A Kind of H_2/H_∞ Filtering Sheme on Deformation Monitoring Data

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Abstract

Based on the research of the Kalman filter, a kind of H_2/H_∞ filter is put forward which depends on the model. H_2 filter assumes that the noise is white noise, but does not require the statistical properties, also ignores color noise. H_∞ filter only considers the non-white and energy- limited noise. It can ensure the accuracy of the filter in worst frequency point, but doesn't consider the influence of white noise. Synthesizing the advantages and disadvantages of H_2 and H_∞ filter scheme, H_2/H_∞ hybrid filter devide the noise into the white noise and non-white noise of limited energy. Baed on norm analysis about the noise and system, the optimization index J is adopted as the optimization goal, and its physical meaning is given. The hybrid filter is designed by solving the corresponding Riccati Equation, the simulation and actual calculation example are given.

Keywords: H_2/H_∞ filter, deformation monitoring, estimate error, filtering

1. Introduction

Nowadays, with the development of economic construction, high-rise building,the projects of water conservancy & hydropower, the large bridge project and open-pit mining are becoming more and more. It is the most common geological environmen,in which the human engineering and the life activities filled. Its deformation can cause the deterioration of human survival geological environment, even bring severe disaster to the human. In the relevant geological disasters, its damages is only next to earthquake. The accurate monitoring play an important patr in landslide disaster analysis and prediction. So data analysis of deformation monitoring have become an important research direction.

In the practice of foundation pit and slope deformation monitoring, the accuracy and efficiency of monitoring is greatly improved with the improvement of automation and measurement precision of monitoring equipment. But some new problems are put forward to the traditional data processing method with the hardware level enhanced [1-5]. The current observation data filtering methods can be divided into several categories [6-7]. The traditional data filtering method include three kinds of method: One is related to gross error, aiming at jumping on larger data processing, such as Layda Criterion, etc; Another is to suppress the observation noise, such as the data point peplced with the average value of few near points instead of itself; Another is signal filtering method. Fourier transform of signal is a filtering method. It convert the signal in time domain into frequency domain, and eliminate the part of high frequency part as noise. In recent years, some of the new filtering method have sprung up. They can be divided into two categories. One kind depends on the model ,and the other one does not rely on model. Wavelet method[8-9], for example, is not depend on the model, mainly is time-frequency decomposition of signals. It set the threshold value and eliminate some frequency parts. The Kalman Filter method is depend on the model [5], and its filtering effect is excellent. But it is needed both to establish the motion model of deformation and movement system and statistical characteristic of noise given. The filtering method which we have referred to is popular in currently. But they generally assume that the observation noise is white noise. such assumption in the practice of the observation is not fully established, at least not accurate enough. A kind of filter is given in this paper, they are dependent on the model. H_∞Filter considers the worst case about the noise component, and minimizie maximum gain of filtering system from noise to estimated error on frequency domain. So It make sure the precision in the

worst points. But it is conservative because it does not pay attention to the other signals outside of the worst point. H_2 Filtering scheme is used to deal with white noise, and is the overall optimal filter when considering the highest probability at the most generally situation. But it does not consider the worst case about noise outside of white, so it cannot ensure the precision of the individual worst point. Synthesizing the advantages and disadvantages of the single optimization index of H_2 filter and H_∞filter, the hybrid filter scheme is proposed. It uses the hybrid optimization index J as the optimization goal based on the norm analysis about the noise signal and model. On the condition of the H_∞ norm of the filter system less than the given valueγ, the H_2 norm of filter system is overall optimized. compareing H_2, H_∞ and H_2/H_∞ three kinds of filter scheme, the advantages and disadvantages of various options is discussed adequately.

2. The Filter Design of H_2/H_∞ Deformation System
2.1. Deformation System Modeling and Filter Design
Considering the distortion system $P(s)$ is described by the following state space model [10].

$$\begin{cases} \dot{x} = Ax + B_1\omega_1(t), x_0 = x(0) \\ \quad z = C_1 x \\ \quad y = C_2 x + D_{21}\omega_2(t) \end{cases}, \tag{1}$$

Where, state variable x is a vector, $x = [\dot{x}_x, \dot{x}_y, \dot{x}_z, x_x, x_y, x_z]^T$, x_x, x_y, x_z respect the relative coordinates of x、y、z direction in observation point relative to the control points respectively. $\dot{x}_x, \dot{x}_y, \dot{x}_z$ are the relative displacement of the observation point in x、y、z direction.t is the observation time, and $\omega(t)$ is the observation noise, which is assumed to be containing gaussian white noise and colored noise, z is estimated deformation vector. y is obsrved displacement value. A is related to model system. B_1, C_1, C_2, D_{21} are the constant of the models. x_0 Is the initial state, which is known. the first observed value will act as the initial state.
Designing a dynamic filter $L(s)$:

$$\begin{cases} \dot{\hat{x}}(t) = A_L\hat{x}(t) + B_L y(t), \hat{x}_0 = 0 \\ \quad \hat{z} = C_L\hat{x}(t) + D_L y(t) \end{cases} \tag{2}$$

Make sure the different norm index of the transfer functionfrom from the noise (system $\omega_1(t)$ and observation noise $\omega_2(t)$)to the estimation error $\tilde{z} = z - \hat{z}$ satisfy certain requirements. Set $\tilde{x} = [x^T(t) \quad \hat{z}^T(t)]$, then the dynamic equation of filtering error is given as:

$$\begin{cases} \dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}\omega(t), \tilde{x}_0 = 0 \\ \quad \tilde{z} = \tilde{C}\tilde{x}(t) + \tilde{D}\omega(t) \end{cases} \tag{3}$$

Where $\tilde{A} = \begin{bmatrix} A & 0 \\ B_L C & A_L \end{bmatrix}$, $\tilde{B} = \begin{bmatrix} B \\ B_L D \end{bmatrix}$, $\tilde{C} = \begin{bmatrix} C_1 - D_f C_2 & -C_L \end{bmatrix}$, $\tilde{D} = -D_L D$.

2.2. Filtering Dynamic System Norm
The norm definition of the following signal and the system transfer function on time domain and frequency domain are introduced.
The norm definition of the observation noise signal $\omega = [\omega_1(t) \quad \omega_2(t)]$, filtering estimate the error output $\tilde{z}$ on time domain are following:

If $\sqrt{\int_{-\infty}^{\infty}\|\omega(t)\|^2 dt} < \infty, ess\sup\|\omega(t)\| < \infty$, the L_2 of observation noise signal $\omega(s)$ are defined as:

$$\|\omega\|_2 = \sqrt{\int_{-\infty}^{\infty}\|\omega(t)\|^2 dt}, \tag{4}$$

$$\|\omega\|_\infty = ess\sup\|\omega(t)\| = ess\sup\sqrt{\omega^T(t)\omega(t)}, \tag{5}$$

If $\sqrt{\int_{-\infty}^{\infty}\|\tilde{z}(t)\|^2 dt} < \infty, ess\sup\|\omega(t)\| < \infty$, the L_2 and L_∞ norm of observation noise signal $\tilde{z}(s)$ are defined as:

$$\|\tilde{z}\|_2 = \sqrt{\int_{-\infty}^{\infty}\|\tilde{z}(t)\|^2 dt}, \qquad , \tag{6}$$

$$\|\tilde{z}\|_\infty = ess\sup\|\tilde{z}(t)\| = ess\sup\sqrt{\tilde{z}^T(t)\tilde{z}(t)}, \tag{7}$$

esssup means essured upper limit.

The norm definition of the observation noise ω、Estimation error output $\tilde{z}$ and the transfer function G of filtering system on complex frequency domain s are following:

if $\sqrt{\int_{-\infty}^{\infty}\|\omega(s)\|^2 ds} < \infty, ess\sup\|\omega(s)\| < \infty$. The signal does not have pulse, The L_2 norm of observation noise $\omega(s)$ is defined as:

$$\|\omega\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\|\omega(s)\|^2 ds} = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\omega^T(s)\omega(s)ds}, \tag{8}$$

if $\sqrt{\int_{-\infty}^{\infty}\|\tilde{z}(s)\|^2 ds} < \infty, ess\sup\|\tilde{z}(s)\| < \infty$. The signal does not have pulse, The L_2 norm of observation noise $\tilde{z}(s)$ is defined as:

$$\|\tilde{z}\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\|\tilde{z}(s)\|^2 ds} = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{z}^T(s)\tilde{z}(s)ds}, \tag{9}$$

the H_∞ norm of G system in the form of transfer function in frequency domain is defined as :

$$\|G(s)\|_\infty = \sup\sigma[G(s)] = \gamma,$$
$$\text{or } \|G(s)\|_\infty = \sup(\|\tilde{z}\|_2 / \|\omega\|_2) \tag{10}$$

Where γ is a positive known number, s ia the complex frequency, σ is the largest singular value. The H_2 norm of system's in the form of transfer function in frequency domain is defined as:

$$\|G(s)\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}trace(G^T(s)*G(s)ds}, \tag{11}$$

Where s is the complex frequency, trace is the matrix's trace. If the description from the state of the system equation, H_2 norm of system G can be expressed as:

$$\|G(s)\|_2 = \sqrt{trace(C_1 L C_1')} \,,$$

Where L is the solution of Lyapnov equation as follows: $AL + LA + B_1 B_1' = 0$. The definition of signal system in time domain and frequency domain is equivalent, they can be convert each other by Fourier Transform.

2.3. H_2 Filter Design

For the filter described by the formula (2), if it can make H_2 norm $\|G\|_2$ of transfer function G from the system noise $\omega(t)$ to estimate the error $\tilde{z} = z - \hat{z}$ to get the minimum norm index, the filter described by the formula (2) is its H_2 filter. Want tp get the H_2 filter (2), following two Hamilton matrix (12-13) in the first place are given:

$$H_2 := \begin{bmatrix} A - B_2 D_{12}^T C_1 & -B_2 B_2^T \\ -C_1^T D_{12}^* (D_{12}^*)^T C_1 & -(A - B_2 D_{12}^T C_1)^T \end{bmatrix} , \qquad (12)$$

$$J_2 := \begin{bmatrix} (A - B_1 D_{21}^T C_2)^T & -C_2^T C_2 \\ -B_1 (D_{21}^*)^T D_{21}^* B_1^T & -(A - B_1 D_{21}^T C_2) \end{bmatrix} , \qquad (13)$$

If the filter solution exists, and matrix H_2: and J_2 have no eigenvalue on the imaginary axis, X_∞, Y_∞ are the two positive semidefinite solution of the Ricatti equation corresponded to Hamilton matrix (12),(13), the H_2 optimized filter is given by (14) [10]:

$$K = \begin{bmatrix} \hat{A}_2 & -L_2 \\ F_2 & 0 \end{bmatrix} . \qquad (14)$$

Where $\hat{A}_2 = A + B_2 F_2 + L_2 C_2$, $F_2 := -(D_{12}^T C_1 + B_2^T X_2)$ $L_2 = -(B_1 D_{21}^T + Y_2 C_2^T)$, $D_{12} D_{12}^* = I$, $D_{21} D_{21}^* = I$, namely D_{12}^*, D_{21}^* as the orthogonal array of D_{12}, D_{21}.

Based on H_2 norm definition of the system G, as the input noise signal is assumed to be gaussian white noise process to H_2 filter, It is not hard to deduced the integral term in H_2 norm definition is the gain function array about frequency (formula 11). The integral result to frequency reflects the gain on the frequency domain. So physical meaning of the system's H_2 norm is that it reflects the condition to suppress noise of filtering system when the noise signal probability is uniform on the whole frequency domain. Accordingly, it is not hard to known its geometric significance is the area surrounded by largest singular value curve and frequency axis.

For H_2 filtered system expressed by the form of transfer function $\tilde{z}(s) = G(s)\omega(s)$, Assume that the noise signals is gaussian white noise process. Without losing generality, assume ω as mean value 0, variance 1, then $\omega_i^T \omega_i$ =I(unit matrix), $\|\tilde{z}\|_2 = \|G\omega\|_2 = \|G\|_2$. So optimizing the $\|\tilde{z}\|_2$ means optimizing the G, The optimization goal is only related to the system G itself. Thus the H_2 filter is to reduce the estimation error $\tilde{z} = z - \hat{z}$ in the frequency domain (or time) on the whole, as far as possible.

2.4. H_∞ Filter Design

For the filter described by the formula (2), if it can make H_∞ norm $\|G\|_\infty$ of transfer function G from the system noise $\omega(t)$ to estimate the error $\tilde{z} = z - \hat{z}$ to get the minimum, the filter described by the formula (2) is its H_∞ filter. Want to get the H_∞ filter (2), following two Hamilton matrix in the first place are given:

$$H_\infty := \begin{bmatrix} A - B_2 D_{12}^T C_1 & \gamma^{-2} B_1 B_1^T - B_2 B_2^T \\ -C_1^T D_{12}^* (D_{12}^*)^T C_1 & -(A - B_2 D_{12}^T C_1)^T \end{bmatrix} \tag{15}$$

$$J_\infty := \begin{bmatrix} (A - B_1 D_{21}^T C_2)^T & \gamma^{-2} C_1^T C_1 - C_2^T C_2 \\ -B_1 (D_{21}^*)^T D_{21}^* B_1^T & -(A - B_1 D_{21}^T C_2) \end{bmatrix} \tag{16}$$

If the filter solution exists, and matrix H_∞ and J_∞ have no eigenvalue on the imaginary axis, and the polar radius $\rho < \gamma^2$, X_∞, Y_∞ are the two positive semidefinite solution of the Ricatti equation corresponded to Hamilton matrix (15-16), the H_∞ optimized filter is given by (17) [10]:

$$K = \begin{bmatrix} \hat{A}_\infty & -Z_\infty L_\infty \\ F_\infty & 0 \end{bmatrix} \tag{17}$$

Where $\hat{A}_\infty = A + \gamma^{-2} B_1 B_1^T X_\infty + B_2 F_\infty + Z_\infty L_\infty C_2$, $F_\infty := -(D_{12}^T C_1 + B_2^T X_\infty)$, $L_\infty = -(B_1 D_{21}^T + Y_\infty C_2^T)$, $Z_\infty = (I - \gamma^{-2} Y_\infty X_\infty)^{-1}$, $D_{12} D_{12}^* = I$, $D_{21} D_{21}^* = I$, namely D_{12}^*, D_{21}^* as the orthogonal array of D_{12}, D_{21}

H_∞ Filter can also be got by solving a set of linear matrix inequality (LMI) for the following:

$$\begin{bmatrix} \bar{A} P + P \bar{A} + \bar{C}^T \bar{C} & PB \\ B^T P & D^T D - \gamma^2 I \end{bmatrix} \leq 0,$$

Where P is asymmetric positive definite matrix, 0 means the negative semi definite.

It also can be seen from the definition of H_∞ norms $\|G\|_\infty$ of filter system. Its physical meaning is the biggest gain of filter system (the worst frequency point), and its geometric meaning is the peak of the largest singular value curve.

On the frequency domain, only when frequency equals a certain value, the gain of filter system G from ω to estimated error $\tilde{z}$ *acquire to maximum. It is* H_∞ norms $\|G\|_\infty$ of filter system. Considering the frequency point which is on the maximum gain system, the H_∞ filter minimized the supremum that from the filter noise input to estimated error $\tilde{z}$ (energy amplification), which frequency is also called as the worst noise case. The H_∞ filter consider the worst possible frequency point (low probability) only in the whole frequency domain. When index $\|G\|_\infty$ is smaller, it means the influence degree to estimate error is smaller on this frequency point. H_∞ filter consider colored noise, and it guarantee the estimate error $\tilde{z}$ minimization on the worst frequency point, but it was not be considered on other frequency point when $L(s)$ is be designed. Therefore it consider that the noise reach the worst-case in a limited range of energy values. The filtered waveform also represents the worst-

case noise waveform, so its filtering is conservative and reliable. It's important to note that H_∞ filter is not designed for white noise, and the influence of white noise on the signal can be treated through other means such as H_2 filter.

2.5. H_2 / H_∞ Filter Design

For the H_2 filter, assume that the noise signal is Gaussian white noise($\omega \in BS$). The optimization goal is only associated with the system itself and is overall optimization. For a colored noise (including the worst-case that of small probability), It is not optimal and cannot assure the H_∞ norm less than a given value γ (suboptimal). Beside, the stability of the filtering performance index $\tilde{z}$ is not considered.

H_∞ filtering method avoided the drawbacks of the H_2 filtering method in statistic process of noise signal. Considering the worst situation that colored noise signal is bounded, propose a filtering method to minimize $\|\tilde{z}\|_2$. However, this method is not optimized for common white noise signal (high probability), and in most situations it shows a conservative and bad effect.

The tradeoff H_2 / H_∞ filtering method overcome this sidedness. It divided the noise signal $\omega(t)$ (including system noise ω_1 and observed noise ω_2) into a deterministic part belongs to a bounded set (colored noise) and a random part described by the random variable (Gaussian white noise). Recorded as: $\omega = \begin{bmatrix} w_0 & w_1 \end{bmatrix}$, $w_0 \in BS$, $w_1 \in P$)。Thus, the distortion system $P(s)$ (formula 1) can be described by the following state space model (formula 18).

$$\begin{cases} \dot{x} = Ax + B_0 w_0 + B_1 w_1 \\ z = C_1 x \\ y = C_2 x + + D_{20} w_0 + D_{21} w_1 \end{cases} \tag{18}$$

The transfer function form can be described by：

$$\tilde{z} = G\omega = [G_0(s) \quad G_1(s)][\omega_0(s) \quad \omega_1(s)]^T . \tag{19}$$

To take $\|\tilde{z}\|_P$ -norm as optimization index, it can be seen from the definition of P-norm that the physical meaning of $\|\tilde{z}\|_P$ -norm is the total energy instead of the total power which differs from L_2 -norm. It's easy to understand that when $\|\tilde{z}\|_P$ is optimal the $\|\|\tilde{z}\|_2$ will be optimal either. Thus, the following optimization index was adopted:

$$J = \sup_{\omega \in w}(\|z\|_p^2 - \gamma^2 \|\omega_1\|_p^2) \text{ ,where.} \|G_1\|_\infty < \gamma . \tag{20}$$

It can be proved that $J = \|G\|_2^2$,when ω_0 is orthogonal to ω_1 , $\omega_0 \in BS$, $\omega_1 \in P$, $\|G\|_\infty <$ γ . Thus, the physical meaning of hybrid filtering performance indicator J is to minimize $\|G\|_2$ in condition of‖ G ‖∞< γ . For systems expressed by the state Equation (18), when the colored noise signal satisfies the condition $\omega_1 \in P$ and ω_0 is orthogonal to ω_1 , a stable optimized filter can be got to minimize the index J in condition of‖ G ‖∞< γ . The hybrid filtering issue will be turned into standard H_2 issue when $\gamma \to \infty$ or standard H_∞ issue when ω_0 =0.

These hybrid filtering methods will have solutions when the following conditions are satisfied:

(1) Hamilton matrix corresponding to the system doesn't have eigenvalues on the imaginary axis and the corresponding Riccati Equations definite.

(2) There is certain L, Y, P to satisfy the follows coupling Riccati equation and Lyapnov equations:

$$Y(LR_0 + B_0 D_{20}^T + PC_2^T + \gamma^{-2} PX_\infty B_1 D_{21}^T + \gamma^{-2} PYLR_1 + \gamma^{-2} PYB_1 D_{21}^T) = 0 \qquad (21)$$

$$YA_{ml} + A_{ml}^T Y + \gamma^{-2} Y(B_1 + LD_{21})(B_1 + LD_{21})^T Y + F_\infty^T F_\infty = 0 \qquad (22)$$

$$Y \geq 0 \text{ and } A_{ml} + \gamma^{-2}(B_1 + LD_{21})(B_1 + LD_{21})^T Y \qquad (23)$$

$$\{A_{ml} + \gamma^{-2}(B_1 + LD_{21})(B_1 + LD_{21})^T Y\}P$$
$$+ P\{A_{ml} + \gamma^{-2}(B_1 + LD_{21})(B_1 + LD_{21})^T Y\}^T + (B_0 + LD_{20})(B_0 + LD_{20})^T = 0 \qquad (24)$$

When these above conditions are satisfied, the hybrid optimization filter (linear fractional transformation form) is given by:

$$K(s) := \begin{bmatrix} A_{ml} + B_2 F_\infty & | & -L \\ ----- & & -- \\ F_\infty & | & 0 \end{bmatrix}$$

Where, $R_0 = D_{20} D_{20}^T, R_1 = D_{21} D_{21}^T, A_{ml} = A + \gamma^{-2} B_1 B_1^T X_\infty + L(C_2 + \gamma^{-2} D_{21} B_1^T X_\infty)_m F_\infty = -(D_{12}^T C_1 + B_2^T X_\infty)$.

The equations above can be solved through the following steps of the algorithm programming in matlab environment:

(1) Initialize the $\gamma > 0$, $L_0 = 0$; To plug L_i in equation (22) then calculate the Y_i; Conditions to plug Y_i in (9), if satisfy the conditions, we can execute the next step, if does not, do the again;

(2) Using L_i、Y_i, plus them into the equation (24) for getting P_i;

(3) Plusing Y_i、P_i into the equation (21) for getting L_{i+1};

(4) If $L_{i+1} - L_i \leq \varepsilon$ (required accuracy), the program is over, L_i, Y_i, P_i are answer, otherwise $L_{i+1} = L_i$, repeat step (2), In the actual calculation, If no convergence of $L_{i+1} - L_i$ iterative calculation, or (23) does not satisfy the conditions, it is likely the difference is too big between the initial value and the actual, we should adjust the initial value, and γ value should be gradually reduced.

3. Filter Effect Analysis

The effect of three kinds of filtering scheme are analysed with a known model examples. As shown in Figure 1, X, Y, Z respectively represent the displacement value of observation points. The abscissa is the observation time sequence, assuming the observation of 100 days, sampled once a day. The total displacement of X, Y, Z is 0.8mm, 0.7mm, 1.6mm respectively, and the deformation is assumed to be uniformand linear. The noise signal is composed of two parts. They are gaussian colored noise (Figure 2(e)) and white noise (Figure 2(d)) respectively. The maximal displacement of the colored noise is 0.5mm. The true value of the displacement and combined with noise represent observed values (as shown in Figure 3 contains noise in the observed curve). Thought three different kinds of filtering scheme, Figure 3 shows the curves filtered. Table 1 shows the three types of performance indicators of the noise before and after filtering. The noise value in curves after filtering is got according to the difference between the waveform data after filtering and the true value.
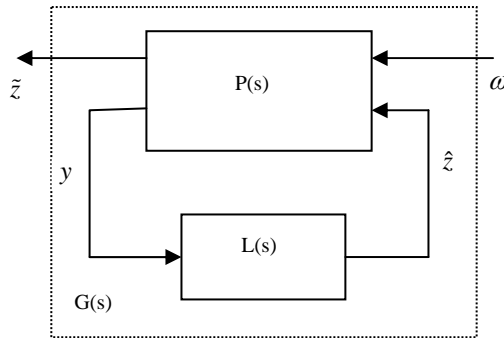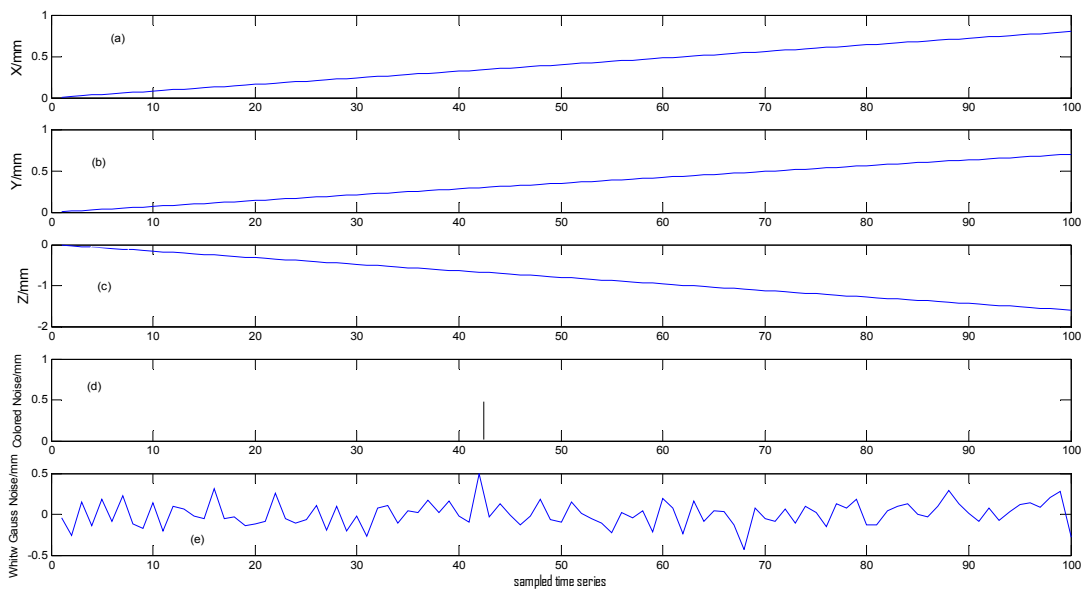
Figure 1. Sketch of Filter System

(a) The true value of X direction displacement, (b) the true value of Y direction displacement, (c) the true value of Z direction displacement, (d) the colored noise, (e) white gauss noise

Figure 2. The Time-displace Curves of True Displace and Noise on Known Model

Table 1. Noise Performance Indicators before and after Filtering

		Before filter rmm)	H2 filtering(mm)	HINF filtering(mm)	Mixed Filter(mm)
The noise standard dev iation	X	0.1484	0.0440	0.0479	0.0446
	Y	0.1484	0.0440	0.0479	0.0447
	Z	0.1484	0.0455	0.05	0.0465
The Noise mean value	X	0.005	0.0115	0.0065	0.009
	Y	0.005	0.0115	0.0065	0.009
	Z	0.005	0.0115	0.0065	0.009
The noise maximum value	X	0.5	0.1471	0.0987	0.1195
	Y	0.5	0.1471	0.0988	0.1195
	Z	0.5	0.1471	0.11	0.1195

The effect of each kind of filter is obvious through the Table 1 and Figure 2, Deformation curve smooths significantly after filtering, and largely in coincident with the displacement deformation law. But compared with the ideal displacement true value, some gap also exist. This is normal because we can not separate the noise from the signal completely. The noise mean is not equal to zero because of containing the colored noise. After filtering by

the H_2 filter scheme, the noise standard deviation is the minimum, the curve is smooth, and filtering to white noise is the most effective. But the noise of average and maximum is the biggest and the result of filtering colored noise is bad, the suppression effect to the colored noise at the 42 point is the worst. After filtering by the H_∞ filter scheme, the noise standard deviation is the highest, the effect of filtering the white noise is not good, but the mean and maximum of noise is the minimum, the result of suppressing and filtering the colored noise is good. The suppression effect to the colored noise at the 42 point is the best, but it is at the cost the filtering effect of other point. The hybrid scheme, not only considers the effective suppression to the colored noise, but also considers to filter the white noise as much as possible.Therefore, in the actual work γ can be choosed according to tolerance of colored noise, so it is a kind of effective filtering scheme.
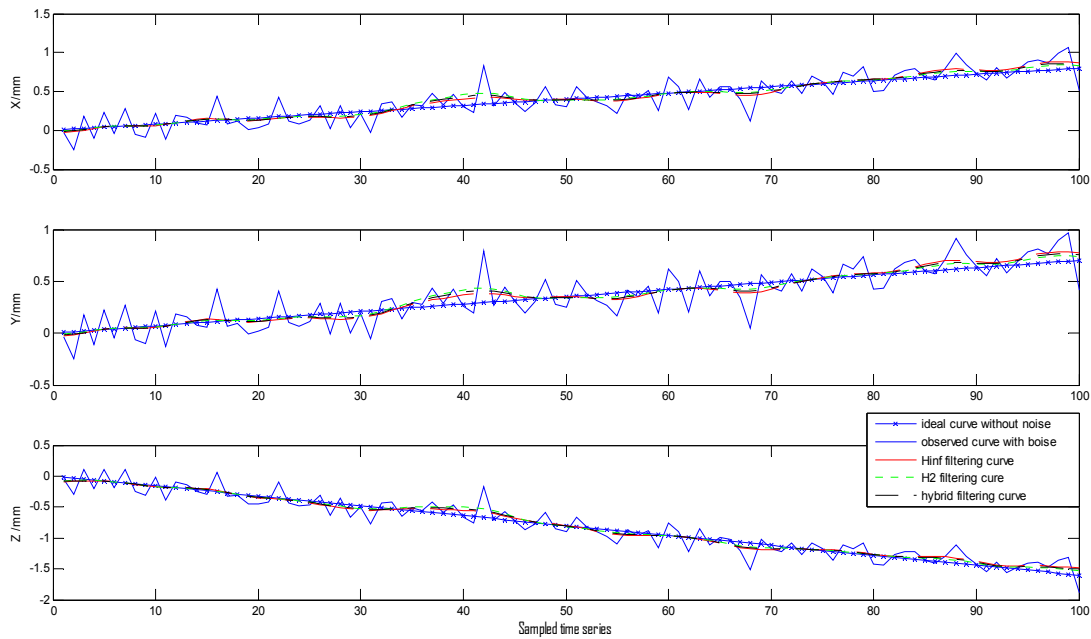

Figure 3. The Time-displace Curves of Observed and Filtered Value on Known Model
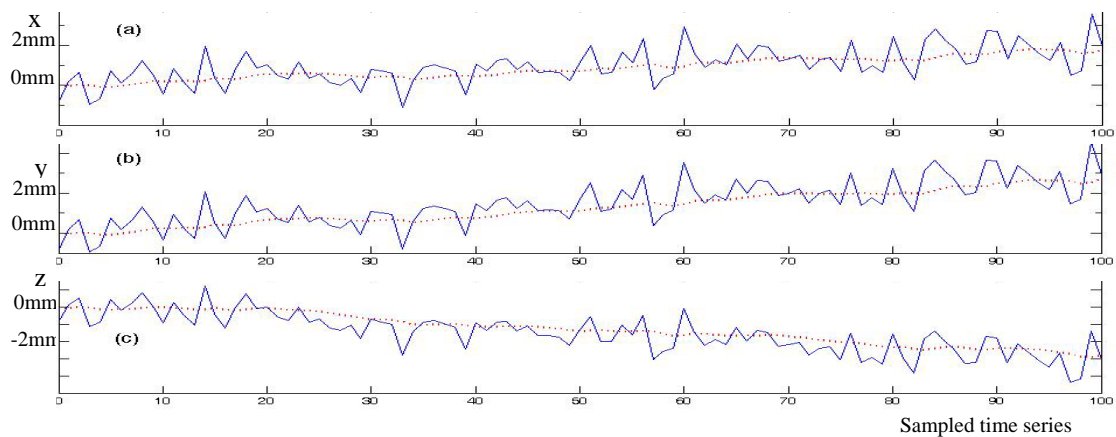
4. Calculation Example

Figure 4. The Time-displace Curves of Observed and Filtered Value on a Strip Mine in Liaoning Province, (a) X displace/mm, (b) Y displace/mm, (c) Z displace

Figure 3 is a deformation observation waveform diagram from a monitoring point on the slope of an open-pit iron ore in Liaoning Province, China. The horizontal axis represents the observation time and x,y, z represent the displacement component of the observation point from the control point in the three coordinate respectively. The solid line is the original observational data. According to the data analysis and colored noise tolerance, the H_2/H_∞ hybrid filtering method was applied to filter the original observation data and the filtered data curve is shown as the dotted line in Figure 4. From the analysis of deformation curves before and after filtering, it can be seen that amplitude of observation noise exceed the deformation true values. In other words, the deformation value is small while the observation noisy is large. Thus, the filtering is very important. A good result has been achieved after using the H_2/H_∞ hybrid filtering method as we can see from the Figure 4. The H_2/H_∞ hybrid filtering method is not only theoretically rigorous but also practicable.

5. Conclusion

A kind of H_2/H_∞ filter scheme is proposed, with respect to the high-precision observed data by measurement robot. Based on the previous work, H_2 and H_∞ filtering scheme are proposed, and the solving method of filter is given. The assumption that noise consists of white gauss noise and color noise of limited energy is proposed. Referring to the H_2 and H_∞ ffiltering algorithm respectively, and based on the analysis of the norm of signals and systems, the hybrid performance index J is given and analysed. For a given γ value, by solving the corresponding riccati equation to get arithmetic to minimize index J. Through theoretical analysis and numerical examples, the advantages and disadvantages of H_2, H_∞ and hybried filter are discussed. It is an effective method to deal with the noise containing non-white noise, under the premise of limiting the impact of the non-white noise component, as far as possible to reduce the influence of white noise.

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