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Forecasting Spatial Migration Tendency with FGM(1,1) and Hidden Markov Model

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Abstract

Population spatial migration tendency forecasting is very important for the research of spatial demography. Traditional approaches are too complex to be used for time series prediction. This paper presents a method combinating Hidden Markov Model (HMM) and Fourier Series Grey Model (FGM) based on Grey Model (GM) to predict the trend of Jiangsu Province's migration in China. There are three parts of forecast. The first one is to build GM from a series of coordinate data, the second uses the Fourier series to refine the residuals produced by the mentioned model and the third uses HMM to refine the residuals of FGM .It is evident that the proposed approach gets the better result performance in studying the population migration. Satisfactory results have been obtained, which improve HMM-FGM reached when only GM was used for the population spatial migration tendency forecasting.

Keywords*:* *spatial demography, grey model, fourier grey model, hidden Markov model, forecast error, gravity center model*

1. Introduction

Demography is an inherently spatial science which involves the study of complex patterns of interrelated social, behavioral, economic, and environmental phenomena [1]. Thus, scholars have increasingly argued that spatial answers to demographic questions including spatial analysis of demographic processes and outcomes. A great deal of attention has been given to the phenomena of migration and population migration prediction. Continued high levels of migration to advanced cities will lead to unprecedented change in demographic and economic compositions of regional populations, especially in China. As a consequence, Chinese provinces will experience a shift towards more uneven distribution, which concerns to most parts of the Chinese public. Although debate focused mainly on the province scale, the city transformations will be the most profound. Johnson et al. used both global and local spatial statistics to look for spatiotemporal patterns in migration of the American Southwest [2]. Borgoni et al. analysis the immigrant residential distribution spatially with particular reference to density and diversity-based methods [3]. Although these approaches allow the investigators to examine dynamic migratory patterns of spatial and temporal clustering, they couldn't be used to make accurate predictions based on the time series for predicting the tendency of migration.

Statistical and artificial intelligence approaches are the two main techniques for time series prediction seen in the literature [4-6], which include simple moving average forcasting (SMAF), autoregressive (AR), autoregressive moving average (ARMA) and neural network. However, they are too complex to be used in predicting future values of a time series and have not prerequisites for time series normality or error calibration [7]. Grey system theory is an interdisciplinary scientific area that was first introduced in early 1980s by Deng. Since then, the theory has become quite popular with its ability to deal with the systems that have partially unknown parameters. As a superiority to conventional statistical models, Grey Model (GM) require only a limited amount of data to estimate the behavior of unknown systems [8]. Hidden Markov Model (HMM) is a widely tool to analyse and predict time series phenomena. HMM has been used successfully to analyse various types of time series including financial time series [9, 10], speech signal recognition [11], and DNA sequence analysis [12] etc.

In this paper, we proposed HMM-FGM combining HMM with GM and Fourier series refining the residuals to achieve better forecasts. In our model, GM was constructed to do the

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calculation of the coordinate of population gravity center,and the residual error of the model was corrected by Fourier series. HMM was used to improve forecasting accuracy.

2.Proposed Method
2.1. GM(1,1)

GM predict the future values of a time series based on a set of the most recent data, and the sampling frequency of the time series is fixed. The main task of grey system theory is to extract realistic governing laws of the system using available data. This process is known as the generation of the grey sequence. In grey system theory, GM(n,m) denotes a grey model, where is the order of the difference equation and is the number of variables. Although various types of grey models can be mentioned, most of the previous researchers have focused their attention an GM(1,1) models in their predictions because of its computational efficiency. It should be noted that in real time applications, the computational burden is the most important parameter after the performance. GM(1,1) type of grey model is the most widely used in the literature, pronounced as “Grey Model First Order One Variable”. This model is time series forecasting model. The differential equations of the GM(1,1) model have time-varying coefficients. The GM(1,1)model can only be used in positive data sequences . In this paper, since all the primitive data are positive, grey models can be used to forecast the future values of the primitive data points.

In order to smooth the randomness, the primitive data obtained from the system to form the GM(1,1)is subjected to an operator, named Accumulating Generation Operator (AGO).The differential equation (i.e. GM(1,1))is solved to obtain the n-step ahead predicted value of the system. Finally, using the predicted value, the Inverse Accumulating Generation Operator (IAGO) is applied to find the predicted values of original data.

Consider a time sequence $X^{(0)}$ that denotes x-coordinate or y-coordinate of demography gravity center.

$$X^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),\dots,x^{(0)}(n)\right), \qquad n\ge 4, \tag{1}$$

Where $X^{(0)}$ is a non-negative sequence and n is the sample size of the data. When this sequence is subjected to the Accumulating Generation Operation (AGO), the following sequence $X^{(1)}$ is monotonically increasing.

$$X^{(1)}=\left(x^{(1)}(1),\ x^{(1)}(2),\dots,\ x^{(1)}(\mathrm{n})\right), \qquad \mathrm{n}\ge 4, \tag{2}$$

$$x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i), \qquad k=1,2,3,\dots,n. \tag{3}$$

The generated mean sequence $Z^{(1)}$ of $X^{(1)}$ is defined as:

$$Z^{(1)}=\left(z^{(1)}(1),z^{(1)}(2),\dots,z^{(1)}(n)\right), \tag{4}$$

Where $z^{(1)}(k)$ is the mean value of adjacent data, i.e.:

$$z^{(1)}(k)=0.5x^{(1)}(k)+0.5x^{(1)}(k-1),\ \ k=2,3,\dots,n \tag{5}$$

The least square estimate sequence of the grey difference equation of GM(1,1) is defined as follows:

$$x^{(0)}(k)+az^{(1)}(k)=b. \tag{6}$$

The whitening equation is therefore, as follows:

$$\frac{dx^1(t)}{dt} + ax^{(1)}(t) = b. \tag{7}$$

In above, $|a,b|^T$ is a sequence of parameters that can be found as follows:

$$|a,b|^T = \left(B^T B\right)^{-1} B^T Y, \tag{8}$$

Where:

$$Y = \left[x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right]^T, \tag{9}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{10}$$

According to equation (6),the solution of $x^{(1)}(t)$ at time k:

$$x_p^{(1)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right]e^{-ak} + \frac{b}{a}. \tag{11}$$

2.2. Gravity Model

Gravity center was introduced to the research, namely, with the balance point that the population spatial distribution reached spatial torque in the research area during a certain time. We can analyze this area's evolution in the population migration and disclose the characteristics and forecast the tendency of the research area's population migration. $\overline{x}$ and $\overline{y}$ are written as:

$$\overline{x} = \frac{\sum_{i=1}^{n} p_i x_i}{\sum_{i=1}^{n} p_i}, \overline{y} = \frac{\sum_{i=1}^{n} p_i y_i}{\sum_{i=1}^{n} p_i} \tag{12}$$

Where, n means the number of the administrative unit; (x_i, y_i) are the geographic gravity center of each basic unit; p_i means the population number; $(\overline{x}, \overline{y})$ are the population gravity center in Jiangsu.

2.3. Hidden Markov Model

Hidden Markov Model (HMM) is composed of a five-tuple: $\lambda = (S, O, A, B, \Pi)$, where $S = (S_1, S_2, \cdots, S_n)$ is a set of distinct states and n is the number of states; $O = (O_1, O_2, \cdots, O_m)$ is an observed sequence and m is the number of observation sequence; $A = \{a_{ij}\}_{1 \le i, j \le N}$ is transition probabilities and $a_{ij} = \mathrm{P}(S_{t+1} = \theta_j \mid S_t = \theta_i)$ is the probability of a transition from state θ_i to state θ_j; $B = \{b_{ij}\}_{1 \le i \le N}^{1 \le j \le M}$ is emission probalilities and $b_{ij} = \mathrm{P}(O_t = v_k \mid S_t = \theta_j)$ is the probability of state θ_j emitting v_k; $\Pi = (\pi_i)_{1 \le i \le N}$ is a vector of initial probalilities.

Forecasting the population gravity center position error is an HMM process. The prediction error is the observation sequence , and the prediction ability of the model is the sequence state. According to the historical forecasting errors distribution, forecast error were devided into the N classes, analysizing the hidden state sequence model. The state transfer matrix and observation probability matrix were estimated to forecast to the forecast errors. Finally, prediction error was corrected according to the predictive value.

2.4. FGM(1,1)

In order to improve the modeling accuracy of GM(1,1), Fourier series and gravity model was used to modify the grey models. The fourier series can make the forecasted results more precise. The forecasting algorithm based on the combined FGM(1,1) model is described as follows:

1) Computing the gravity center of spatial data.
2) Testing the quasi-exponential and the quasi-smoothness of the series. If it is quasi-exponential and quasi-expoential, then goto 3;else do the smooth process or IAGO process.
3) Considering the original data series processed by step 1 and step 2,

$$X^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),\ldots,x^{(0)}(n)\right),\qquad n\geq 4.$$

Computing the 1-AGO of serial $X^{(0)}$,

$$X^{(1)}=\left(x^{(1)}(1),\ x^{(1)}(2),\ldots,\ x^{(1)}(\mathrm{n})\right),\qquad \mathrm{n}\geq 4,$$

where $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i),\qquad k=1,2,3,\ldots,n.$

4) Constructing the matrix

$$B=\begin{bmatrix}-z^{(1)}(2) & 1\\ -z^{(1)}(3) & 1\\ \vdots & \vdots\\ -z^{(1)}(n) & 1\end{bmatrix}$$

Where $z^{(1)}(k)=0.5x^{(1)}(k)+0.5x^{(1)}(k-1),\ \ k=2,3,\ldots,n$.

5) According to $|a,b|^{T}=\left(B^{T}B\right)^{-1}B^{T}Y$,the estimated value a and b were calculated, where $Y=\left[x^{(0)}(2),x^{(0)}(3),\ldots,x^{(0)}(n)\right]^{T}$.

6) Calculating the simulated value $x^{(1)}(t)$ according to the equation (10).

7) Calculating the first-order residual error series

$$E^{(0)}=\left(\varepsilon^{(0)}(1),\varepsilon^{(0)}(2),\ldots\varepsilon^{(0)}(n)\right)^{T},$$

Wherein, $\varepsilon^{(0)}(k)=x^{(0)}(k)-\hat{x}^{(0)}(k)$.

8) Modeling the residual series based on the Fourier series according to the following formula:

$$E(k)=\frac{1}{2}a_{0}+\sum_{i=1}^{N}\left[a_{i}\cos\left(\frac{k2\pi}{T}t\right)+b_{i}\sin\left(\frac{k2\pi}{T}t\right)\right],\ for\quad k=2,3,\ldots,n. \qquad (13)$$

Where T indicates the length (period) of the residual series, which is equal to $(n-1)$; a_k and b_k are coefficients to be determined by the least square methods; a_0 is the average value of the function in the used range; N is the number of harmonics of the series; t is the order number given in the series. The least square method was used to calculate the coefficients a_0 , a_k and b_k expressed as follows:

$$C=\left(P^T P\right)^{-1} P^T E^{(0)} \tag{14}$$

Where, $C=\left[a_0,a_1,b_1,a_2,b_2,\ldots,a_N,b_N\right]^T$, and

$$P=\begin{bmatrix} 1/2 & \cos\left(\frac{2\pi\times 1}{T}2\right) & \sin\left(\frac{2\pi\times 1}{T}2\right) & \cdots & \cos\left(\frac{2\pi\times N}{T}2\right) & \sin\left(\frac{2\pi\times N}{T}2\right) \\ 1/2 & \cos\left(\frac{2\pi\times 1}{T}3\right) & \sin\left(\frac{2\pi\times 1}{T}3\right) & \cdots & \cos\left(\frac{2\pi\times N}{T}3\right) & \sin\left(\frac{2\pi\times N}{T}3\right) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1/2 & \cos\left(\frac{2\pi\times 1}{T}n\right) & \sin\left(\frac{2\pi\times 1}{T}n\right) & \cdots & \cos\left(\frac{2\pi\times 2}{T}n\right) & \sin\left(\frac{2\pi\times N}{T}n\right) \end{bmatrix}$$

9) Calculating the forecasted value of series as the following formula:

$$\hat{X}(1)=X^{(0)}(1) \quad and \quad \hat{X}^{(0)}(k)=\hat{X}^{(0)}(k)+E(k) \quad for \quad k=2,3,\ldots,n \tag{15}$$

2.5. HMM-FGM(1,1)

HMM were employed to improve the modeling accuracy of FGM(1,1). To apply the HMMS, the number of states, the types of models and the parameters to be modeled must be decided. The forecasting algorithm based on the combined HMM-FGM(1,1) model is described as follows.

1) Processing the forecasting error series of FGM(1,1) in the discrete way. The states and observation sequence can be decided according to the thresholds.

2) Estimating the transition probabilities and the emission probalilities.

3) Get the prediction of forecasting error series according to the current state as follows:

$$e_i=S_i T_i H \tag{16}$$

Where S_i is the current state; T_i is the probability of a transition from state i to the other state; H is the threshold discreting forecasting error series. In this paper, the threshold is denoted as $H=\left[P_{0.2},P_{0.4},P_{0.5},P_{0.6},P_{0.8}\right]^T$.

4) Calculating the forecasted value of series as the following foumula:

$$X_i^{'}=X_i+e_i \tag{17}$$

Where X_i is x-coordinate or y-coordinate of the population gravity center; $X_i^{'}$ is forecasting x-coordinate or y-coordinate of the population gravity center.

3.Application of the Proposed Method

3.1. Data Set

According to algorithm described above, the combined models were based on the population of Jiangsu province in China (1991-2010) collected from from the Jiangsu Statistics

Almanac, which is shown in Figure 1. According to Jiangsu Statistics Almanac in 2008, Jiangsu has 13 prefecture-level cities, 54 urban districts, 27 county-level cities and 25 counties. Suzhou, Wuxi, Changzhou, Nanjing and Zhenjiang along with the districts and counties subordinate to them consist of Southern Jiangsu; Yangzhou, Taizhou and Nantong with the districts and counties subordinate to them consist of Central Jiangsu; Xuzhou, Huai'an, Suqian, Yancheng and Lianyungang with their boroughs and counties--Northern Jiangsu.And the modeling, forecasting and analysis process are as follows:

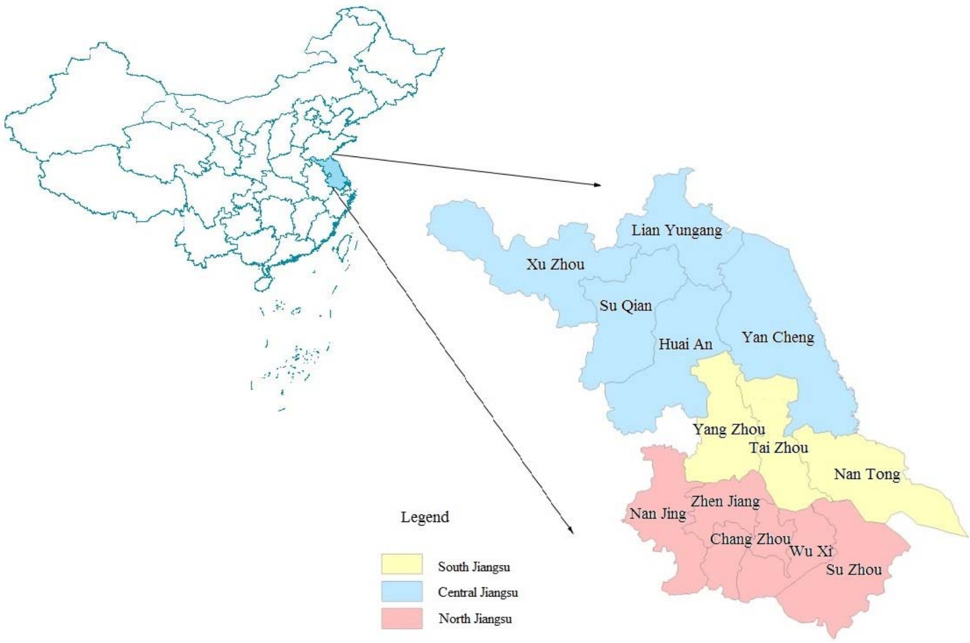


Figure 1. Economic Sketch Map of Jiangsu in 2008

3.2. Experimental Methodology

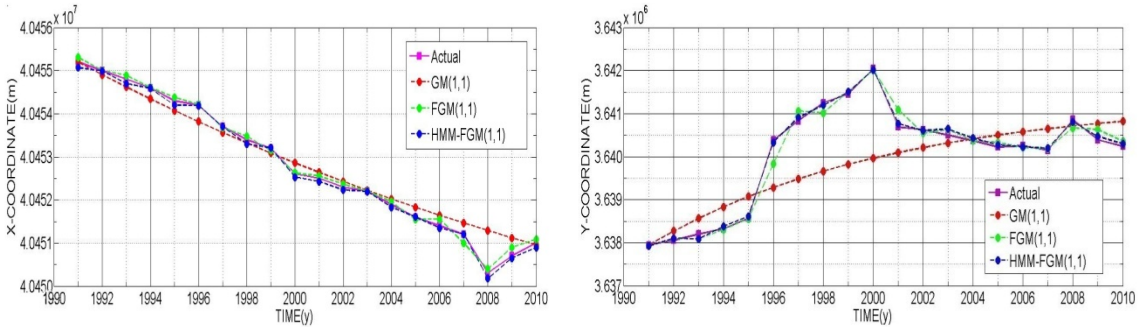
The gravity center coordinates of population distribution were computed according to the demography data.The population gravity center coordinate was achieved based on equation (12), which disclosed the spatial distribution and tendency of population migration. GM(1,1) was constructed based on the population gravity center coordinates. As shown in Figure 2, although the precision of applying GM(1,1) to forecast the coordination seems to be acceptable, the prediction performance could still be improved. In order to improving the precision. In order to improve the accuracy, Fourier series was used to modeling the residual series of GM(1,1) according to Equation (13), and the coefficients a_0, a_k and b_k were calculated according to Equation (14). To apply HMM to the residual series of FGM(1,1), the residual series must be transformed into the distinct states and observation sequence. The number n of states S was 3 according to classification. The number m of observation sequence O was 5 according to the percentiles of forecasting error as Equation(18).

$$O = \begin{cases} 1 & e < P_{0.2} \\ 2 & P_{0.2} \le e < P_{0.4} \\ 3 & P_{0.4} \le e < P_{0.6} \\ 4 & P_{0.6} \le e < P_{0.8} \\ 5 & e \ge P_{0.8} \end{cases} \quad (18)$$

The state transition probabilities matrix T and emission probabilities of a HMM for sequence O with known states S were calculated. The prediction of forecasting error series were calculated according to the current state as Equation (16) . The forecasted value of HMM-FGM is shown in Table 1. The results of the forecasting accuracy for coordinate of gravity center by using GM(1,1), FGM(1,1) and HMM-FGM(1,1) are shown in Table 1. As shown in Figure 2 and Figure 3, GM(1,1) cannot predict the acute parts of the coordinate of population gravity in Jiangsu satisfactorily. However, the forecasting errors are obviously reduced by FGM(1,1) and HMM-FGM(1,1). The HMM-FGM(1,1) is more accurate than FGM(1,1) approach.

Table 1. Forecasting Results of Different Models

Year	Actual Value		GM(1,1)		FGM(1,1)		HMM-FGM(1,1)	
	X (m)	Y (m)	X (m)	Y (m)	X (m)	Y (m)	X (m)	Y (m)
1991	40455200	40455308	40455200	3637940	40455308	3637921	40455075	3637923
1992	40455000	40455008	40454905	3638267	40455008	3638096	40454994	3638106
1993	40454800	40454892	40454619	3638562	40454892	3638090	40454701	3638085
1994	40454600	40454606	40454342	3638828	40454606	3638310	40454594	3638376
1995	40454300	40454383	40454075	3639069	40454383	3638554	40454201	3638616
1996	40454200	40454210	40453816	3639286	40454210	3639832	40454194	3640328
1997	40453700	40453675	40453565	3639482	40453675	3641061	40453704	3640908
1998	40453400	40453478	40453322	3639659	40453478	3641017	40453301	3641198
1999	40453200	40453150	40453088	3639818	40453150	3641507	40453204	3641506
2000	40452600	40452622	40452861	3639962	40452622	3642003	40452529	3642023
2001	40452500	40452554	40452641	3640093	40452554	3641087	40452429	3640768
2002	40452300	40452370	40452428	3640210	40452370	3640539	40452229	3640603
2003	40452200	40452202	40452222	3640316	40452202	3640622	40452194	3640641
2004	40451900	40451953	40452023	3640412	40451953	3640372	40451829	3640426
2005	40451600	40451546	40451831	3640498	40451546	3640321	40451604	3640276
2006	40451400	40451554	40451644	3640576	40451554	3640208	40451353	3640233
2007	40451200	40450994	40451464	3640646	40450994	3640188	40451204	3640196
2008	40450300	40450395	40451289	3640710	40450395	3640669	40450175	3640808
2009	40450700	40450896	40451120	3640767	40450896	3640627	40450653	3640468
2010	40451000	40451088	40450957	3640819	40451088	3640353	40450901	3640296



(a) X-coordinate of gravity center (b) Y-coordinate of gravity center

Figure 2. Coordinate Forecasted Results

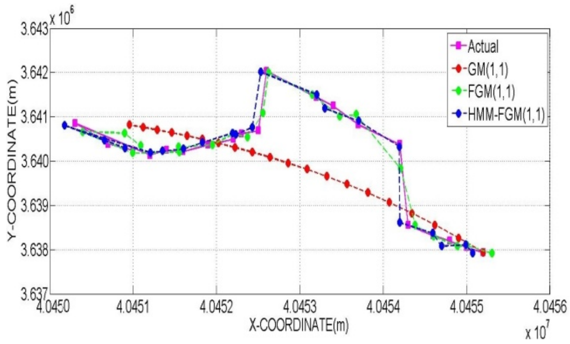


Figure 3. Comparison of GM(1,1), FGM(1,1) and HHM-FGM(1,1) from 1991 to 2010

3.3. Results Accuracy

The utimate goal of any forecasting endeavor is to provide an accurate and unbiased forecast. Forecast error is the difference between actual quantity and the forecasted. This study makes a comparison of the results from 1991 to 2010 to assess the forecast performance of GM(1,1), FGM(1,1) and HHM-FGM(1,1) by way of the mean absolute error (MAE), mean absolute percentage error $(MAPE)$ and root mean squre error $(RMSE)$. MAE is a quantity used to meansure how close forecasts or predictions are to the eventual outcomes. $MAPE$ is a measure of accracy in a fitted time series value in statistics,which usually expresses accuracy as a percentage. $RMSE$ a way to quantify the difference between an estimator and the true value of the quantity being estimated. The indicators were expressed as follow:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|A_t - F_t| \quad (19)$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \quad (20)$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(A_t - F_t)^2} \quad (21)$$

Where A_t is actual value for period t , and F_t is forecast value for period t.

From Table 2, GM(1,1) provided x-corrdinate prediction accuracy to 300m in $RMSE$ which FGM(1,1) and HMM-FGM(1,1) achieved accuracy to 93m and 69m in $RMSE$, respectivly. As above, the FGM(1,1) used an integral approach to improve the forecasting value of GM(1,1) further and was improved by Fourier series. The HMM-FGM(1,1) was improved by HMM. HMM-FGM(1,1) is better in comparison to all studied approaches regardless of the adapting index of MAE, $MAPE$ or $RMSE$.

Table 2. Simulation Results of Coordinate Forecasted

Name	GM(1,1)			FGM(1,1)			HMM-FGM(1,1)		
	MAE[m]	MAPE	RMSE[m]	MAE[m]	MAPE	RMSE[m]	MAE[m]	MAPE	RMSE[m]
X-coordinate	217	0.00054	300	73	0.00018	93	53	0.00013	69
Y-coordinate	639	0.01756	856	92	0.00361	191	58	0.00160	65

4. Conclusion

GM is very common techniques used for time series forecasting. However, the HMM-FGM which combined use of HMM and Fourier series based on GM in population migration forecasting is a novel approach, which has been proved to provide an adequate performance. In this paper, the HMM-FGM is presented to use the grey model to roughly predict the next datum from a set of the most recent data. The model use HMM and the Fourier series to fit the residual errors produced by the GM. It is evident that the proposed approach HMM-FGM has a higher forecasting accuracy than GM in population spatial migration tendency forecasting.

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