -\dfrac{a + bh_0}{|b|} \end{cases} \tag{18}$$

Where $\mu = k / \operatorname{sgn}(b)$.

The results are shown in Figure 2. with plant parameters a locating [-100 100] and b being equal to -5,-1, 1,5,respectively. Thestable region is the area offold line upper-right side.

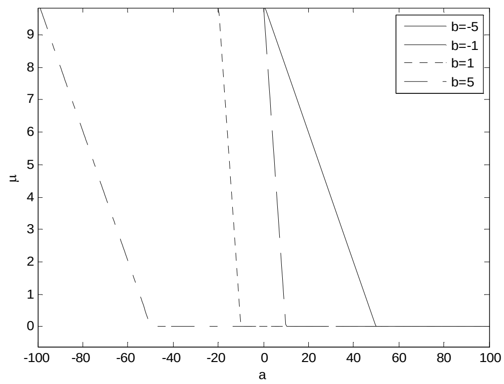

Figure 2. Stable Region of 1-order SISO NRC

3.2.2. 2-order Plant

We consider a general 2-order plant:

$$\ddot{y} = -a_1\dot{y} - a_0 y + bu \tag{19}$$

Where constants a_1, a_0 and b are all unknown.

Using model estimator, the 2-order NRC is designed as follows [8]:

$$\begin{cases} u = -h_0 y - h_1\dot{y} - \hat{d} \\ \hat{d} = \xi + k\dot{y} \\ \dot{\xi} = -k\xi - k^2\dot{y} - ku \end{cases} \tag{20}$$

The transfer function of this 2-order NRC is computed as follows:

$$G_c(s) = (h_1 k + h_0) + \frac{h_0 k}{s} + (h_1 + k)s \quad (21)$$

Whose structure is similar to PID controller.

The analysis method is similar to the 1-order plant. Because of the limited article length, we omit many analysis steps. By defining new state variable vector $\xi = [\xi_1, \xi_2, \xi_3]^T = [y, \dot{y}, \tilde{d}]^T$, the close-loop system can be described as follows:

$$\dot{\xi} = A_z \xi \quad (22)$$

Where $A_z = \begin{bmatrix} 0 & 1 & 0 \\ -h_0 & -h_1 & 1 \\ a_1 h_0 + b h_0 h_1 - h_0 h_1 & b h_1^2 - h_1^2 + a_1 h_1 + h_0 - b h_0 - a_0 & h_1 - b h_1 - a_1 - bk \end{bmatrix}$.

Hence, System (22) is asymptotically stable only if:

$$\begin{cases} a_1 + b h_1 + bk > 0 \\ a_0 + b h_0 + b h_1 k > 0 \\ b h_0 k > 0 \\ (a_0 + b h_0 + b h_1 k)(a_1 + b h_1 + bk) > b h_0 k \end{cases} \quad (23)$$

Because $h_0 > 0$, $h_1 > 0$ and $b \neq 0$, the stable condition becomes:

$$\begin{cases} \mu > -\dfrac{a_1 + b h_1}{|b|} \\ \mu > -\dfrac{a_0 + b h_0}{|b| h_1} \\ (a_0 + b h_0 + |b| h_1 \mu)(a_1 + b h_1 + |b| \mu) > |b| h_0 \mu \end{cases} \quad (24)$$

Where $\mu = k / \mathrm{sgn}(b)$.

In order to simplify the tuning process, the parameters, which determine desired dynamics are selected as $h_1 = 2\omega_c$ and $h_0 = \omega_c^2$. In other words, the closed-loop roots are placed at $-\omega_c$.

The results are shown in Figure 3. with plant parameters a_0 locating [-2000 1000], a_1 locating [-100 100], $b = 1$ and $\omega_c = 10$, where the area to the upper side of the mesh surface is the stable region.

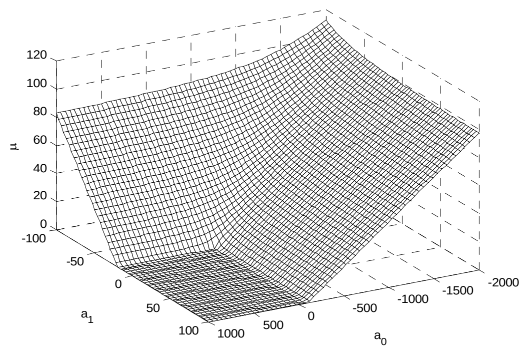

Figure 3. Stable Region of 2-order SISO NRC

4. Integrity Theorem and Stable Region of MIMO System
4.1. Integrity Theorem

The MIMO system has more sensors and actuators than SISO one. Because of interaction, each element failure will change the performance of rest subsystem, even cause instability. The closed-loop system should exhibit stability with accepted dynamic properties even if some elements or channels break down. This property is called integrity. The system has high-integrity property if it is stable under any possible loop failures. We denote the static gain matrix of process $G_p(s)$ as:

$$G_{p0} = \begin{bmatrix} g_{110} & g_{120} & \cdots & g_{1n0} \\ g_{210} & g_{220} & \cdots & g_{2n0} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n10} & g_{n20} & \cdots & g_{nn0} \end{bmatrix} \tag{25}$$

Then the following lemma on the integrity of decentralized PI form control system is given.

Lemma1 [9]: Consider a stable linear system (3), and assume that static gain matrix $g_{0ii} > 0, i = 1,...,n$. Then a necessary and sufficient condition for the existence of decentralized PI controllers that ensure closed-loop stability for all possible ε is given by the requirement that all principal minors of G_{p0} are positive.

If the control system is presented as transfer function form (of course including PI form), we have the following lemma.

Lemma 2 [9]: Consider a stable linear system (3), and assume that decentralized controllers ensure the stability of the closed-loop system. When interaction decreases or breaks down, or some loops are disconnected from the system, if the following condition is satisfied, the overall closed-loop system will be stable and still remain stable:

$$d(s) = \lambda_{\text{p}}[V_2 \text{diag}\,|\,(I + G_{ci}(s) g_{ii}(s))^{-1} G_{ci}(s)\,|] < 1 \tag{26}$$

Where:

$$V_2 = \begin{pmatrix} 0 & |g_{12}(s)| & \dots & |g_{1n}(s)| \\ |g_{21}(s)| & 0 & \dots & \vdots \\ \vdots & & \ddots & |g_{n-1,n}(s)| \\ |g_{n1}(s)| & \dots & |g_{n,n-1}(s)| & 0 \end{pmatrix} \tag{27}$$

$\lambda_p(\square)$ denotesPerron-Frobenius eigenvalue of matrix $(\square)$; $|\square|$ denotes absolute matrix, in which all elements are the absolute value of primary matrix $(\square)$.

Because the 1-order NRC has similar PI form, we can examine the integrity of a designed NRC using this two lemmas. Also, lemma 1 is used to ensure the existence of decentralized 1-order NRC with high-integrity. If condition in lemma 1 is not satisfied, the pairings should be re-organized, or much attention should be paid on when and how to switch to other controllers in case some loops disconnect from the system.

If the designed NRC has 2-order or higher order, lemma 1 cannot be used directly. We adopt equivalent transformation, which replaces $g_{ii}(s)$ with $g_{ii}^{'}(s)$ in the calculation.

$$\begin{aligned}(\frac{1}{s}\sum_{j=0}^{\rho_i} K_{j,i} s^j) g_{ii}(s) &= \frac{1}{s}(K_{0,i} + K_{1,i}s)(\frac{\sum_{j=0}^{\rho_i} K_{j,i} s^j}{K_{0,i} + K_{1,i}s}) g_{ii}(s) \\ &= \frac{1}{s}(K_{0,i} + K_{1,i}s) g_{ii}^{'}(s)\end{aligned} \tag{28}$$

4.2. Stable Region

As mentioned above, the stable regions of 1-order and 2-order controller for SISO system can be figured in two-dimensional and three-dimensional space, respectively. For a MIMO system, the stable region is quite complicated. From system (3), the characteristic equation of the decentralized control system is:

$$\begin{aligned}&\det\{I+G_p(s)\cdot\varepsilon\cdot G_c(s)\}\\&=\det\{\begin{bmatrix}1+\varepsilon_1 g_{11}(s)G_{c1}(s) & \varepsilon_2 g_{12}(s)G_{c2}(s) & \cdots & \varepsilon_n g_{1n}(s)G_{cn}(s)\\ \varepsilon_1 g_{21}(s)G_{c1}(s) & 1+\varepsilon_2 g_{22}(s)G_{c2}(s) & \cdots & \varepsilon_n g_{2n}(s)G_{cn}(s)\\ \vdots & \vdots & \ddots & \vdots\\ \varepsilon_1 g_{n1}(s)G_{c1}(s) & \varepsilon_2 g_{n2}(s)G_{c2}(s) & \cdots & 1+\varepsilon_n g_{nn}(s)G_{cn}(s)\end{bmatrix}\}\\&=0\end{aligned} \quad (29)$$

It can be seen that the stable regions are correlative, and the variety of each controller changes the stable regions of all other controllers. So it is difficult to express and calculate the stable regions of MIMO NRCs.

In fact, the main reason for calculating stable regions of NRCs is to provide areasonable search space for parameter tuning. Since integrity is one of our objectives, and at the existence of integrity property, it implies that each controller can stabilize the corresponding local feedback loop independently. For the purpose of simplicity, the union sets of stable region of each diagonal element are used as NRC parametersreferencestable region for MIMO system.

Meanwhile, combining desired dynamics and simulation, the parameters set of NRC $\{k_i, h_{0,i}, h_{1,i}, \cdots, h_{r-1,i}\}(i=1,\cdots,n)$ are determined finally.

5. Design Procedure

To sum up, the design procedure of MIMO decentralized NRCs can be carried out as following steps:

1) Confirm appropriate structure of control system with *n* SISO suitable order NRC according to process information.

2) Examine the integrity of the designed control system using lemma 1, otherwise, re-analysis the system or decompose it, and design new control system.

3) Calculate the stable region of each diagonal element as the parameters reference tuning space.

4) Determine controller parameters combining above region, desired dynamics and simulation.

5) Check the integrity of the resulted closed-loop system with lemma 2.

6) If satisfies, design is finished. Otherwise, check the integrity by simulation.

Remark 1: In many conditions, the exact relative degrees of controller process are not easy to obtain because of the complexity and uncertainties of model. We can design appropriate decentralized control system, whose diagonal elements are 1-order or 2-order NRCs.

Remark 2: The proposed method is based on nominal model. Under plant uncertainties or operating point variation, robustness can also be ensured because NRC with suitable parameters has strong disturbance rejection.

Remark 3: We can also optimize the controller parameters by defining specific objective function. The parameter optimization can always be carried out as long as the objective function is quantitatively valuable. It will be discussed in our following paper.

6. Illustrative Examples

In this section, we demonstrate the superior control ability and robust performance to great gain perturbations in cross-couplings of NRC approach through six typical multivariable processes [5].

6.1. 2×2 Models

Wood and Berry (WB) controlled process is considered firstly. The transfer function matrixis given as follows:

$$G_1(s) = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \tag{30}$$

The method in Section 5 is carried out to tune the decentralized NRC. In order to simplify analysisprocess, the time-delay is not considered when we design subsystem controller. First, two 2-order subsystem controllers are designed according to the relative degrees of corresponding diagonal transfer function,

$$G_{ci}(s) = \frac{1}{s}(k_i h_{0,i} + (k_i + h_{0,i})s),\ i = 1,2 \tag{31}$$

Where $k_i = \mathrm{sgn}(g_{ii}(s))\mu_i$.

Then the control for overall system is:

$$G_c(s) = \begin{bmatrix} G_{c1}(s) & 0 \\ 0 & G_{c2}(s) \end{bmatrix} \tag{32}$$

By checking the process with lemma 1, it can be found that all principal minors of static gain matrixes are positive, so it satisfies the sufficient and necessary conditions of existing 2-order decentralized NRC with integrity.

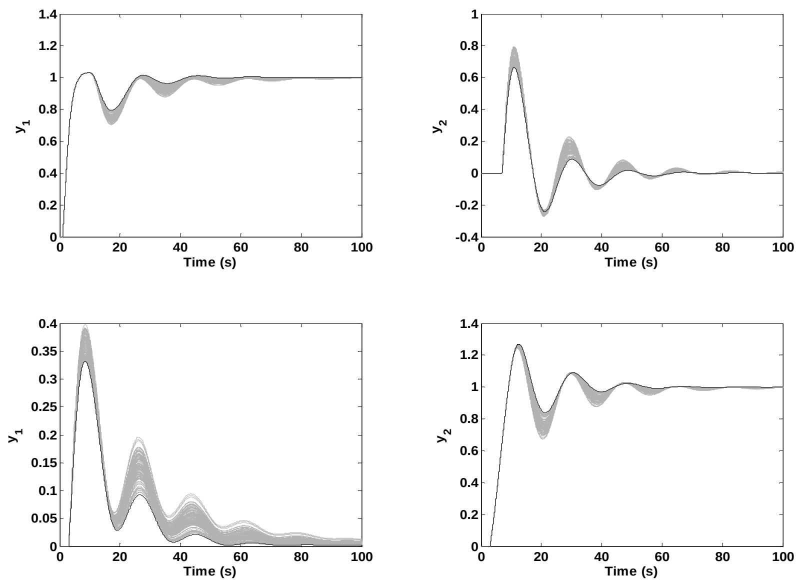

Figure 4. Dynamic Response of Different Step Inputs (WB model)

The stable region of each 2-order controller is calculated using the method presented in section 3.2. We get $\mu_1 > 0$ and $\mu_2 > h_{0,2} - 0.0515$. In these two loops, the desired poles are all placed at -0.1, that is, $h_{0,1} = h_{0,2} = 0.1$. Also, we select $\mu_1 = 0.35$, $\mu_2 = 0.2$.

When the set-point signal of two channels has unit step disturbances respectively, the dynamic responses are illustrated in Figure 4.

If there are +20% gain perturbations in cross-couplings, new cross-couplings become:

$$g_{12}^{'}(s) = \delta_1 \times g_{12}(s) \quad (33)$$

$$g_{21}^{'}(s) = \delta_2 \times g_{21}(s) \quad (34)$$

Where δ_1 and δ_2 both vary randomly in the area of [1, 1.2]. The black lines represent dynamic responses in normal condition, and gray lines represent dynamic responses when perturbations exist.

The 2-order decentralized NRC for three more 2×2 processes demonstrated as Table 1, using the same configuration.

Table 1. Transfer Function Matrices of 2×2 Systems

Model	Transfer Function
Vinante-Luyben model (VL)	$G_2(s) = \begin{bmatrix} \frac{-2.2e^{-s}}{7s+1} & \frac{1.3e^{-0.3s}}{7s+1} \\ \frac{-2.8e^{-1.8s}}{9.5s+1} & \frac{4.3e^{-0.35s}}{9.2s+1} \end{bmatrix}$
Wardle-Wood model (WW)	$G_3(s) = \begin{bmatrix} \frac{0.126e^{-6s}}{60s+1} & \frac{-0.101e^{-12s}}{(45s+1)(48s+1)} \\ \frac{0.094e^{-8s}}{(38s+1)} & \frac{-0.12e^{-8s}}{35s+1} \end{bmatrix}$
Ogunnaike-Ray model (OR2)	$G_4(s) = \begin{bmatrix} \frac{22.89e^{-0.2s}}{4.572s+1} & \frac{-11.64e^{-0.4s}}{1.807s+1} \\ \frac{4.689e^{-0.2s}}{2.174s+1} & \frac{5.8e^{-0.4s}}{1.801s+1} \end{bmatrix}$

The dynamic responses in both normal condition and existing +20% gain perturbations in cross-couplings condition for the three processes are illustrated in Figure 5-Figure 7.

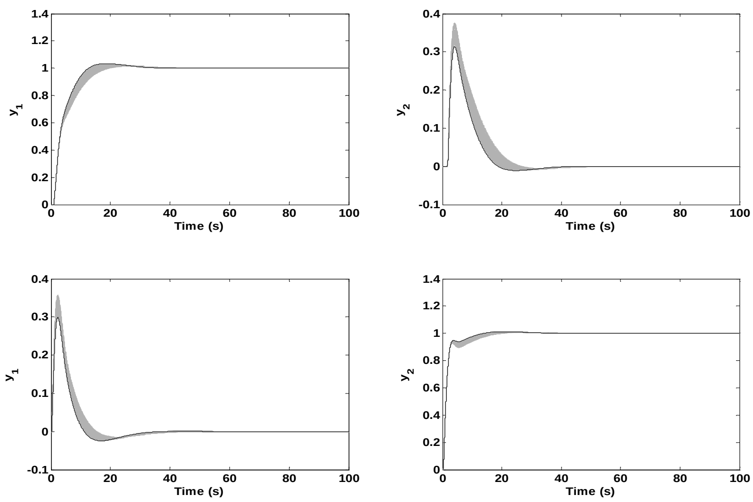

Figure 5. Dynamic Response of Different Step Inputs (VL Model)

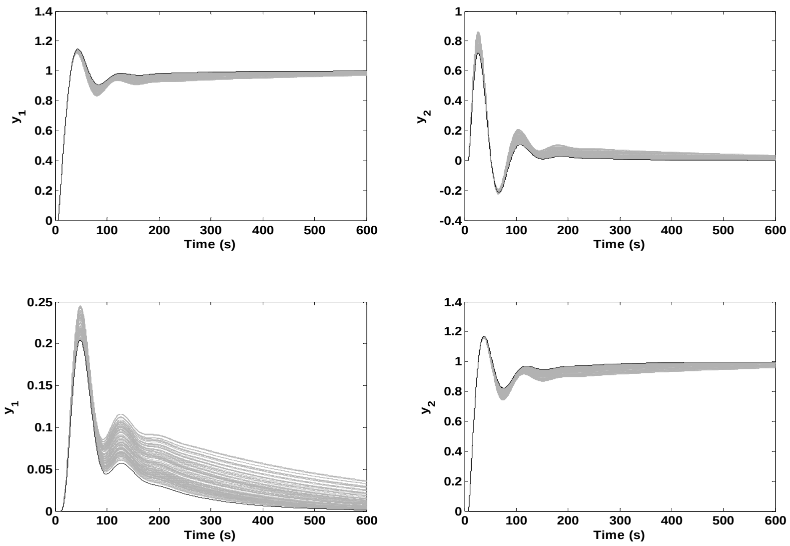

Figure 6. Dynamic Response of Different Step Inputs (WW Model)

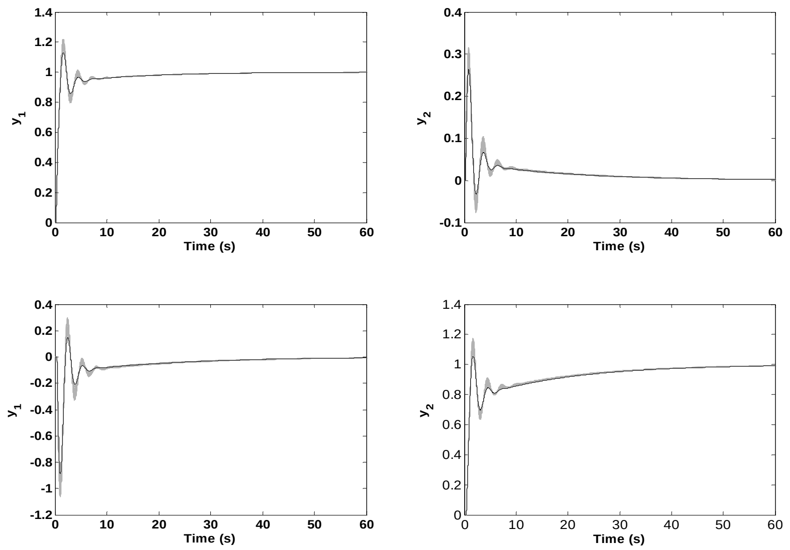

Figure 7. Dynamic Response of Different Step Inputs (OR2 Model)

On the whole, the resulted NRC parameters of four 2×2 processes are summarized in Table 2.

Table 2. Parameters of Decentralized NRC

Model	k_1	$h_{0,1}$	k_2	$h_{0,2}$
WB	0.35	0.1	-0.2	0.1
VL	-1	0.2	1.2	0.2
WW	20	0.05	-16	0.05
OR2	0.2	0.1	0.25	0.1

The Perron-Frobeniuseigenvalue λ_p with changing of ω is shown in Figure 8.

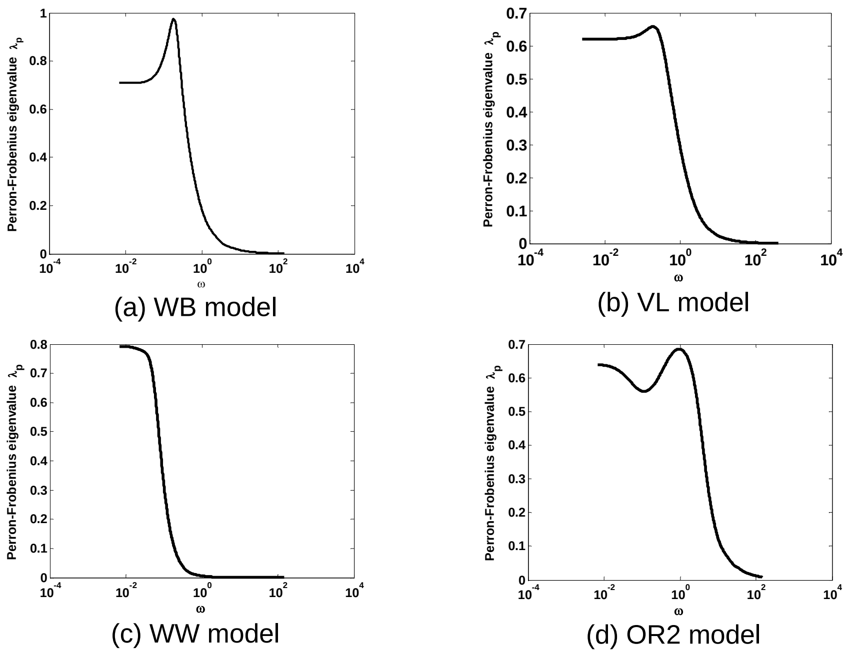

Figure 8. Perron-Frobeniuseig Value

It is clear that the sufficient condition $\lambda_p < 1$ in integrity theorem is always satisfied for four resulted NRC systems. So the closed-loop overall systems are stable and remain stable under the decentralizedNRC(32) if cross-couplings within the plant are decreased or broken down or if some control stations are disconnected from the plant.

6.2. 3×3 Model Ogunnaike-Ray Model (OR3)

$$G_5(s) = \begin{bmatrix} \frac{0.66e^{-2.6s}}{6.7s+1} & \frac{-0.61e^{-3.5s}}{8.64s+1} & \frac{-0.0049e^{-s}}{9.06s+1} \\ \frac{1.11e^{-6.5s}}{3.25s+1} & \frac{-2.36e^{-3s}}{5s+1} & \frac{-0.01e^{-1.2s}}{7.09s+1} \\ \frac{-34.68e^{-9.2s}}{8.15s+1} & \frac{46.2e^{-9.4s}}{10.9s+1} & \frac{0.87(11.61s+1)e^{-s}}{(3.89s+1)(18.8s+1)} \end{bmatrix} \quad (35)$$

For 3×3 processes, the design procedure is similar with precedent. The controller is given as the following form:

$$G_c(s) = \begin{bmatrix} G_{c1}(s) & 0 & 0 \\ 0 & G_{c2}(s) & 0 \\ 0 & 0 & G_{c3}(s) \end{bmatrix} \quad (36)$$

Where $G_{ci}(s), i = 1, \cdots, 3$ are designed according to their relative degrees of corresponding diagonal transfer function. Here we have:

$$G_{ci}(s) = \frac{1}{s}(k_i h_{0,i} + (k_i + h_{0,i})s),\ i = 1, \cdots, 3 \quad (37)$$

Where $k_i = \text{sgn}(g_{ii}(s))\mu_i$, with μ_i being a suitable positive constant in its stable region. The parameter $h_{0,i}$ are suitable positive constants, which determine the desired dynamic reponses.

In these three loops, the desired poles are all placed at -0.1, that is, $h_{0,1} = h_{0,2} = h_{0,3} = 0.1$. Also, we select $\mu_1 = 0.35$, $\mu_2 = 0.2$, $\mu_3 = 0.2$, respectively.

The dynamic responses are illustrated in Figure.9. The black lines represent dynamic responses in normal condition, and gray lines represent dynamic responses when +20% gain perturbations in cross-couplings $g_{ij}, i \neq j$ exist. It is clear that the NRC ensure the stability of the closed-loop subsystems and the dynamic response has its strong points. Here, the step input of third loop brings least affection to other two loops.

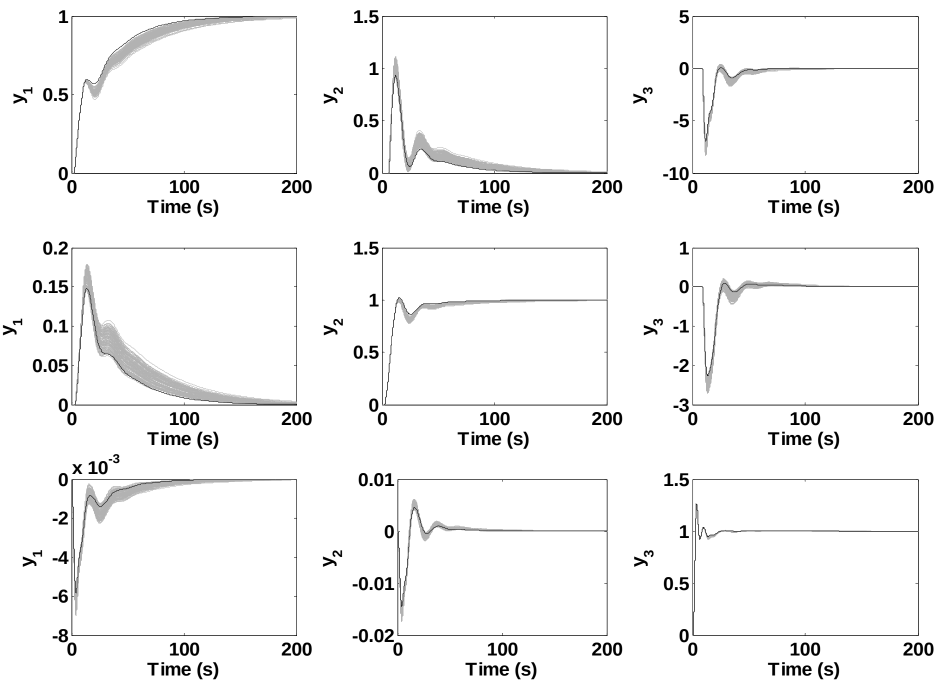

Figure 9. Dynamic Response of Different Step Inputs (OR3 Model)

The Perron-Frobenius eigenvalue λ_p with changing of ω is shown in Figure 10. Here, the sufficient condition in integrity theorem can not be satisfied any longer. But the system integrity can be easily evaluated by a great number of simulations.

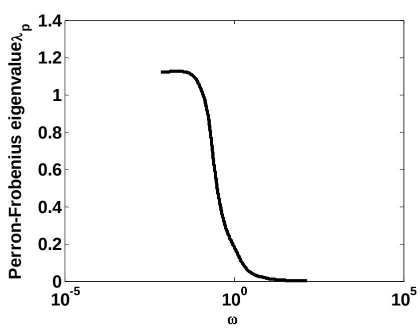

Figure 10. Perron-Frobeniuseig value (OR3 model)

6.3. 4×4 Model Alatiqi Case 1(A1)

$$G_6(s)=\begin{bmatrix} \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)} & \frac{-2.94(7.9s+1)e^{-0.05s}}{(23.7s+1)^2} & \frac{0.017e^{-0.2s}}{(31.6s+1)(7s+1)} & \frac{-0.64e^{-20s}}{(29s+1)^2} \\ \frac{-2.33e^{-5s}}{(35s+1)^2} & \frac{3.46e^{-1.01s}}{32s+1} & \frac{-0.51e^{-7.5s}}{(32s+1)^2} & \frac{1.68e^{-2s}}{(28s+1)^2} \\ \frac{-1.06e^{-22s}}{(17s+1)^2} & \frac{3.511e^{-13s}}{(12s+1)^2} & \frac{4.41e^{-1.01s}}{16.2s+1} & \frac{-5.38e^{-0.5s}}{17s+1} \\ \frac{-5.73e^{-2.5s}}{(8s+1)(50s+1)} & \frac{4.32(25s+1)e^{-0.01s}}{(50s+1)(5s+1)} & \frac{-1.25e^{-2.8s}}{(43.6s+1)(9s+1)} & \frac{4.78e^{-1.15s}}{(48s+1)(5s+1)} \end{bmatrix} \quad (38)$$

The design procedure is similar to the precedent. The controller is given as the following form:

$$G_c(s)=\begin{bmatrix} G_{c1}(s) & 0 & 0 & 0 \\ 0 & G_{c2}(s) & 0 & 0 \\ 0 & 0 & G_{c3}(s) & 0 \\ 0 & 0 & 0 & G_{c4}(s) \end{bmatrix} \quad (39)$$

Where $G_{ci}(s), i=1,\cdots,4$ are designed according to their corresponding relative degrees of diagonal transfer function. Here we have:

$$\begin{cases} G_{ci}(s)=\frac{1}{s}(k_i h_{0,i}+(k_i h_{1,i}+h_{0,i})s+(k_i+h_{1,i})s^2),\ i=1,4 \\ G_{ci}(s)=\frac{1}{s}(k_i h_{0,i}+(k_i+h_{0,i})s),\ i=2,3 \end{cases} \quad (40)$$

Where $k_1=\mathrm{sgn}(g_{11}(s))\mu_1$, $k_2=\mathrm{sgn}(g_{22}(s))\mu_2$, $k_3=\mathrm{sgn}(g_{33}(s))\mu_3$, and $k_4=\mathrm{sgn}(g_{44}(s))\mu_4$, with μ_i being a suitable positive constant in its stable region. The parameters $h_{0,i}$ and $h_{1,i}$ are suitable positive constants, which determine the desired dynamic responses.

In these four loops, the desired poles are all placed at -0.16, that is, $h_{0,1}=h_{0,4}=0.0256$, $h_{1,1}=h_{1,4}=0.32$, $h_{0,2}=h_{0,3}=0.16$. Also, we select $\mu_1=1$, $\mu_2=1$, $\mu_3=2$, $\mu_4=20$, respectively.

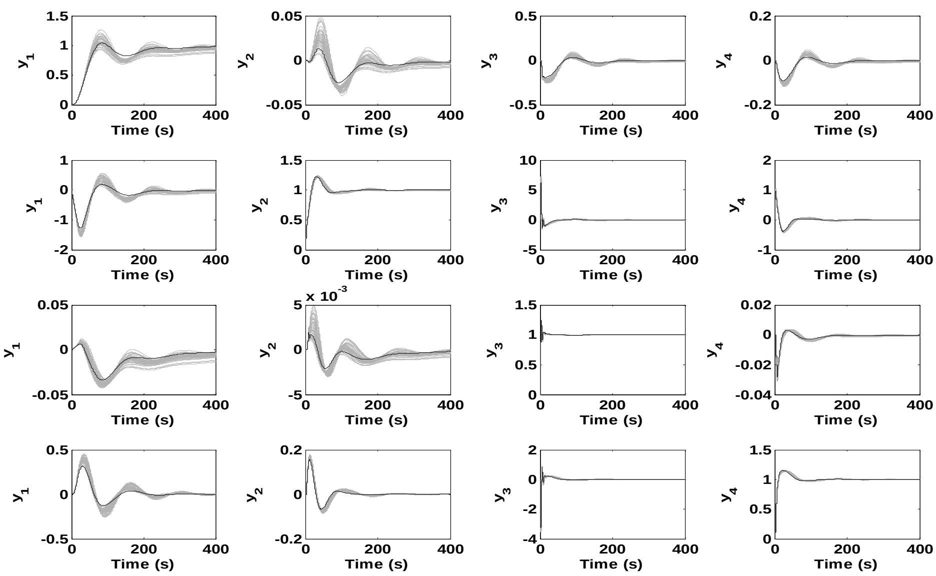

Figure 11. Dynamic Response of Different Step Inputs (A1 Model)

The dynamic responses are illustrated in Figure 11. The black lines represent dynamic responses in normal condition, and gray lines represent dynamic responses when +10% gain perturbations in cross-couplings $g_{ij}, i \neq j$ exist. It is clear that the NRC ensure the stability of the closed-loop subsystems. Here, the step input of third loop brings least affection to other three loops.

The Perron-Frobenius eigenvalue λ_p with changing of ω is shown in Figure 12. Here, the sufficient condition in integrity theorem can not be satisfied any longer. But the system integrity can be easily evaluated by a great number of simulations.

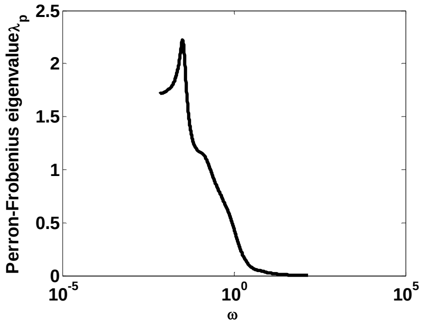

Figure 12. Perron-Frobeniuseig Value (A1 Model)

From analysis and simulations, it can be concluded that the designed NRC approach with simple structure have superior control ability and robust performance to great gain perturbations in cross-couplings. The parameters have specific meaning and can be tunedeasily. So this NRCapproach is more perspective in practical environments.

7. Conclusion

In this paper, a nonlinear robust control (NRC) scheme based on integrity theory is presented to MIMO systems with strong uncertainly cross-coupling. Unlike most available methods, the propose controller is able to estimate and compensate for the plant dynamics that is entirely unknown, thus eliminating the need for the controlled model. And the stable region of NRC for SISO system is easy to obtain, which brings great convenience to tune parameters combining with desired dynamics and simulation. The performance of decentralized NRC system is analyzed via six simulations applied to illustrative examples. The results demonstrate effectiveness, robustness and feasibility of the method.

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