TELKOMNIKA Indonesian Journal of Electrical Engineering
Vol.12, No.4, April 2014, pp. 3208 ~ 3213
DOI: http://dx.doi.org/10.11591/telkomnika.v12i4.4932

Iterative Integral Equation Method for Efficient Computation of Multiple Scattering

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Abstract

An iterative integral equation method (IEM) is proposed for calculation of the electromagnetic (EM) scattering field from geometries with multiple reflections, such as rough surface, dihedral and trihedral. The first reflection is computed by physical optics and the coupling effects are computed by integral equations. The average size of the triangular meshes used in the proposed method is a constant value while that in method of moment is a linear function of wavelength. As a result, compared with method of moment, the proposed method will lead to less number of unknowns for electrically large geomety. Accordingly, this method is more efficient and suitable for fast computation of scattering from electrically large geometry. Further more, when compared with high frequency asymptotic method, the proposed method is more accurate. The numerical results demonstrate that this method is accurate for computation scattering with multiple reflections and efficient for electrically large object.

Keywords: *iterative integral equation method, multiple scattering, electromagnetics*

1. Introduction

Electromagnetic wave scattering problems address the physical issue of detecting the diffraction pattern of the electromagnetic radiation scattered from a large and complex body when illuminated by an incident incoming wave [1-2]. A good understanding of these phenomena is crucial to radar cross section (RCS) calculation, antenna design, electromagnetic compatibility, and so on. Generally, the scattering of arbitrary geometry can be accurately computed by method of moment (MoM) [3]. However, since the multiple reflections exist, the coupling effects will lead to a slow convergence history. Besides, the electrically large geometry will lead to a large number of unknowns, which makes the memory requirement of MoM incredible large. Both the convergence and the memory requirement are the main bottleneck for MoM. Accordingly, an numerical method named iterative integral equation method is proposed for efficient calculation of scattering from any electrically large geometries with coupling effects, such as rough surface, dihedral, trihedral and so on.

At present, numerical method such as the method of moments (MoM) in [3] has the specialties of computational complexity and slow speed while with the high precision. Influenced by the computer computing ability, only good conditoned system with not very large number of unknowns could be analyzed by traditional method of moment. For example, the scattering from 3D target on a rough surface with the scale of no more than 30λ × 30λ [4], λ represent the wavelength. Actually, the rough surface is usually hundreds of thousands of wavelengths. By contrast, the aymptotic method shows low precision, such as kirchhoff approximation (KA) [5], physical optics (PO) [6], etc. Due to the small computational complexity and fast speed, it can be further simplified into the approximate mathematical expressions when studying the far-field characteristics of rough surface scattering. Hybrid algorithm which combined Analytical method with numerical method was put forward by G. A. Thiele and others in1975. Followed by hybrid method had been in continuous development, and it had made significant effect on improving the efficiency of algorithm. In 2008, Jin Yaqiu proposed the hybrid method which combined kirchhoff approximation method with the method of moments (KA-MoM) in [7]. This method implemented the general rough surface, such as soil and sea, efficiency calculation for target EM scattering field. However, KA or PO is only applicable to large-scale rough surface with low-rough and smooth. When the roughness increases, the coupling effects increas either. Using

Received September 21, 2013; Revised November 20, 2013; Accepted December 10, 2013

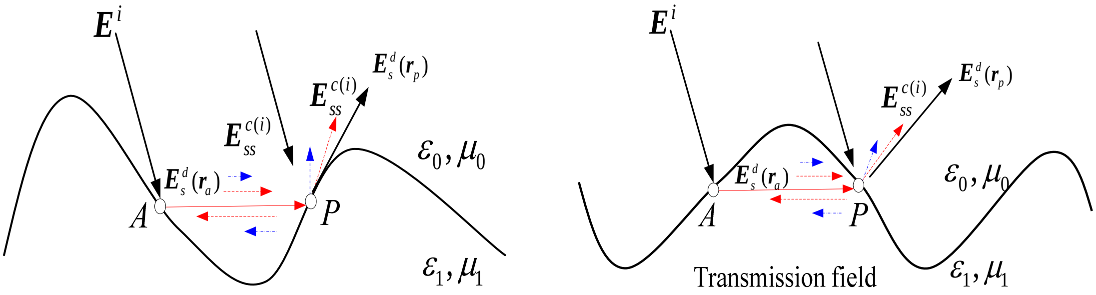
KA or PO only will not lead to reasonable results. Fung and others combined KA with SPM, putting forward the integral equation method (IEM) from [8-9] which trial scope broader in 1992. In 2003, the algorithm is improved by Chen. The improved AIEM can be used to a wider rough parameters range. It is applied to characteristics of terrain features study in [10].

Whether the traditional IEM or the AIEM, the efficiency mainly embodies in considering only one time about the effect of mutual coupling scattering field between the rough area elements. Actually, the increase of the several repeated mutual coupling scattering field between the rough area elements can't be ignored while the rough surface roughness increased. In this paper, based on the traditional algorithms of IEM, an efficient iterative integral equation method (IEM) is proposed, which can calculate the EM scattering for the dielectric target. This algorithm considers several times about the effect of mutual coupling scattering field between the rough area elements. It uses the near-field radition formular to compute the coupling field. The characteristic of grid subdivision could not affected by the influence of the dielectric constant. Therefore, IEM has an advantage over MoM when calculating the EM scattering problem for dielectric target. The mathematical theory of IEM is introduced in section 2 and then the numerical experiment proves the effectiveness of this algorithm in section 3. The conclusion is made in section 4.

2. Iterative IEM for Multiple Reflections

Suppose that the geometry surface is divided into *m* facets and $A(\boldsymbol{r}_a)$ is any one of the elements while $\mathrm{P}(\boldsymbol{r}_p)$ is the detecting point. When the incident EM wave $\boldsymbol{E}_i$ illuminates the area element, it will be formed the equivalent surface current $\mathbf{J}_{ei}$, where $i = 1, 2, 3....n$. At this time, apart from the induction to produce direct scattering $\boldsymbol{E}_{\mathrm{s}}^{\mathrm{d}}$, the coupling induction between $\boldsymbol{J}_{ei}$, will also produce the coupling field $\boldsymbol{E}_{\mathrm{ss}}^{\mathrm{c}(i)}$, as is shown in Figure 1(a). Therefore, the scattering field of any point including point *P* and *A* on the surface of the target can be represented as:

$$\boldsymbol{E}_{\mathrm{s}}^{\mathrm{s}} = \boldsymbol{E}_{\mathrm{s}}^{\mathrm{d}} + \sum_{i=1}^{n} \boldsymbol{E}_{\mathrm{ss}}^{\mathrm{c}(i)} \tag{1}$$



(a) The top rough surface field (b) The bottom rough surface field

Figure 1. Schematic Diagram of IEM Algorithm

Here, n refers to the coupling number. And $\boldsymbol{E}_{\mathrm{s}}^{\mathrm{d}}$ can becalculated according to the following formula:

$$\boldsymbol{E}_{s}^{d} = -jk_0\eta_0 \iint_s \left(\overline{\overline{\mathbf{I}}} + \frac{1}{k_0^2} \nabla \nabla \right) G_0(\boldsymbol{r}, \boldsymbol{r}') \cdot \mathbf{J}_{ei}(\boldsymbol{r}') ds' \tag{2}$$

By the formula of (2), we can get the direct scattering field $\boldsymbol{E}_{\mathrm{s}}^{\mathrm{d}}(\boldsymbol{r}_p)$ of the point *P* and the direct scattering field $\boldsymbol{E}_{\mathrm{s}}^{\mathrm{d}}(\boldsymbol{r}_a)$ of the point *A*. When the effect of the EM mutual coupling field between the area elements cannot be ignored, $\boldsymbol{E}_{\mathrm{s}}^{\mathrm{d}}(\boldsymbol{r}_a)$ is a new incident field for the area elements *P*. Then $\boldsymbol{E}_i$ should be replaced by $\boldsymbol{E}_{\mathrm{s}}^{\mathrm{d}}(\boldsymbol{r}_a)$, and then the coupling field $\boldsymbol{E}_{\mathrm{ss}}^{\mathrm{c}(1)}$ could be calculated. This process will be countinued until the coupling fields between the elements become weak. Finally, by the formulate of (1) , we can get $\boldsymbol{E}_{\mathrm{s}}^{\mathrm{s}}$.

Above is targeted for a perfect electrical conductor (PEC), the surface does not exist the equivalent magnetic current, and the scattering field calculation is relatively simple. According to the equivalence principle, there exists the equivalent electric current $\mathbf{J}_e$ and magnetic current $\mathbf{J}_m$ on the homogeneous medium target surface. They meet the demands of (3) as follows:

$$\begin{aligned} \boldsymbol{n}\times\left(\boldsymbol{E}^i+\boldsymbol{E}^s\right) &= -\mathbf{J}_m \\ \boldsymbol{n}\times\left(\boldsymbol{H}^i+\boldsymbol{H}^s\right) &= \mathbf{J}_e \end{aligned} \tag{3}$$

The scattering EM fields, $\boldsymbol{E}^s$ and $\boldsymbol{H}^s$, are listed below:

$$\boldsymbol{E}^{\mathrm{s}} = -jk_0\eta_0\iint_s\left(\bar{\bar{\mathbf{I}}}+\frac{1}{k_0^2}\nabla\nabla\right)G_0\left(\boldsymbol{r},\boldsymbol{r}'\right)\cdot\mathbf{J}_{ei}(\boldsymbol{r}')ds' - \iint_s\nabla G_0\left(\boldsymbol{r},\boldsymbol{r}'\right)\times\mathbf{J}_{mi}(\boldsymbol{r}')ds' \tag{4}$$

$$\boldsymbol{H}^{\mathrm{s}} = -j\frac{k_0}{\eta_0}\iint_s\left(\bar{\bar{\mathbf{I}}}+\frac{1}{k_0^2}\nabla\nabla\right)G_0\left(\boldsymbol{r},\boldsymbol{r}'\right)\cdot\mathbf{J}_{mi}(\boldsymbol{r}')ds' + \iint_s\nabla G_0\left(\boldsymbol{r},\boldsymbol{r}'\right)\times\mathbf{J}_{ei}(\boldsymbol{r}')ds' \tag{5}$$

The scattering characteristics of the dielectric objects are different from those of the PEC objects. For the dielectric objects, the electromagnetic wave can transmit in the inner of the objects, which will cause the transmission field. And the transmission filed could occur the multiple coupling field further. While, for the PEC objects, the total reflection occurs on the PEC surface of the target, so there is no transmission field. In this work, only the scattering above the surface of the objects is considered, therefore, not all of the transmission filed should be considered. However, when the angle between the incident direction of the EM wave and the normal direction of the target surface is an acute angle, as is shown in Figure 1(b), the contribution of the scattering field for the point P should be from the bottom space of the dielectric target. To calculate the coupling EM field, the inner surface of incidence EM field and the normal of the every area element should be relevant in the opposite direction, and the reflection angle should follow the snell's law. Here, the coupling EM field which spreads from the second half of space is the transmission field.

3. Results and Discussion

Figure 2 is a model of 3D rough surface with roughness. Let *L* = 1.0m and *σ* =0.0m, 0.2m, 0.5m, where *L* is the correlation length and σ is the root mean square (RMS) height of the rough surface. Calculate the RCS of the horizontal polarization radar random rough surface which is mentioned above. The plane incident wave's frequency is 300MHZ, along the vertical direction. Figure 3 shows the comparison of the numerical results of the proposed iterative IEM method in this paper with MoM and KA with the incident pitch angles range from -90 to 90deg. From the numerical calculation results, it is not hard to see, when the surface roughness increases, the calculation precision characteristics of IEM algorithm shows better than KA and close to MoM. This is because KA does not take into account the mutual coupling field when calculating the EM scattering of rough surface, while large calculation error will be introduced because of the higher roughness. And the error will increase by following the mutual coupling strength. In order to demonstrate the suitable for use, the contrast of the three algorithms' results displays in Figure 4 for dihedral angle and trihedral angle model with the incident pitch angles range from 0 to 90deg. The calculation results showed that IEM calculation results are more consistent than KA with MoM.

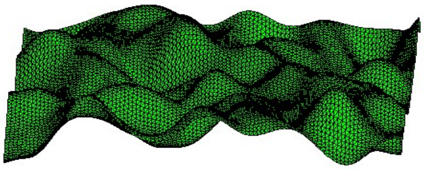
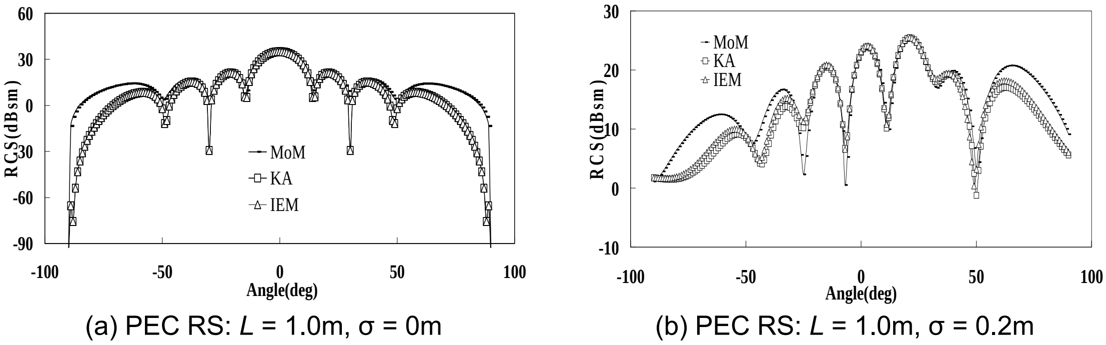
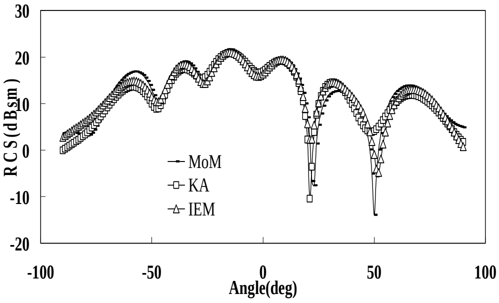


Figure 2. 3D Rough Surface Model



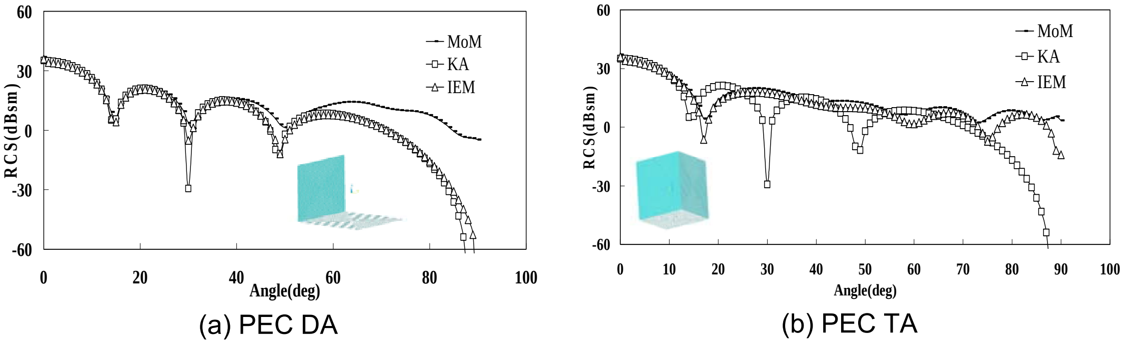
(a) PEC RS: L = 1.0m, σ = 0m

(b) PEC RS: L = 1.0m, σ = 0.2m



(c) PEC RS: L = 1.0m, σ = 0.5m

Figure 3. The Numerical Contrast of MoM, KA, IEM for PEC Rough Surface (RS)

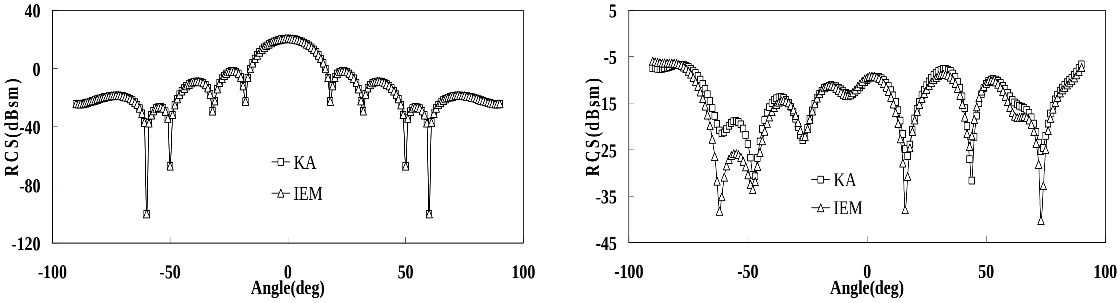


(a) PEC DA

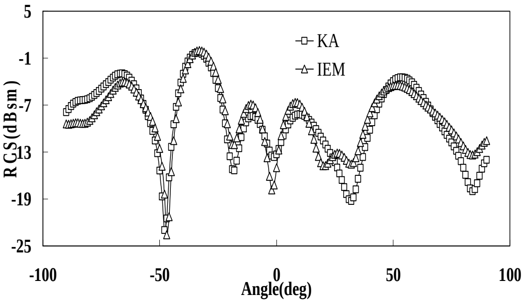
(b) PEC TA

Figure 4. The Numerical Contrast of MoM, KA, IEM for PEC Dihedral Angle (DA) and Trihedral Angle (TA)

L = 1.0m is changeless, by *σ* = 0.0m, 1.0m, 1.5m three groups of dielectric rough surface on the background of ocean as an example, while the relative dielectric constant was 4.0. Assume that the incident wave for horizontal polarization is pyramidal wave and the width is 2.0λ. The incident direction remained unchanged just like the previous example. It shows the numerical results of KA and IEM with the different surface roughness in Figure 5. It's not difficult to see, when the surface roughness increases, the difference of the numerical results between KA and IEM is greater. As a result, the coupling field about the area elements gradually enhancement is not allowed to ignore. IEM is also applicable to other dielectric model. Figure 6 shows the numerical results of the dihedral angle and trihedral angle in the same conditions.

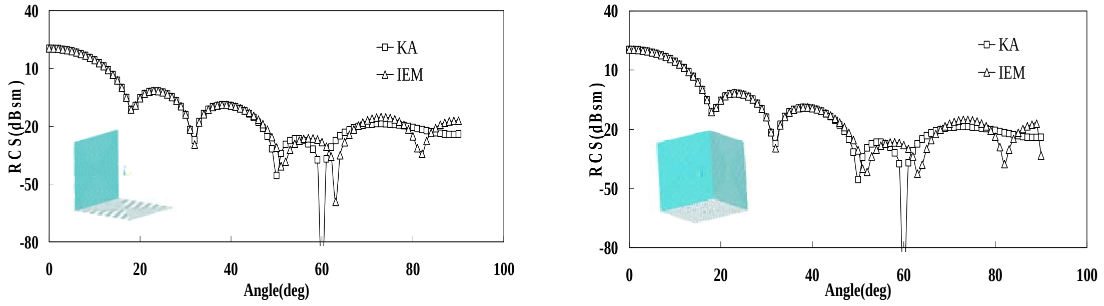


(a) Die-RS: *L* = 1.0 m, σ = 0.0 m (b) Die-RS: *L* = 1.0 m, σ = 1.0 m



(c) Die-RS: *L* = 1.0 m, σ = 1.5 m

Figure 5. The Numerical Contrast of KA, IEM for Different Dielectric Rough Surface(Die-RS)



(a) Die-DA (b) Die-TA

Figure 6. The Numerical Contrast of KA, IEM for Dielectric Dihedral Angle (Die-DA) and Trihedral Angle (Die-TA)

4. Conclusion

The proposed IEM has more extensive applicability and higher accuracy due to considering the coupling field between the target surface area elements. This proposed algorithm applies to calculate the target scattering field not only for the PEC model computation, but also for dielectric model computation. The numerical experiment results show that even for a smooth PEC target EM scattering calculation, while KA produces larger calculation error, IEM calculation error is small with MoM, and the calculation results are in good agreement. Also, IEM are not affected by the influence of the dielectric constant while meshing, so it's less time-consuming and more advantage to calculate the target EM scattering problem. As for large scale dielectric rough surface model, its roughness do not meet the applicable condition of KA, so the IEM is more applicability than KA.

Acknowledgements

The authors would like to thank the support of National Science Foundation of China (No: 61061002, 61261005); Open Project of State Key Laboratory of Millimeter Wave (No: K201325, K201326); Youth Foundation of Jiangxi Provincial Department of Science and Technology (No: 20122BAB211018); Youth Foundation of Jiangxi Provincial Department of Education (No: GJJ13352, GJJ13321, GJJ13320).

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