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Joint Optimization of Sensing Time and Fusion Rules for Cognitive Radios

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Abstract

In Cognitive Radios, sensing-time and fusion rules affect the performance of spectrum sensing when cooperative sensing is applied. Besides, the more unlicensed users are involved in cooperative sensing, the higher spectrum utilization the channel can achieve, while from the unlicensed users' perspective, the lower average throughput the unlicensed users can obtain. In this paper, we explore the issue on the sensing-time and the fusion rules to optimize the average throughput of the unlicensed users under the constraint that the licensed users are sufficiently protected. At first, we formulate this issue as an optimization problem, and showed the unimodal characteristics of the unlicensed users' average throughput as a function of the sensing-time and the fusion parameter. Then a numerical optimization algorithm is proposed to obtain the optimal solution. At last, by theoretical analysis and performance comparisons we derived the optimal fusion rule selection scheme under different scenarios. Computer simulations show that significant improvement in the average throughput of unlicensed users is achieved when the sensing-time and the fusion rule are jointly optimized.

Keywords: *cognitive radios, sensing-time, fusion rule, spectrum utilization*

1. Introduction

Spectrum sensing is an important aspect of cognitive radios **Error! Reference source not found.**. There are two probabilities of interest in spectrum sensing. The first is, the probability of detecting the licensed users when they are active. It indicates how well the licensed users are protected since the unlicensed users have to vacate the channel once the licensed users are detected. The other is the probability of detecting the licensed users when the they are not active, which is referred to as probability of false alarm. If the false alarm probability is high, the usability of an unoccupied channel by the unlicensed users are low, since the unlicensed users will still have to vacate the channel when there is no licensed users. In short, they capture the reliability and efficiency of the overall cognitive system **Error! Reference source not found.**, and both of them are the functions of the sensing-time and the parameters of the fusion rule in cooperative sensing. Besides, a longer sensing-time will improve the sensing performance, but with a fixed frame size, the longer sensing-time will shorten the allowable data transmission time of the unlicensed users. Hence, a sensing-throughput tradeoff problem was formulated in **Error! Reference source not found.**to find the optimal sensing-time and fusion rule that maximizes the unlicensed users' throughput while providing adequate protection to the licensed users.

Cooperative spectrum sensing is an important technique in spectrum sensing **Error! Reference source not found.**. It can improve the performance of spectrum sensing and combatting the negative effects caused by shadowing and fading to the transmitter detection accuracy **Error! Reference source not found.**. Besides, the more unlicensed users are involved in sensing, the higher spectrum utilization the channel can achieve **Error! Reference source not found.**. However, from the unlicensed users' perspective, the average throughput of the unlicensed users decreases with the increase in the number of unlicensed users. In order to increase the average throughput of unlicensed users, in this paper, we deisgned a new frame structure and derived the objective function of it. Based on the proposed frame structure, we prove that there exists an optimal sensing-time to maximize the average throughput of

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unlicensed users under the Logical-AND and Logical-OR fusion rules, and compared the performances between them. We show that the Logical-AND fusion rule has poor performance in this scenario. Besides, we proved the unimodal characteristics of the unlicensed users' average throughput as a function of the sensing-time and the fusion parameter k when the k-out-of-N fusion rule is used, and proposed a numerical optimization algorithm to obtain the optimal solution in 2-D space. Furthermore, by theoretical analysis and comparing the performance between the Logical-OR and the k-out-of-N fusion rule, we derived the optimal fusion rule selection scheme in different scenarios.

The remainder of this paper is organized as follows. In Section 2, we present the sensing frame structure we proposed and derive the expression of the average throughput of unlicensed users. In Section 3, we compare the average throughput of unlicensed users between the Logical-AND and Logical-OR fusion rule, and propose a numerical optimization algorithm to obtain the optimal solution under the k-out-of-N fusion rule. Based on the new frame structure, computer simulations are provided in Section 4 to get insight into the effectiveness of proposed algorithm and validate the optimal fusion rule selection scheme. At last, we draw our conclusions in Section 5.

2. Sensing Frame Structure and Problem Formulation

We consider a cognitive radio network where there are N unlicensed users and one fusion center that act as sensor nodes to cooperatively detect the presence of the licensed users. In licensed user network, there are M licensed channels. The licensed users have the priority to use the channel. But they do not always occupy the channel, which lead to the channels being underutilized in the time domain. Each unlicensed user senses the spectrum periodically and makes a local decision about the presence of the licensed users based on its observations. The local decisions are to be sent to the fusion center in different sub-carrier based on OFDM scheme. The final decision is then made at the fusion center. Several fusion rules have been studied in literature **Error! Reference source not found.**, of which we consider a hard fusion rule due to its improved energy and bandwidth efficiency **Error! Reference source not found.**.

In this system model, each unlicensed user employs periodic time frames of length T for sensing and transmission. The frame structure we proposed is shown in Figure 2. Each frame comprises two parts namely a sensing-time (T_s) required for observation and decision making and a transmission time denoted by T_t for transmission in case the licensed user is absent. The sensing-time can be further divided into a time which depends on the number of the licensed channels M and is denoted by T_s / M . Between the sensing-time and the transmission time, there is a reporting time for unlicensed users to send the local decisions to fusion center and the fusion center publishes the spectrum allocation information to unlicensed users. Here, we employ OFDM approach as the basis for reporting. Hence, the reporting time can be negligible compared with the sensing period.

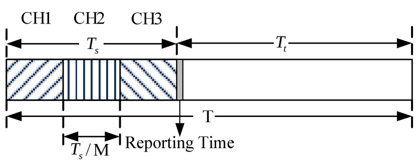

Figure 1. Sensing Frame Structure for Cognitive Radios

Suppose that we are interested in a frequency band with bandwidth *W* and the received signal is sampled at sampling frequency f_s . Then denote H_0 and H_1 as the hypotheses of the absence and the presence of the licensed user, respectively. The sampled signals that are received at the j_{th} unlicensed user during the divided sensing-time T_s / M , are given as:

$$\begin{cases} y_j(n) = u(n) & H_0 \\ y_j(n) = s_j(n) + u(n) & H_1 \end{cases} \quad (1)$$

Where $s_j(n)$ denotes the signal received at the j_{th} unlicensed user from the licensed user, $j \in [1, N]$, and each sample is assumed to be an independent identically distributed (i.i.d.) random process with zero mean and variance $E[|s_j(n)|^2] = \sigma_s^2$. The noise $u(n)$ is also assumed to be i.i.d. real Gaussian with zero mean and variance $E[|u(n)|^2] = \sigma_u^2$. Denote *L* as the number of samples for one of the licensed channels, where *L* is the maximum integer not greater than $T_s f_s / M$. Based on energy detection, the test statistic for energy detector is given by **Error! Reference source not found.**:

$$p_{s,j}(y) = \frac{1}{L}\sum_{n=1}^{L} |y_j(n)|^2 \quad (2)$$

Based on the Central Limit Theorem (CLT) **Error! Reference source not found.**, for a large *L*, the PDF of $p_{s,j}(y)$ under hypothesis H_0 and H_1 can be approximated by a Gaussian distribution **Error! Reference source not found.**

$$p_{s,j}(y) \sim \begin{cases} N(\sigma_u^2, \frac{2}{L}\sigma_u^4) & H_0 \\ N[(\gamma_j + 1)\sigma_u^2, \frac{2}{L}(\gamma_j + 1)^2 \sigma_u^4] & H_1 \end{cases} \quad (3)$$

Where $\gamma_j = \sigma_{s,j}^2 \big/ \sigma_u^2$ is the signal to noise ratio (SNR) of the j_{th} unlicensed user. For a given threshold ε, the probability of detection and false alarm are, respectively, given as:

$$P_{d,j}(\varepsilon, \frac{T_s}{M}) = Q[(\frac{\varepsilon}{\sigma_u^2(\gamma_j + 1)} - 1)\sqrt{\frac{T_s f_s}{2M}}] \quad (4)$$

$$P_{f,j}(\varepsilon, \frac{T_s}{M}) = Q[(\frac{\varepsilon}{\sigma_u^2} - 1)\sqrt{\frac{T_s f_s}{2M}}] \quad (5)$$

Where $Q(x) = (2\pi)^{-1/2}\int_x^{\infty} e^{-t^2/2}dt$. By combining equations (4) and (5), the false alarm probability can be expressed by the detection probability:

$$P_{f,j} = Q[Q^{-1}(P_{d,j})(\gamma_j + 1) + \gamma_j\sqrt{\frac{T_s f_s}{2M}}] \quad (6)$$

Let $C_{0,i}$ and $C_{1,i}$ denote as the throughput of the i_{th} licensed channel when unlicensed user operates in the absence and presence of licensed user, $i \in [1, M]$, then $C_{0,i}$ and $C_{1,i}$ are expressed as **Error! Reference source not found.**

$$\begin{cases} C_{0,i} = W\log_2(1 + SNR_{s,i} / \sigma) & H_0 \\ C_{1,i} = W\log_2(1 + SINR_{p,i} / \sigma) & H_1 \end{cases} \quad (7)$$

Where $SINR_{p,i} = p_{s,j} / (p_{p,i} + \sigma_u^2)$, and $p_{p,i}$ is the interference power of licensed user measured at the unlicensed user receiver and σ is considered as the SNR gap to channel capacity, so we have $C_{0,i} > C_{1,i}$.

Let $Q_{d,i}$ and $Q_{f,i}$ denote as the global detection probability and the global false alarm probability on the i_{th} licensed channel. Both of them are the functions of $P_{d,j}$ and $P_{f,j}$, separately. For a targeted $\bar{Q}_d$, the total throughput of unlicensed user is given as:

$$\begin{aligned} &R_{NET}(T_s,k) = \sum_{i=1}^{M} (T - T_s)[P(H_0)(1 - Q_{f,i})C_{0,i} + P(H_1)(1 - Q_{d,i})C_{1,i}] \\ &s.t.\ Q_{d,i} \geq \bar{Q}_d \end{aligned} \tag{8}$$

Thus the average throughput of unlicensed users for per unit frequency and per unit time is given by:

$$\begin{aligned} &\bar{R}(T_s,k) = \frac{1}{NW}\sum_{i=1}^{M} \frac{T - T_s}{T}[P(H_0)(1 - Q_{f,i})C_{0,i} + P(H_1)(1 - Q_{d,i})C_{1,i}] \\ &s.t.\ Q_{d,i} \geq \bar{Q}_d \end{aligned} \tag{9}$$

We suppose that the activity probability of licensed user is less than 0.3, so it is economically advisable to explore the secondary usage for that frequency band. Since $C_{0,i} > C_{1,i}$, the first term in the left hand side of (9) dominates $\bar{R}(T_s,k)$. Therefore the above optimization problem can be approximated by:

$$\begin{aligned} &\hat{R}(T_s,k) = \frac{1}{NW}\sum_{i=1}^{M} \frac{T - T_s}{T} P(H_{0,i})(1 - Q_{f,i})C_{0,i} \\ &s.t.\ Q_{d,i} \geq \bar{Q}_d \end{aligned} \tag{10}$$

3. Fusion Rules and Numerical Optimization Algorithm
3.1. Logical-OR and Logical-AND Fusion Rule

In Logical-OR fusion rule, the global detection probability and global false alarm probability of the i_{th} licensed channel are given as:

$$\begin{cases} Q_{d,i} = 1 - \prod_{j=1}^{N}(1 - P_{d,j}) \\ Q_{f,i} = 1 - \prod_{j=1}^{N}(1 - P_{f,j}) \end{cases} \tag{11}$$

From (10) and (11), the average throughput of unlicensed users is given by:

$$\begin{aligned} &\hat{R}_{or}(T_s,k) = \frac{1}{NW}\sum_{i=1}^{M} \frac{T - T_s}{T} P(H_{0,i})\left(\prod_{j=1}^{N}(1 - P_{f,j})\right)C_{0,i} \\ &s.t.\ Q_{d,i} \geq \bar{Q}_d \end{aligned} \tag{12}$$

In the following, we will prove that there exists an optimal sensing-time with which the highest average throughput of unlicensed users can be achieved under the Logical-OR fusion rule. It can be verified from (12) that:

$$\frac{d\breve{R}_{or}}{dT_s} = \frac{C_0 P(H_0)}{NW}\sum_{i=1}^{M}\{[\frac{-M}{T}(1-Q\{Q^{-1}[1-(1-Q_{d,j})^{1/N}](g_j+1)+g_j\sqrt{\frac{T_s f_s}{2M}}\})^N$$
$$+\frac{T-T_s}{T}N(1-Q\{Q^{-1}[1-(1-Q_{d,j})^{1/N}](g_j+1)+g_j\sqrt{\frac{T_s f_s}{2M}}\})^{N-1}\frac{g_j}{4} \quad (13)$$
$$\times\sqrt{\frac{f_s}{M\pi T_s}}e^{-[Q^{-1}[1-(1-Q_{d,j})^{1/N}](g_j+1)+g_j\sqrt{\frac{T_s f_s}{2M}}]^2/2}\}$$

Obviously, we have $\lim_{t\to T/M}\breve{R}'_{or}<0$ and $\lim_{t\to 0}\breve{R}'_{or}=+\infty$. It means that $\breve{R}_{or}(T_s,k)$ increases when T_s is small and decreases when T_s approaches T/M. It illustrates that there is a maximum point of $\breve{R}_{or}(T_s,k)$ between 0 and T/M. Hence, $\breve{R}_{or}(T_s,k)$ is a unimodal function in the range $(0,T/M)$. In the same way, we can get a similar conclusion in Logical-OR fusion rule.

3.2. k-out-of-N Fusion Rule

Under the k-out-of-N fusion rule, note that the parameter *k* is an integer between 1 and *N*. The global detection probability and global false alarm probability of the i_{th} licensed channel are given as:

$$\begin{cases} Q_{d,i}=\sum_{u=k}^{N}C_N^u(1-P_{d,j})^{(N-u)}P_{d,j}^u \\ Q_{f,i}=\sum_{u=k}^{N}C_N^u(1-P_{f,j})^{(N-u)}P_{f,j}^u \end{cases} \quad (14)$$

As is shown in (14), under the k-out-of-N fusion rule, the expressions of $Q_{d.i}$ and $Q_{f,i}$ are relatively complex, thus the above method can't be used in it. However, it can be observed from (6), (11) and (14) that when the value of *k* is taken as 1 and *N*, the k-out-of-N fusion rule becomes the Logical-OR and Logical-AND fusion rules, respectively. So the average throughput of unlicensed users maybe improved by optimizing the parameter *k* and sensing-time jointly.

In the next, we will formulate an optimization problem using the sensing-time and the parameter *k* as variables to show the unimodal characteristics of the unlicensed users' average throughput as a function of the two parameters, while giving adequate protection to licensed users. Then, we will introduce the proposed numerical optimization algorithm to obtain both the optimal sensing-time and *k* for the optimization problem in 2-D space. From (6) (10) and (14), for a targeted global detection probability ($\bar{Q}_d=0.9$) and 5 unlicensed users participate in cooperative sensing, the numerical analysis result of the unlicensed users' average throughput is shown in Figure 2.

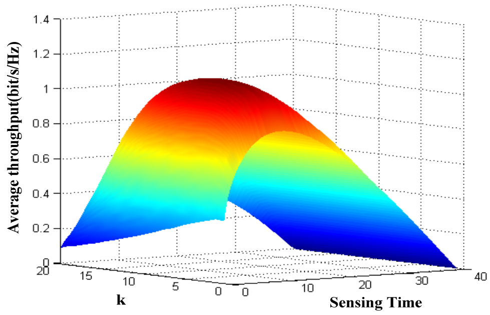

Figure 2. Average Throughput of Unlicensed Users under k/N Fusion Rule

It illustrates that there indeed exists an optimal *k* and T_s which achieves a higher average throughput of unlicensed users than other two fusion rules. Besides, it shows that the optimization problem is concave in the 2-D space. Thus, we propose a numerical optimization algorithm as follows. At first, select a *k* between 1 and *N*, randomly; then fix it, and find an optimal T_s from 1 and $T\ /\ M$ to maximize $\widehat{R}_{k/N}(T_s,k)$ by (13). The next, fix the T_s and find an optimal *k* between 1 and *N* to maximize $\widehat{R}_{k/N}(T_s,k)$. Repeat the aforementioned steps until the previous *k* equals to the current one, the converged solutions T_s and *k* are the optimal solutions to $\widehat{R}_{k/N}(T_s,k)$.

4. Computer Simulations

In this section, simulation results are presented to get insight into the effectiveness of the proposed algorithm.

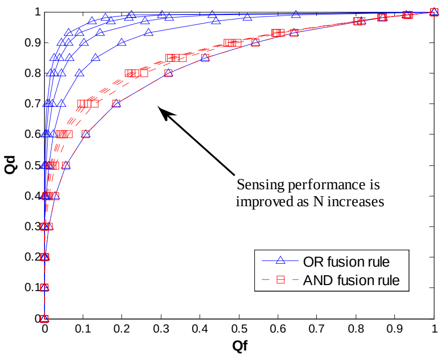

Figure 3. Comparison of ROC for Logical-OR and Logical-AND fusion rule

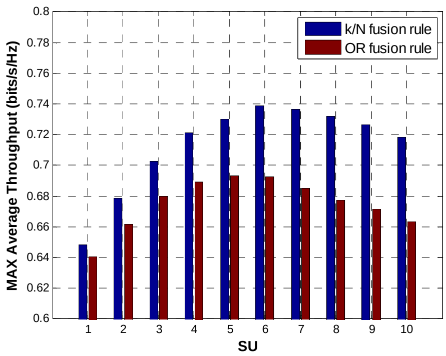

Figure 4. Comparisons between k-out-of-N and OR fusion rule

Figure 3 compares the performance of the Logical-OR fusion rule with the Logical-AND fusion rule in terms of ROC curves **Error! Reference source not found.**. Since Q_f dominates the average throughput of unlicensed users. From Figure 3, for a given Q_d, (Q_d = 0.9), the performance of Logical-OR fusion rule is much better than Logical-AND fusion rule. Figure 4 compares the performance between Logical-OR fusion rule and the optimal k-out-of-N fusion rule. We can see that, when sensing SNR is low and multiple unlicensed users participate in cooperative sensing the *k*-out-of-*N* fusion rule is batter, while in other scenarios, the Logical-OR fusion rule should be chosen, because it is much simpler and does not require much more computation than the optimal k-out-of-N fusion rule.

5. Conclusion

In this paper, we studied the sensing-time and fusion rules to improve the average throughput of unlicensed users. Based on the proposed frame structure, we compared the performances between the Logical-AND and Logical-OR fusion rules, and proved that there indeed exists an optimal sensing-time with which the highest average throughput of unlicensed users are achieved. Besides, under the k-out-of-N fusion rule we proposed a numerical optimization algorithm to obtain the optimal solution in 2-D space. Computer simulations have shown that the average throughput of unlicensed users using the proposed algorithm is more than 10% over the Logical-OR fusion rule, which are in excellent agreement with the simulation estimates.

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