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The Design of PID Controller Based on Hopfield Neural Network

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Abstract

With the complexity increase in industrial production process, the traditional Proportion-Integration-Differentiation (PID) control can not meet the requirements of the control system performance. Because neural network has the ability of adaptive, self-learning and nonlinear function approximation, control equality of system is improved if it is combined with traditional PID. In the paper, Hopfield neural network based on Hebb rules is used to identify the parameters of system, and then the state space model is established. Hopfield Neural network has the function of optimal calculation, PID controller based on Hopfield neural network is designed for a system can optimize the parameter of PID in real-time and improve control accuracy. Simulation result show the performance index is greatly improved.

Keywords: *Hopfield neural network, control system, PID*

1. Introduction

With the improvement of the level of industrial production, the process becomes more and more complicated, high control performance of system is required increasingly, especially when the system involves parameter uncertainties, traditional PID control can not meet the requirements of the control system [1]. Neural network has the properties of self-learning and self-organizing, it also can approximate any nonlinear function [2-4]. If neural network is combined with traditional PID controller, applied range of PID can be enlarged, at the same time, control effect can be improved greatly. In many neural networks, Hopfield network has good performance in optimal calculation [5-9]. Feedback connection is selected among neurons, time-delay between the inputs and outputs is considered, the dynamic process of the system can be described. In this paper, system parameters can be identified by continuous Hopfield neural network (CHNN). Furthermore, CHNN is used to optimize control parameters of PID in real-time. For controlled process, the PID controller based Hopfield neural network can be designed to realize effective control.

2. Continuous Hopfield Neural Network (CHNN)

2.1. Network Model

Hopfield network is a highly interconnected collection of simple processing neurons. The physicist Hopfield designed model used by analog circuit, the structure of Hopfield model is shown in Figure 1.

In Figure 1, The resistance R_i and capacitance C_i in parallel simulate time- delay characteristics of biological neurons. Operational amplifier simulates nonlinear properties of neurons, that is $v_i = f(u_i)$, where u_i is the internal state of ith neuron, whose output is v_i, $f(\cdot)$ is the activation function of neuron, which is continuously differentiable and $f'(\cdot) > 0$; I_i is the external input; w_{ij} is the connection strength between jth neuron and ith neuron, $w_{ij} = w_{ji}$; N is the number of neurons in the Hopfield network.

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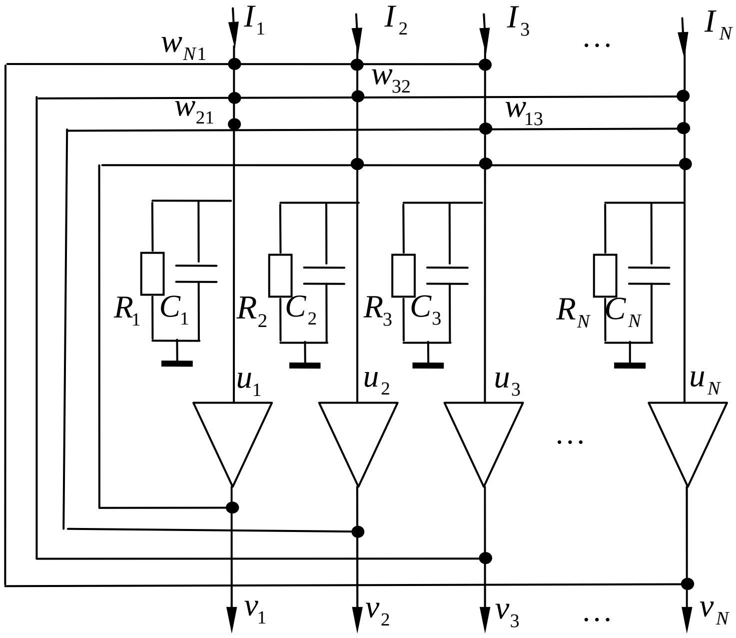

Figure 1. The Structure of CHNN Model

According to Kirchhoff's laws, the dynamics formulation of CHNN can be obtained:

$$C_i \frac{du_i}{dt} = \sum_{j=1}^{N} w_{ij} v_j - \frac{u_i}{R_i} + I_i \tag{1}$$

The activation function of neuron $f(\cdot)$ is selected in the following:

$$v_i = f_i(u_i) = \frac{1 - e^{-u_i/u_0}}{1 + e^{-u_i/u_0}} \tag{2}$$

Where u_0 represents the initial value of input voltage. The dynamic process of CHNN is described by Equation (1) and Equation (2)

2.2. Energy Function of Network

The stable analysis of Hopfield network bases on energy function, training of Hopfield network is to minimize energy function E ,

$$E = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij} v_i v_j - \sum_{i=1}^{N} v_i I_i + \sum_{i=1}^{N} \frac{1}{R_i}\int_0^{v_i} f_i^{-1}(v)dv \tag{3}$$

If $w_{ij} = w_{ji}$, the time derivative of energy function E is:

$$\frac{dE}{dt} = \frac{dE}{dv_i}\frac{dv_i}{dt} = -\sum_{i=1}^{N} C_i \frac{d[f^{-1}(v_i)]}{dv_i}\left(\frac{dv_i}{dt}\right)^2 \le 0 \tag{4}$$

So the equilibrium point of asymptotic stability is the minimum value of energy function. Though it will always settle to a stable state from any initial state, a Hopfield network usually gets trapped into local minimum states.

In the high-gain limit, the energy function can be approximated to Equation(5).

$$E = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij} v_i v_j - \sum_{i=1}^{N} v_i I_i \quad (5)$$

Through a simple analysis of the energy function, the method of solving optimization problems used by CHNN can be gotten. Objective functions of the problem is converted to energy function of network and the variable of problem corresponds with the status of network, the optimal solution of the problem has been obtained the optimal value could be solved as the energy function converges to the minimal value of network.

3. System Identification Based on CHNN

The state equation of linear discrete system is formulated in the following,

$$\begin{aligned} &x(k+1) = Ax(k) + Bu(k) \\ &x(k) = [x_1(k), x_2(k), \cdots, x_n(k)] \\ &A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nm} \end{bmatrix} \end{aligned} \quad (6)$$

Where $u \in R^m$ represents system input, $x \in R^n$ represents system state vector, $A \in R^{n\times n}$ is the state-transition, and $B \in R^{n\times m}$ is input matrix, whose estimated values is represented by $\hat{A}$ and $\hat{B}$. The purpose of system identification is to estimate the value of each element in matrix [10-12].

Because there are $n\times n + n\times m$ elements in matrix $A \in R^{n\times n}$ and $B \in R^{n\times m}$, Hopfield network require $n\times n + n\times m$ neurons in order to estimate the elements, in which each steady state output corresponds with the estimation of matrix A and B.

Assume the identification model is:

$$\begin{aligned} &\hat{x}(k+1) = \hat{A}x(k) + \hat{B}u(k) \\ &\hat{A} = \begin{bmatrix} \hat{a}_{11} & \cdots & \hat{a}_{1n} \\ \vdots & \ddots & \vdots \\ \hat{a}_{n1} & \cdots & \hat{a}_{nn} \end{bmatrix} \quad \hat{B} = \begin{bmatrix} \hat{b}_{11} & \cdots & b_{1m} \\ \vdots & \ddots & \vdots \\ \hat{b}_{n1} & \cdots & \hat{b}_{nm} \end{bmatrix} \end{aligned} \quad (7)$$

Let Equation (7) subtract Equation (6), identification error $e(k+1)$ could be obtained,

$$\begin{aligned} e(k+1) &= x(k+1) - \hat{x}(k+1) \\ &= x(k+1) - \hat{A}x(k) - \hat{B}u(k) \end{aligned} \quad (8)$$

Define the objective function:

$$\begin{aligned} E(k) &= \frac{1}{2} e^T(k+1) e(k+1) \\ &= \frac{1}{2}[x(k+1) - \hat{A}x(k) - \hat{B}u(k)]^T [x(k+1) - \hat{A}x(k) - \hat{B}u(k)] \end{aligned} \quad (9)$$

Parameters to be indentified correspond with outputs of Hopfield network, that is:

$$V(k) = [v_1(k), v_2(k) \cdots v_{n\times n+n\times m}(k)]^T$$
$$= [\hat{a}_{11}, \cdots, \hat{a}_{n1}; \hat{a}_{n1} \cdots \hat{a}_{nn}; \hat{b}_{11} \cdots \hat{b}_{1n}; \hat{b}_{n1} \cdots \hat{b}_{nm}]^T$$

After objective function is transformed appropriately, it can be connected with energy function Equation (5), and then connection weight W and threshold I are determined. For second linear discrete system:

$$w = \begin{bmatrix} x_1^2 & x_1x_2 & 0 & 0 & u_1x_1 & 0 \\ x_1x_2 & x_2^2 & 0 & 0 & u_2x_2 & 0 \\ 0 & 0 & x_1^2 & x_1x_2 & 0 & u_1x_1 \\ 0 & 0 & x_1x_2 & x_2^2 & 0 & u_2x_2 \\ u_1x_1 & u_2x_2 & 0 & 0 & u^2 & 0 \\ 0 & 0 & u_1x_1 & u_2x_2 & 0 & u^2 \end{bmatrix}$$
$$I = \begin{bmatrix} I_1 & I_2 & I_3 & I_4 & I_5 & I_6 \end{bmatrix}$$
$$= \begin{bmatrix} x_1(k+1)x_1(k) & x_1(k+1)x_2(k) & x_2(k+1)x_1(k) & x_2(k+1)x_2(k) & x_1(k+1)u(k) & x_2(k+1)u(k) \end{bmatrix}$$

The activation function of neuron is:

$$v_i = f(u_i) = k\frac{1-e^{-\beta u_i}}{1+e^{-\beta u_i}} = \frac{2k}{1+e^{-\beta u_i}} - k \quad (10)$$

Where $\beta = \frac{1}{u_0}$, k represents gain. The following formulation can be derived from above equation:

$$1+e^{-\beta u_i} = \frac{2k}{V_i + k} \quad (11)$$

The time derivative of output of network v_i is:

$$\frac{dv_i}{dt} = \frac{dv_i}{du_i}\cdot\frac{du_i}{dt}$$

Set $C_i = c$, then:

$$\frac{dv_i}{du_i} = \frac{2k\beta(-e^{-\beta u_i})}{(1+e^{-\beta u_i})^2} = \frac{\beta(k^2 - V_i^2)}{2k}$$
$$\frac{du_i}{dt} = \sum_{j=1}^{N} w_{ij} + I_i, i = 1, 2, \cdots N$$
$$\frac{dv_i}{dt} = \frac{dv_i}{du_i}\cdot\frac{du_i}{dt} = \frac{\beta(k^2 - V_i^2(k))}{2kc}\left(\sum_{j=1}^{N} w_{ij} + I_i\right)$$
$$i = 1, 2, \cdots N$$

Discretization is shown as:

$$v_i(k+1) = v_i(k) + \frac{\beta(k^2 - v_i^2(k))}{2kc}\left[\sum_{j=1}^{N} w_{ij} v_j(k) + I_i(k)\right]\Delta t \tag{12}$$

A new variable is introduced,

$$\alpha = \frac{\beta \Delta t}{2kc}$$

The formula above can be changed into,

$$v_i(k+1) = v_i(k) + \alpha[\sum_{j=1}^{N} w_{ij} v_j(k) + I_i(k)](k^2 - v_i^2) \tag{13}$$

If we bring the weight w_{ij} and threshold I_i into the above equation, assuming $v_i(0) = 0$, we can get the parameter estimation of the system, when $v_i(k)$ converge to balance point by iteration.

4. PID Control Based on CHNN

PID control system structure based on CHNN is shown as in Figure.2,

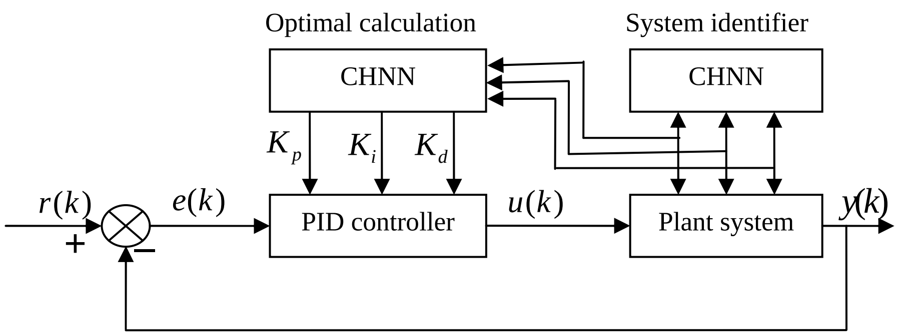

Figure 2. PID Control Structure Based on Continuous HNN

Figure 2 shows that the system structure has two Hopfield neural networks. One HNN is used to identify the parameters, when the state space model of controlled plant is uncertain sometimes, another one is used to optimize the parameters of traditional PID controller, that is the factor of proportion K_p, integration K_i and differentiation K_d, define network output is:

$$V = [v_1 \; v_2 \; v_3 \;]^T = [K_p \; K_i \;\; K_d \;]^T$$

Define the objective function of optimization as follow,

$$\begin{aligned} E(k) &= \frac{1}{2} e^2(k+1) \\ &= \frac{1}{2}\left(r(k+1) - C\left(Ax(k) - Bu(k)\right)\right)^T \left(r(k+1) - C\left(Ax(k) - Bu(k)\right)\right) \end{aligned} \tag{14}$$

Where:

$$e(k+1) = r(k+1) - y(k+1) = r(k+1) - C\left(Ax(k) + Bu(k)\right)$$

From the above Equation (5) and Equation (14), connection weight W and threshold I are determined. Assuming:

$$r(k) = 1, x^T(0) = \begin{bmatrix} 0 & 0 \end{bmatrix},$$
$$V^T(0) = [v_1(0) \quad v_2(0) \quad v_3(0)]$$
$$= [0 \quad 0 \quad 0]$$

The output of HNN can be obtained from Equation (12), when the output of network achieved permitted range.

5. Simulation

State control of order linear discrete system is described as:

$$x(k+1) = Ax(k) + Bu(k)$$
$$y = Cx(k)$$
$$A = \begin{bmatrix} 0.368 & 0 \\ 0.632 & 1 \end{bmatrix} B = \begin{bmatrix} 0.632 \\ 0.368 \end{bmatrix} C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

When the step function is selected as input of the discrete system, the controller is designed based on HCNN, simulation result is shown from Figure 3 to Figure 7.

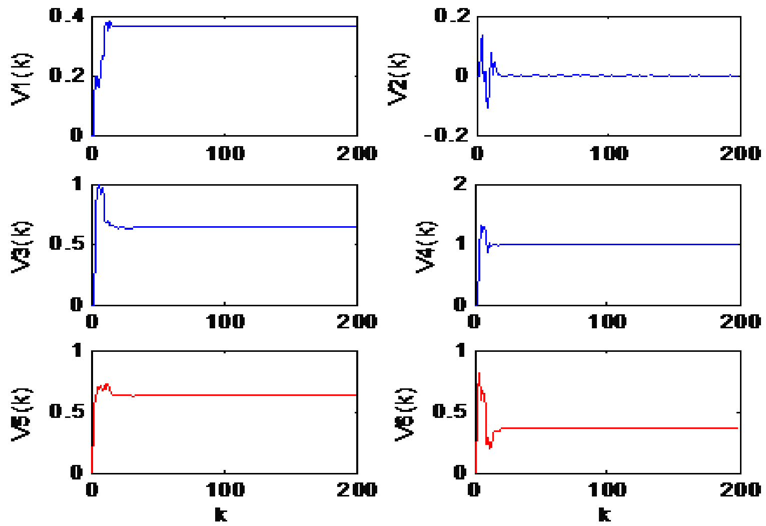

Figure 3. Parameter Estimation of System Model

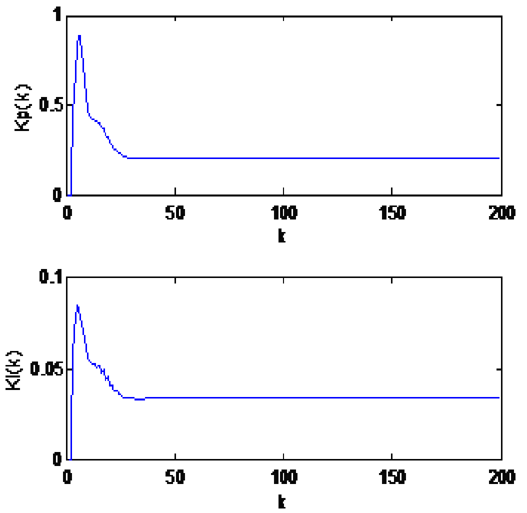

Figure 4. Optimization Process Parameter K_p and K_i

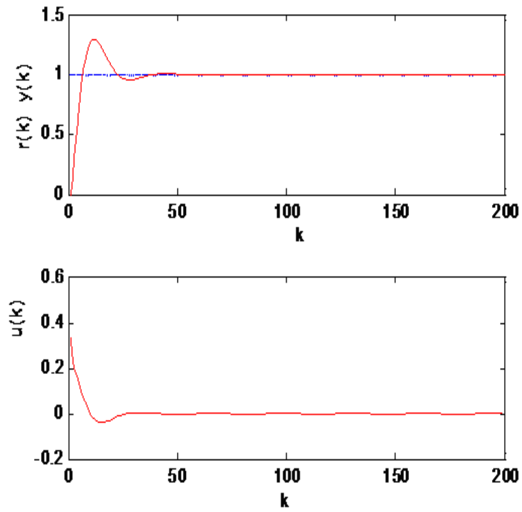

Figure 5. Control Resunlt by using Optimized K_p and K_i

To illustrate control results of the neural network PID control is better than the traditional PID control, we put the same PI parameters ($k_p = 0.2096$, $k_i = 0.0341$) on the traditional PID and make simulation on the same system, the results are shown in Figure 9 and Figure 10.

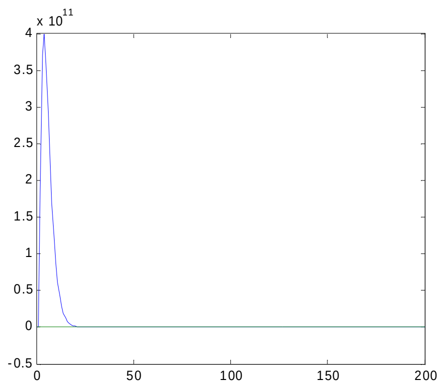

Figure 6. Step Response of the Simulation System

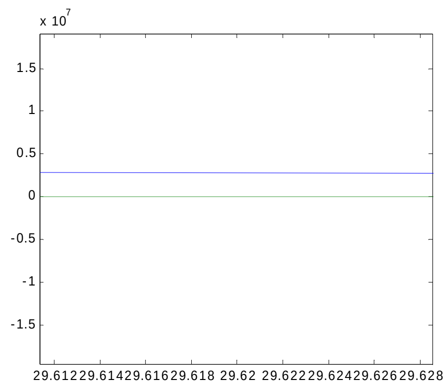

Figure 7. Enlarged Partial Response of Traditional PID

Simulation result shows that control effect based on Hopfield neural network is better than that based on traditional PID. The specific merits include small overshoot, fast response time, less time adjustment, high precision control and etc., it doesn't rely on a fixed system model. Comparison between the two controllers is shown in Table 1.

Table 1. Comparison of Performance Index

Controller type	Regulation time (step)	Steady-state error
Traditional PID controller	35	0.6
PIDcontroller based on HNN	30	0.1

6. Conclusion

A PID controller based on HNN is proposed in this paper. Hopfield neural network based on Hebb rules can converge rapidly, it used to identify and optimize the parameters of system PID, One HNN is used to identify the parameters, when the state space model of controlled plant is uncertain, another one is used to optimize the parameters of traditional PID controller, system parameters can be well optimize in real-time. Simulation result show the performance index of control system is greatly improved.

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