# **Multi-Scale Harris Corner Detection Based on B-Spline**

# Zhu Wenqiu\*, Xu Keke

School of Computer and Communication, HuNan University of Technology, HuNan ZhuZhou, China Taishan Road West, Tianyuan District, Zhuzhou City, Hunan Province, 0731-22813313 \*Corresponding author, e-mail: wnqiu\_zhu@126.com

# Abstract

The existing Harris corner detection algorithm using mostly Gaussian low-pass filter to smooth image, and there are some phenomena about loss of information and location of the corner offset in images, at the same time, the single-scale Harris corner detection algorithm does not have the scale invariance. B-spline function converges to a Gaussian function, therefore we combined B-spline wavelet multi-scale theory and Harris, and proposed multi-scale Harris corner detection method based on B-spline. Firstly, we used B-spline function to smooth filter image at different scales. Secondly image and B-spline convolution template were calculated by convolution operator. Finally, we extracted alternative corner from the different scale images, and searching for the extreme value of scale space as the location and characteristics scale of the feature points within the search window template that has belonged to a fixed size at the center of Harris corner. The experiments show that the proposed method not only maintains the good performance of Harris operator, but also has scale invariance.

Keywords: B-spline function, multi-scale, feature detect, invariance

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#### 1. Introduction

Corner detect is an important operator used frequently in image and computer vision, which is applied in important fields, such as the recognition of the object and scene, image retrieval, stereo matching, image stitching, image registration and target tracking. Therefore, the performance of the corner detection to the subsequent treatment or even the entire processing system has a great impact. The corner points, have the dramatic change of the brightness of the two-dimensional image points of the edges of the image which have a maximum curvature, which have enough information that can be provide adequate constraint for the visual processing, and the corner point number compare to the total number of image pixel is small, therefore, which can greatly improve the calculation speed. The current corner detection method is divided approximately into three class: (1) the corner detection based on template [1-3], (2) the corner detection based on the image edge of image [4-7], (3) the detection method based on the change in brightness [11-15].

There are many corner detection methods used frequently, such as Moravec algorithm [8, 9], Harris algorithm [10, 11], MIC algorithm [14], SUSAN algorithm [15] and Log-Gabor algorithm [16] et al. Where Harris operator own good stability and high precision of corner detection in the L-shaped, is the most widely operator used currently. According to the research of Harris algorithm, the disadvantages of the positioning accounting is not high existed in it, thus it cannot meet the accuracy requirements when precise positioning is needed. In addition, traditional Harris operator, has used Gaussian filter, whose computation speed is relatively slow. There are influences that the information of corner was loss and its position was offset, that clustering was come when corner was extracted. Harris operator that is sensitive to noise has not the scale invariance, According to this deficience of Harris algorithm; this paper proposed a new algorithm of multi-scale Harris corner detection of B-spline based on local image structure. Firstly, we introduced the B-spline function that has the features of data fitting and the low-pass characteristics to the new algorithm. Secondly, we used B-spline to interpolate in local image. Thirdly, we got the sub-pixed coordinates of corner through the secondary angle detection on the image after the interpolation. Finally, we extracted the local extreme value of feature points through the window search algorithm which is fast and partial.

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# 2. The Local Structure Tensor of Image

Local structure tensor, proposed firstly by W.Forstner and E.Gulch [17], is a positive semi-definite matrix that described through the gradient operator. Although the gradient direction of the image also contains a partial structure information, but the average in the neighborhood of each pixed was gained when we calculate the partial direction of the image. If a local neighborhood of a pixel contains simultaneously the gradient of the rising edge and the falling edge (such as two sides of a thin edge), the gradient direction of the positive and negative were offset, therefore, it cannot show its local direction. The local structure tensor avoid this deficiency in the gradient direction, meanwhile can distinguish between a variety of different structures in the image. We let here I(i,j) represents the gray of the image midpoint (i,j), thus the point of the gradient structure tensor is expressed as:

$$GST(i, j) = \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$
(1)

In the above equation  $I_x = \frac{\partial I}{\partial x}$ ,  $I_y = \frac{\partial I}{\partial y}$ ,  $I_x^2 = I_x I_x$ ,  $I_y^2 = I_y I_y$ . The original image

was smooth preprocessed by B-spline function in order to reduce the impact of noise on the edge detection. At the same time, the convolution operation between the gradient structure tensor and B-spline function when the changes of the image gray in its neighborhood, the partial structure tensor after smoothing is:

$$LST(i, j) = B \otimes \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$
(2)

We can get feature vector {u,v} of the partial structure tensor and characteristic values,  $\{\lambda_u, \lambda_v\}(\lambda_u \ge \lambda_v)$ , of their own respective corresponding through principle compact analysis that applied to its local gradient vector. The corresponding to the maximum eigenvalue  $\lambda_u$ , eigenvector u represents the direction of a large gray change, and vice versa. Eigenvalue  $\{\lambda_u, \lambda_v\}$  not only reflects the gray variation at its own respective direction , and also contains the

# 3. Harris Corner Extraction of B-spline Multi-scale

Harris corner detection operator has some invariance of gray change and rotation, but it does not have the scale invariance. Therefore, the method of multi-scale Harris corner extraction based on B-spline function was proposed.

#### 3.1. Harris Corner

Harris operator use autocorrelation function to detect corner and use first-order partial derivatives to describe the change of brightness. The differential operator can reflect gray changes of pixel in any direction, therefore it can effectively distinguish between the corner and edge. Harris matrix:

$$M = G(\sigma) \otimes \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$
(3)

Where  $G(\sigma)$  is Gaussian two-dimensional discrete functions; Ix and Iy are gradient. The two eigenvalues of the matrix M is the main curvature of the self-correlation function, therefore, the two eigenvalues of M is positive, and the maximum value in the local area of the feature point as

the center are obtained. Therefore, all the feature points of an image can be evaluated by the following function:

$$R = det(M) - k \cdot tr^{2}(M) \qquad (4)$$

Where det(M) is the determinant of M, and tr is the trace of M, and k is a constant whose value is generally taken between 0.04 and 0.06 in terms of the experience. R is an angle point value of the corresponding pixel point in the diagram, when a corner in its neighborhood is the largest one and is greater than a threshold R0, thus that point as a feature point.

## 3.2. B-spline Function

Definition: n order central B-spline function under conditions of equidistant and the singlet node is defined as follows:

$$\beta^{n}(x) = \sum_{j=0}^{n+1} \frac{(-1)^{j}}{n!} \cdot {\binom{n+1}{j}} \cdot {\binom{n+1}{2}} \cdot {\binom{x+\frac{n+1}{2}}{j}}^{n} \cdot u {\binom{x+\frac{n+1}{2}}{j}} , (x \in R) \quad (5)$$

$$u(x) = \begin{cases} 0, x < 0 \\ 1, x > 0 \end{cases}$$

Where  $\lfloor 1, x \ge 0$  is a unit step function. The function can be obtained by spline function convolution:

$$\beta^{n}(x) = \overbrace{\beta^{0} \ast \beta^{0} \ast ... \beta^{0}(x)}^{n+1\&} \quad (6)$$

According to the nature of the B-spline function and the requirements of the smooth function, it is not difficult to prove that  $\beta^n(x)$  meet the requirements of the smooth function. When higher order B-spline function, its image is more approximate the Gaussian function. The smoothing and approximation of a smoothing function is contradiction. Therefore smooth performance, with increasing values of n with regard to  $\beta^n(x)$ , also will be changed to the better, and to eliminate the noise, but at the same time support interval change, whereby the processing time also increases. When the value of n is relatively small, the approximation property of  $\beta^n(x)$  is better, thus the edge point (singular point) was positioned rightly, the ability de-noising is relatively weak [18].

It has been proven that optimal smoothing filter impulse response function is a cubic Bspline function or approximate Gaussian function, therefore chosen here cubic B-spline function as a smooth function.

When n=3, we can get the expression of  $\beta^3(x)$  from (5) as follows:

$$\beta^{3}(x) = \begin{cases} \frac{2}{3} - x^{2} + |x|^{3}/2, & 0 < |x| \le 1\\ (2 - |x|)^{3}/6, & 1 < |x| \le 2\\ 0, & \text{otherwise} \end{cases}$$
(7)

# 3.3. B-spline Multi-scale Analysis

The main idea of the scale-space theory is to get the representative sequence of the scale space of the multi-scale in image through the scale transformation of the original image. We extracted the main contour of space scale from this sequences and use it as a feature vector, and to finish the detection of edge and corner and the extraction of feature from different resolution. Generally believed that larger scale can reliably eliminate false detection and testing to the true corner, but the corner is not easy to accurately locate. In contrast, the real feature

corner positioning is more accurate in a smaller scale, but the proportion of false detection will increase. The results that 3 order continuous B-spline was expanded through a scale factor is expressed as  $\beta_s^{3}(x)$ , that is:

$$\beta_s^3(x) = \frac{1}{s}\beta^3(\frac{x}{m}) \tag{8}$$

Samely, we can get the expression from the formula (5), that is:

$$\beta_{s}^{3}(x) = \frac{1}{6s^{3}} \begin{cases} -2 |x|^{3} + 12 |x|^{2} s - 24 |x| s^{2} + 16s^{3}, & s \le |x| \le 2s \\ 6 |x|^{3} - 12 |x|^{2} s + 8s^{3}, & 0 \le |x| \le s \\ 0, & otherwise \end{cases}$$
(9)

We introduce here scale parameter  $s = 2^{j}$  to two-dimensional image, and let

 $\theta_s(x, y) = \frac{1}{s^2} \theta\left(\frac{x}{s}, \frac{y}{s}\right)$ . The smoothing of the image f(x, y) was finished through making convolution  $f(x, y) \otimes \theta_s$  between f(x, y) and  $\theta_s(x, y)$  on different scale, as follows:

$$(f \otimes \theta_s)(x, y) = \int_R \int_R f(x - u, y - v)\theta_s(u, v)dudv \quad (10)$$

The given digital image that can be seen as a uniform sampling of surface was described approximately from the following surface:

$$U(x, y) = \sum_{(i,j)\in\beta_{k,l}(x,y)} f(i,j)\beta_k(x-i)\beta_l(y-j)$$
(11)

Assuming take the size of the neighborhood  $3 \times 3$ , according to the local property of  $\beta^{\,_3}(x)$  , we can deduce discrete expression of U(x,y) .

$$U(k,l) = \sum_{(i,j)\in\beta_{k,l}(x,y)} f(i,j)\beta_k(x-i)\beta_l(y-j)$$
  
=  $Af(k-1,l-1) + Bf(k-1,l)$   
+  $Cf(k-1,l+1) + Bf(k,l-1)$   
+  $Df(k,l) + Bf(k,l+1)$   
+  $Cf(k+1,l-1) + Bf(k+1,l)$   
+  $Af(k+1,l+1)$  (12)

Where  $A = C = \frac{1}{36s^3}$ ,  $B = \frac{1}{9s^3}$ ,  $D = \frac{4}{9s^3}$ , the above formula can be expressed as the convolution of the original image and the B-spline template. When the filter window size is  $3 \times 3$ ,

we can get the B-spline templates as follows:

$$\begin{bmatrix} 1/36 & 1/9 & 1/36 \\ 1/9 & 4/9 & 1/9 \\ 1/36 & 1/9 & 1/36 \end{bmatrix}$$
(13)

Samely, we can get the convolution template at the x-direction and the y-directional derivative as follows:

 $\begin{bmatrix} -1/12 & -1/3 & -1/12 \\ 0 & 0 & 0 \\ 1/12 & 1/3 & 1/12 \end{bmatrix} and \begin{bmatrix} -1/12 & 0 & -1/12 \\ -1/3 & 0 & -1/3 \\ 1/12 & 0 & 1/12 \end{bmatrix} (14)$ 

# 3.4. The Multi-scale Extraction of Image

We Pre-defined a set of scales  $s = 2^{j}$  (j=0,1,2,3,4) using the concept of the B-spline function scale space. In order to make Harris operator scale invariance, we combined Harris corner detector and B-spline scale space.

$$M_{s_j} = 36s_j^{3} \begin{bmatrix} I(s_j)_x^2 & I(s_j)_x I(s_j)_y \\ I(s_j)_x I(s_j)_y & I(s_j)_y^2 \end{bmatrix}$$
(15)

Where  $I(s_j)$  is a image smoothed by three B-spline function  $\theta_s(x, y)$ , which is different from that it did not make convolution of  $\theta_s(x, y)$  to image by  $I_x I_x$ ,  $I_x I_y$ , and  $I_y I_y$ . Its procedure as follows: (1) The scale of scale spatial of angle points is determined entirely by the S; (2) Which avoided excessive blurring the image from multiple convolution operation, and resulted in large-scale space corner is difficult to extract.

Harris feature point can be defined as a maximum value of the local area, if the two feature values  $\{\lambda_u, \lambda_v\}(\lambda_u \ge \lambda_v)$  of the local structure tensor are large enough, we detected the pixels and made as corners. In order to make the simple computation, we define on the scale  $s_j$  the response function of corner:

$$Crn(s_j) = \frac{\det(M_{s_j})}{tr(M_{s_j}) + \varepsilon} \quad (16)$$

Where  $\mathcal{E}$  is very small number because it can make its denominator unequal to zero.

If  $Crn(s_i)$  is larger than the given threshold value  $T_{s_i}$ , thus we thought it as the corner

of the image. Compared to the response function  $R = \det(M) - k \cdot tr^2(M)$  proposed from the Harris corner detection, meanwhile avoiding the selection of the parameter k, and reducing the randomness of selection of k, and therefore more practical.

# 3.5. To Determine the Direction of the Corner Points

Wu used the distribution characteristics of gradient direction of corner's neighborhood pixel for each corner point specified direction. For the corner points on the image f (x, y), the coordinates of the neighborhood of (x, y) at the direction is calculated as follows:

$$\varphi(x, y) = \tan^{-1} \left[ \frac{f(x, y+1) - f(x, y-1)}{f(x+1, y) - f(x-1, y)} \right]$$
(17)

# 3.6. Algorithm Implementation

Implementation steps of the proposed algorithm as follows

- 1) We used B-spline function on different scale  $s_i$  to smooth the original image.
- 2) According to the pre-proposed threshold  $T_{s_i}$ , we used the formula (15) and (16) to

calculate the candidate feature corners, and marked them in the array C(i,j). Where C(i, j)=1 is a candidate corner.  $i \in M$ ,  $j \in N$ , M, N are respectively the width and height of the image block.

3) The precise positioning of the location and scale of the corner. Because candidate points of the position of the space does not necessarily become a candidate point on the scale space. Therefore, we can find the value of the point scale through the scale space search.

a) At each scale, first, the image can be divided into blocking in accordance with the size of the m \* n. We detected the corner exist in each of image block, and stored it in M [Sum], where the Sum for the size of an array, i.e., the number of angular point.

b) The value of the corner in descending order, whose top 25% of corner were made as the candidate corner.

c) To each conner, we used local and non-maxima suppression method which can improve positioning accuracy to get the approximately corner, under the condition that the size of B-spline convolution kernel is equal to the non-maxima suppression window's on that scale. If there is not only a point, where we made the maximum point as a corner and got the final corner on that scale.

d) Starting at the second scale, we checked every corner of the current scale to see whether it appeared or not on the previous scale. If there is not, it is reserved; If there is, then eliminated.

4) Through the calculation of all corners, we recorded the coordinates of all corners, and used graphics to mark the location of corners in image.

# 4. Experimental Results and Analysis

In this section, we used experiments to test the effectiveness of the algorithm proposed. Figure 1 shows the results of the corner detection algorithm that was applied to gray scale and color images under different lighting conditions. The experimental results show that, in terms of light conversion, the proposed algorithm can maintain a certain invariance, where the detected results of the color image is better.



Figure 1. The Result of Corner Detection of Four Images in Different Illumination

(a) of Figure 2 is a corner point extracted from an original image; (b), (c) and (d) are the results of the corner points extracted starting from different scales. It can clearly be seen that the effect of extracting corner whose detection algorithm exist in different scales, and then to verify the algorithm is able to detect the corner in a larger scale and position precisely the corner in a small scale.



Figure 2. The Result of Corner Detection of Four Images in Scale-variant

In Figure 3 (a) is a image rotated clockwise 45° from the original image and extracted corner;(b) is a image rotated clockwise 45° from the original image counterclockwise and extracted of the corner points; (c) is corner image rotated 90° from the original image counterclockwise and extracted; (d) is corner image rotated 90° from the original image extracted corner. As can be seen, the algorithm has rotation invariance, which is that image rotation is able to more accurately extract the corner.



Figure 3. The Result of Corner Detection of Four Images in Rotation

# 5. Conclusion

Corner extraction have important applications in the field of image registration, image stitching, pattern recognition. And so we proposed new multi-scale corner detection algorithm based on B-spline function of Harris. In order to improve the detection performance of the corner detection operator, this paper introduced the idea of multi-resolution analysis to single-scale Harris corner detection algorithm, and build a new corner detection algorithm framework-B-

spline multi-scale Harris corner detection. The paper presents a set of experimental data, and analyzed the results. Experimental results show that the algorithm is a effective corner detection algorithm, with good invariance, strong noise immunity and high positioning accuracy. But the use of multi-scale thinking has increased the amount of calculation, need more time to complete the testing process. One of the future objectives is to find a more effective corner detection algorithm, and then analyze quantitatively effectiveness of its algorithm. The other is to study a new invariant feature description method, and use the proposed algorithm to apply for multivideo image stitching.

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