CLF Based Stabilization of Chaos in PMSM with Uncertain Parameters

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Abstract

The robust stabilization problem of chaos suppression for permanent magnet synchronous motors (PMSM) drive system is investigated in presence of parametric uncertainties. Based on control Lyapunov function (CLF) approach, a new state feedback controller is designed to realize the state of the system globally asymptotically stable. The stability of the proposed control scheme is verified via Lyapunov stable theory. Finally, simulation results illustrate the effectiveness of the presented method.

Keywords: permanent magnet synchronous motor, chaos control, control Lyapunov function, state feedback control

1. Introduction

With the advantages of high torque/inertia ratio, high torque/weight ratio, compact size and no rotor loss, permanent magnet synchronous motor (PMSM) drive system has been widely used in industrial application, such as in robotic system, CNC system, diskdrive systems, and so on [1-3]. During the past years, the stability of the motor drive system, which is an essential requirement for industrial automation manufacturing, has received considerable attenuation. Up to now, it has been found that chaos was widely existed in all kinds of motor drive systems, such as induction motors, DC motors, and switched reluctance motors [4]. Chaotic behavior in permanent magnet DC motor open drive system was first addressed by Hemati [5]. Li in [6] has found that chaos was also existed in permanent magnet synchronous motor (PMSM). Without considering power electric switching, PMSM drive system can be transformed into a typical Lorenz system, which is well known exhibiting chaotic behavior. In most engineering applications, this undesirable chaotic oscillation, which will extremely destroy the stabilization of the system or even induce system collapse, should be suppressed or even eliminated.

Recently, numerous methods have been successfully used to control PMSM chaotic system, such as feedback control [7, 8], passivity control [9], dynamic surface control [10], adaptive control [11, 12], sliding mode control [13], finite time control [14], Lyapunov exponents approach [15, 16] and fuzzy control [17, 18]. However, most of those methods either do not consider uncertain (such as feedback control and passivity control) parameters or have complicated control structure (such as Lyapunov exponents approach and fuzzy control), which have prevented the application of those methods in practice.

Control Lyapunov function is one of the most powerful tool to design controller for nonlinear systems, which was first introduced by Artstein [19] and Sontag [20]. This method, converting stability descriptions into tools for solving stabilizations, has made tremendous impact on stabilization theory. Up to now, CLF method has been successfully applied to control manipulators [21], converter systems [22], high-voltage direct current (HVDC) systems [23], switched systems [24] and some nonlinear systems with time delay [25]. Wang et al. [26, 27] first used this method to achieve chaotic synchronization of two chaotic systems. But it has one disadvantage that the response speed of the proposed control system is not tunable, because it has no adjustable control parameters in the controller. In fact, it is necessary to provide adjustable control parameters for users. To solve this problem, Yang et al. [28] proposed an improved controller with time-vary parameters for nonlinear system. The introduction of adjustable time-vary parameter has not only lessened the controller's dependence on the

selection of CLF, but also made the controller optimal for some adjustable control performance indexes. To the best of our knowledge, there has no result reported in the literature so far to apply this method to stabilize chaotic systems. In this paper, we attempt to use this approach to control PMSM chaotic system with uncertain parameters.

This paper is organized as follows. Section 2 introduces the basic concepts and lemmas of CLF. In Section 3, we present the chaos model of PMSM drive system first. And then the controller is designed based on CLF theory and the the stability of the controlled closed systems is verified according to Lyapunov stability theory. Section 4 presents the simulation results to illustrate the effectiveness of the method. Finally, Section 5 concludes.

2. Basic Concepts and Lemmas

Important concepts and lemmas necessary for controller design are given below.

Definition1: [29] Consider the following affine nonlinear system.

$$\dot{x} = f(x) + g(x)u , \tag{1}$$

Where $x \in R^n$ denotes the state vector of the system, $u \in R^m$ denotes the control input vector, $f(x): R^n \to R^n$ and $g(x): R^n \to R^m$ are smooth vector fields with nonlinear $f(0) = 0$. A positive definite function $V(x)$ is a CLF of the system (1) if it is smooth, proper, and satisfies the following condition:

$$L_g V(x) = 0, x \neq 0 \Rightarrow L_f V(x) < 0 , \tag{2}$$

Where $L_g V(x)$ and $L_f V(x)$ denotes the Lie derivative of $V(x)$ along $g(x)$ and $f(x)$, respectively.

Remark 1: $V(x)$ positive means that $V(0) = 0$ and $V(x) > 0$ for $x \neq 0$, and $V(x)$ proper means that $V(0) \to \infty$ as $\|x\| \to \infty$.

Definition 2: [29] Assume that $k(x): R^n \to R^+$ is a function with $k(0) = 0$. $u = k(x)$ is almost smooth on R^n if it is smooth away from the origin and continuous at the origin of R^n.

Lemma 1: [29] Let $V(x, \mu)$ is a CLF of system (1). Then there exists an almost smooth feedback control law $u = k(x)$ such that system (1) is globally asymptotically stable. And the control law μ is:

$$u = k(x) = -p(x)\beta(x)^T , \tag{3}$$

$$p(x) = \begin{cases} [\alpha(x) + \sqrt{\|\alpha(x)\|^2 + ((\|\beta(x)\|)^4}]/\|\beta(x)\|^2 , \beta(x) \neq 0 \\ 0, \beta(x) = 0 \end{cases} , \tag{4}$$

Where $\alpha(x) = L_f V(x)$ and $\beta(x) = L_g V(x)$.

Proof: If $\beta(x) = 0$, note that $V(x, \mu)$ is a CLF of system (1), then $\dot{V}(x, \mu) = \alpha(x) = L_f V(x) < 0$;

If $\beta(x) \neq 0$, $\dot{V}(x, \mu) = \alpha(x) + \beta(x)k(x, \mu) = -\sqrt{\|\alpha(x)\|^2 + ((\|\beta(x)\|)^4} \leq -\|\alpha(x)\| < 0$.

Thus, $\dot{V}(x, \mu) < 0$ for all states *x*.

Moreover, $\alpha(x) = L_f V(x)$ is continuous and $(\|\beta(x)\|)^4 = o(\|\beta(x)\|^2)$, so $u = k(x)$ is continuous at the origin. Thus $u = k(x)$ an almost smooth control and it is globally asymptotically stable at the equilibrium *x*=0.

Remark 2: In control theory, a control Lyapunov function $V(x, \mu)$ is a generalization of the notion of Lyapunov function $V(x)$ used in the stability analysis. The ordinary Lyapunov function is used to test whether a dynamical system is stable (more restrictively, asymptotically stable). That is, whether the system starting in a state $x \neq 0$ in some domain Γ will remain in Γ, or for asymptotic stability will eventually return to $x = 0$. The control-Lyapunov function is used to test whether a system is feedback stable, that is, whether for any state x there exists a control $u = k(x)$ such that the system can be brought to the zero state with the control μ.

3. Controller Design for PMSM Chaotic System

In this section, it is given the chaos model of the PMSM drive system and at the same time the controller is designed in detail. Then, The stability of the proposed control scheme is verified via Lyapunov stable theory.

3.1. Dymanic Model and Chaotic Characteristics of PMSM Chaotic System

The transformed model of PMSM with the smooth air gap can be expressed as follows [3]:

$$
\begin{cases}
\dot{i}_d = -i_d + w i_q + v_d \\
\dot{i}_q = -i_q - w i_d + \gamma w + v_q \\
\dot{w} = \sigma(i_q - w) - T_L
\end{cases}
\tag{5}
$$

Where v_d, v_q, i_d, and i_q are the transformed stator voltage components and current components in the d-q frame, w and T_L are the transformed angle speed and external load torque respectively, and γ and σ are the motor parameters.

Considering the case that, after an operation of the system, the external inputs are set to zero, namely, $v_d = v_q = T_L = 0$, system (5) becomes an autonomous system:

$$
\begin{cases}
\dot{i}_d = -i_d + w i_q \\
\dot{i}_q = -i_q - w i_d + \gamma w \\
\dot{w} = \sigma(i_q - w)
\end{cases}
\tag{6}
$$

The modern nonlinear theory such as bifurcation and chaos has been used to study the nonlinear characteristics of PMSM drive system in [6]. It has found that, with the operating parameters γ and σ falling into a certain area, PMSM will exhibit complex dynamic behavior, such as periodic, quasi periodic and chaotic behaviors. In order to make an overall inspection of dynamic behavior of the PMSM, the bifurcation diagram of the angle speed w with increasing of the parameter γ is illustrated in Figure 1(a). We can see that the system shows abundant and complex dynamical behaviors with increasing parameter γ. The typical chaotic attractor is shown in Figure 1(b) with $v_d = v_q = T_L = 0$, $\gamma = 25$, and $\sigma = 5.46$.

According to chaos theory, the Lyapunov exponents and power spectrum are two effective methods to determine whether a continuous dynamic system is chaotic. In general, a three-dimensional nonlinear system has one positive Lyapunov exponents, implying that it is chaotic. Figure 1(c) and (d) show the Lyapunov exponents and power spectrum of PMSM chaotic system (6) with $\gamma = 25$, and $\sigma = 5.46$. When the parameters are set as obove, calculated Lyapunov exponents are: $L_{E1} = 0.479453$, $L_{E2} = -0.024905$, $L_{E3} = -7.914548$, and the Lyapunov dimension is $D_L = 2.057432$, which means the system is chaotic.
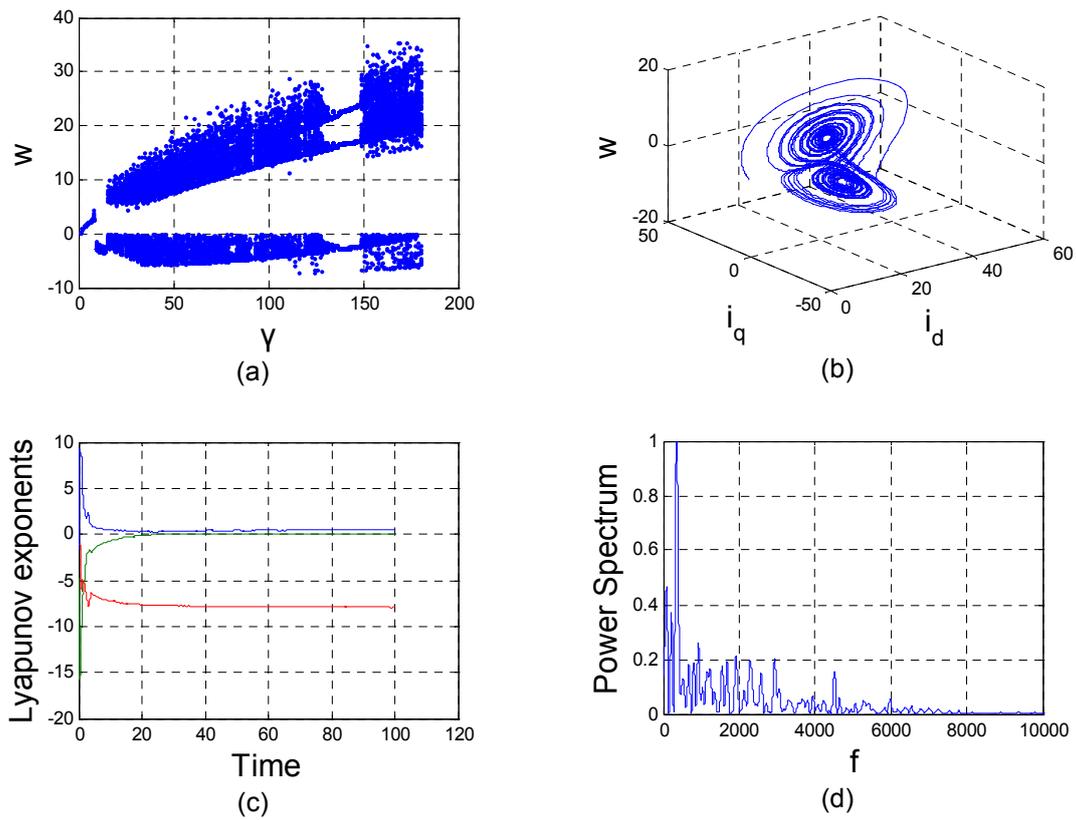
Figure 1. Bifurcation Deagram and the Characterizations of Chaos in PMSM (a) Bifurcation diagram of state variable w with the parameter γ, (b) typical chaotic attractor, (c) Lyapunov exponents, (d) power spectrum of state variable w

With uncertain parameters, the dynamic model of the system can be described as follows:

$$\begin{cases} \dot{i}_d = -i_d + w i_q \\ \dot{i}_q = -i_q - w i_d + (\gamma + \Delta_\gamma)w , \\ \dot{w} = (\sigma + \Delta_\sigma)(i_q - w) \end{cases} \qquad (7)$$

Where Δ_γ and Δ_σ represent the uncertainty of γ and σ respectively and are both bounded.

Because the parameters γ and σ are related to the parameters of PMSM drive system, such as resistors, inductors, magnetic, which will change in a certain the range of temperature. Therefore, this article assumes that the fluctuation range of system parameters is 30%, that is, $\left\| \Delta_\gamma \right\| \le \delta_1 \le 0.3\gamma$, $\left\| \Delta_\sigma \right\| \le \delta_2 \le 0.3\sigma$.

3.2. Controller Design

System (6) indicates three equilibrium points: $S_0(0,0,0)$, $S_1(\gamma-1, \sqrt{\gamma-1}, \sqrt{\gamma-1})$, and $S_2(\gamma-1, -\sqrt{\gamma-1}, -\sqrt{\gamma-1})$. Given that $\gamma = 25$, S_0 is locally stable, and S_1 and S_2 are both locally unstable [6] . Assuming that one equilibrium point of system (5) is $S(i_{dd}, i_{qd}, w_d)$, then:

$$\begin{cases} \dot{i}_{dd} = -i_{dd} + w_d i_{qd} = 0 \\ \dot{i}_{qd} = -i_{qd} - w_d i_{dd} + \gamma w_d = 0 \\ \dot{w}_d = \sigma(i_{qd} - w_d) = 0 \end{cases} \tag{8}$$

To quickly stabilize to equilibrium point $S(i_{dd}, i_{qd}, w_d)$, u_1 and u_2 are used to control the system (7). Under the control efforts u_1 and u_2, the controlled system can be represented as:

$$\begin{cases} \dot{i}_d = -i_d + wi_q + u_1 \\ \dot{i}_q = -i_q - wi_d + (\gamma + \Delta_\gamma)w + u_2 \\ \dot{w} = (\sigma + \Delta_\sigma)(i_q - w) \end{cases} \tag{9}$$

Let $e_1 = i_d - i_{dd}$, $e_2 = i_q - i_{qd}$ and $e_3 = w - w_d$, we can obtain the dynamic error equations of the system:

$$\begin{cases} \dot{e}_1 = -e_1 + e_2 e_3 + e_2 w_d + e_3 i_{qd} + u_1 \\ \dot{e}_2 = -e_2 - e_1 e_3 - e_1 w_d - e_3 i_{dd} + \gamma e_3 + \Delta_\gamma(e_3 + w_d) + u_2 \\ \dot{e}_3 = (\sigma + \Delta_\sigma)(e_2 - e_3) \end{cases} \tag{10}$$

In order to design the controller, system (10) can be rewritten in a compact form as:

$$\dot{e} = f(e) + g(e)\Delta + Bu \tag{11}$$

Where $e = (e_1, e_2, e_3)^T$, $\Delta = (\Delta_\gamma, \Delta_\sigma)^T$ and $\|\Delta\| \le \delta$, $f(e) = \begin{bmatrix} -e_1 + e_2 e_3 + e_2 w_d + e_3 i_{qd} \\ -e_2 - e_1 e_3 - e_1 w_d - e_3 i_{dd} + \gamma e_3 \\ \sigma(e_2 - e_3) \end{bmatrix}$,

$g(e) = \begin{bmatrix} 0 & 0 \\ e_3 + w_d & 0 \\ 0 & e_2 - e_3 \end{bmatrix}$, $u = (u_1, u_2)$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ is control input matrix.

The control objective is to stabilize the system (9) at the equilibrium point $S(i_{dd}, i_{qd}, w_d)$, that is, we design the controller to stabilize the error system (10) at $e=0$. So we will focus on the controller designing for system (11).

Theorem 1. Consider error dynamic system (11). If the positive function $V(e)$ is defined by:

$$V(e) = \tfrac{1}{2}(e_1^2 + e_2^2 + e_3^2), \tag{12}$$

Then there exists an almost smooth feedback control law $u = k(e, \mu)$ such that system (11) is globally asymptotically stable. And the control law u is:

$$u = k(e, \mu) = -p(e, \mu)\beta(e)^T, \tag{13}$$

$$p(e, \mu) = \begin{cases} [\alpha(e) + \delta\|\eta(e)\| + \sqrt{(\|\alpha(e)\| + \|\delta\eta(e)\|)^2 + ((\mu\|\beta(e)\|)^4}] / \|\beta(e)\|^2, & \beta(e) \ne 0 \\ 0, & \beta(e) = 0 \end{cases} \tag{14}$$

Where $\alpha(e) = L_f V(e)$, $\beta(e) = L_B V(e)$, $\eta(e) = L_g V(e)$ and, μ is an adjustable parameter.

Proof. $\alpha(e) = L_f V(e) = [e_1 \quad e_2 \quad e_3] \cdot f(e),\quad \eta(e) = L_g V(e) = [e_1 \quad e_2 \quad e_3] \cdot \begin{bmatrix} 0 & 0 \\ e_3 + w_d & 0 \\ 0 & e_2 - e_3 \end{bmatrix}$,

$\beta(e) = L_B V(e) = [e_1 \quad e_2 \quad e_3] \cdot B = [e_1 \quad e_2 \quad 0]$.

Case 1) If $\beta(e) = L_B V(e) = 0$ and $e \neq 0$, we can obtain that $e_1 = e_2 = 0$ and $e_3 \neq 0$.

Then, $\dfrac{\partial V}{\partial e}(f(e) + g(e)\Delta) = L_f V(e) + \eta(e)\Delta = -(\sigma + \Delta_\sigma)e_3^2 < 0$, so $V(e)$ is one CLF of system (11).

$$\dot{V}(e) = \frac{\partial V}{\partial e}(f(e) + g(e)\Delta + Bu) = \alpha(e) + \eta(e)\Delta = -(\sigma + \Delta_\sigma)e_3^2 < 0$$

Case 2) If $\beta(e) \neq 0$,

$$V(e) = \frac{\partial V}{\partial e}(f(e) + g(e)\Delta + Bu) = \alpha(e) + \eta(e)\Delta + \beta(e)k(e,\mu)$$

$$= \eta(e)\Delta - \delta\|\eta(e)\| - \sqrt{(\|\alpha(e)\| + \|\delta\eta(e)\|)^2 + ((\mu\|\beta(e)\|)^4}$$

$$\leq -\sqrt{(\|\alpha(e)\| + \|\delta\eta(e)\|)^2 + ((\mu\|\beta(e)\|)^4} < 0$$

Thus, for all (x, μ), the positive and proper function V(e) decrease along the trajectory of the error system (11).

Moreover, $\alpha(e) = L_f V(e)$ and $\eta(e) = L_g V(e)$ are both continuous, and $(\mu\|\beta(e)\|)^4 = o(\|\beta(e)\|^2)$, so $u = k(e,\mu)$ is continuous at the origin. Thus $u = k(e,\mu)$ an almost smooth control and it is globally asymptotically stable for system (11) under control effect u.

4. Simulation results

We use SIMULINK of MATLAB to verify the feasibility of the proposed controller for a PMSM chaotic system. In the simulation, the fourth-order Runge–Kutta method is used to solve the systems with time step size 0.001. The parametric values of PMSM are the same as in Section 3. Without loss of generality, we select S_1 ($\gamma - 1$, $\sqrt{\gamma - 1}$, $\sqrt{\gamma - 1}$) as the desired equilibrium point. When $\gamma=25$, the desired equilibrium point is S_1 (24, $\sqrt{23}$, $\sqrt{23}$). The control method takes effect after $t=20$ s and the adjustable parameter µ=5.
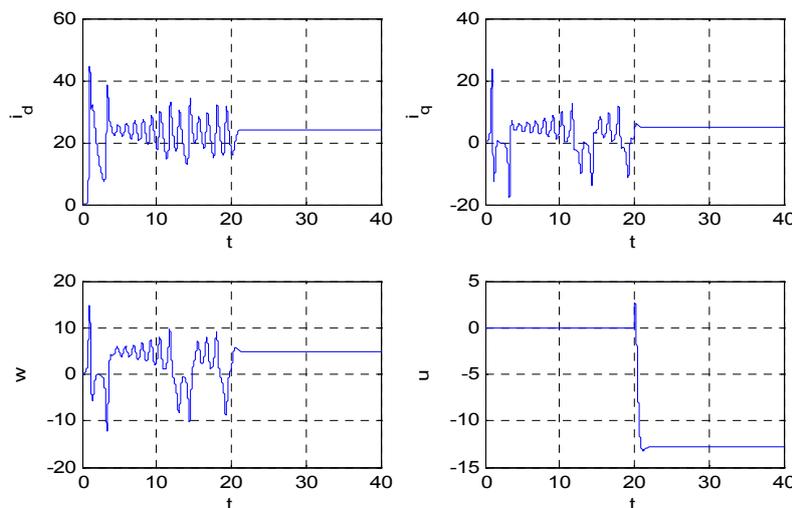
Figure 2. State Trajectories (i_d, i_q and w) and Control Input u of System (9) without Considering Uncertain Parameters.
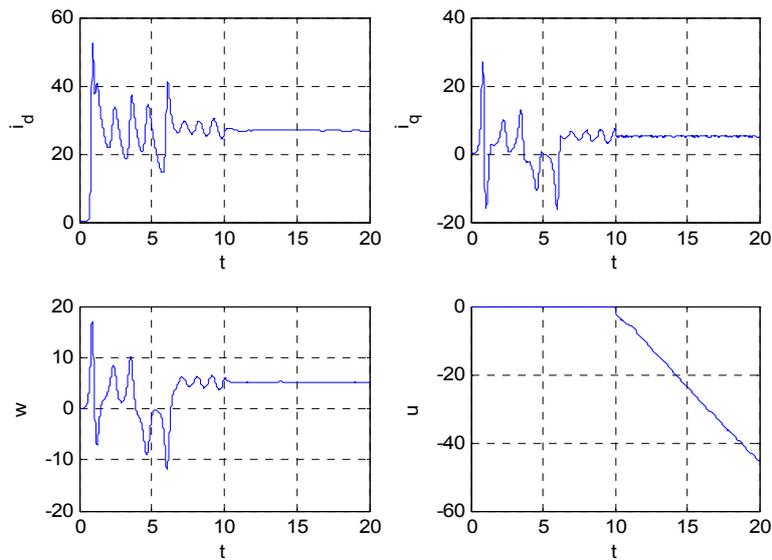
Figure 3. State Trajectories (i_d, i_q and w) and Control Input u of System (9) Considering Uncertain Parameters

The simulation results with only one control input, that is, $B = [0 \quad 1 \quad 0]^T$, is shown in Figure 2 (without considering uncertain parameters) and Figure 3 (without considering uncertain parameters). It can be seen from Figure 2 and Figure 3 that the state trajectories and control input of the closed loop PMSM chaotic system with only one control input can quickly stabilize to its unstable equilibrium point S_1. We can seen from Figure 2 and Figure 3 that it takes about 2s to stablize them to S_1, but there are strong chattering phenomenon when the uncertain parameters is considered.

Figure 4 and Figure 5 show the the state trajectories and control inputs of the PMSM chaotic system with two control inputs. We can see from Figure 4 that the states of the system stabilize to S1 within 1s, and there are no overshoot in the control inputs. Moveover, when the uncertain parameters are taken into consideration, the states of the systems can also stabilize to their equilibrium with no chattering. So, the performance of the PMSM chaotic system with two control inputs is significantly better than that with just one control input. It has fast, accurate and robust performance of the second method (with two control inputs).
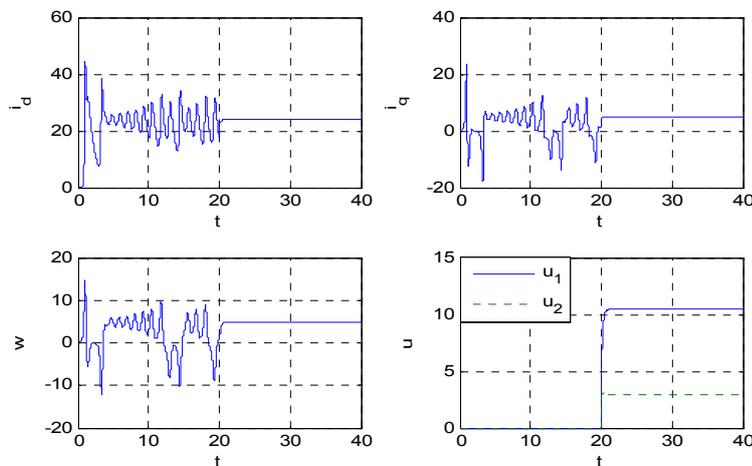
Figure 4. State Trajectories (i_d, i_q and w) and Control Input (u_1 and u_2) of System (9) without Considering Uncertain Parameters
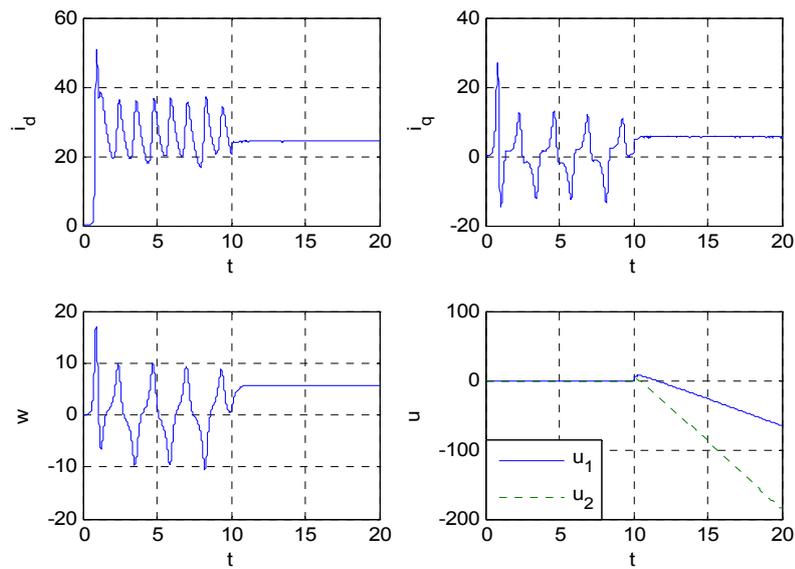
Figure 5. State Trajectories (i_d, i_q and w) and Control Input (u_1 and u_2) of System (9) Considering Uncertain Parameters

Figure 6 shows the state trajectories i_d of the proposed controller with different adjustable parameter μ. It shows that with the increasing of parameter μ, the transition time is reduced accordingly. So we can choose the parameter μ according to the design requirement of the system performance.
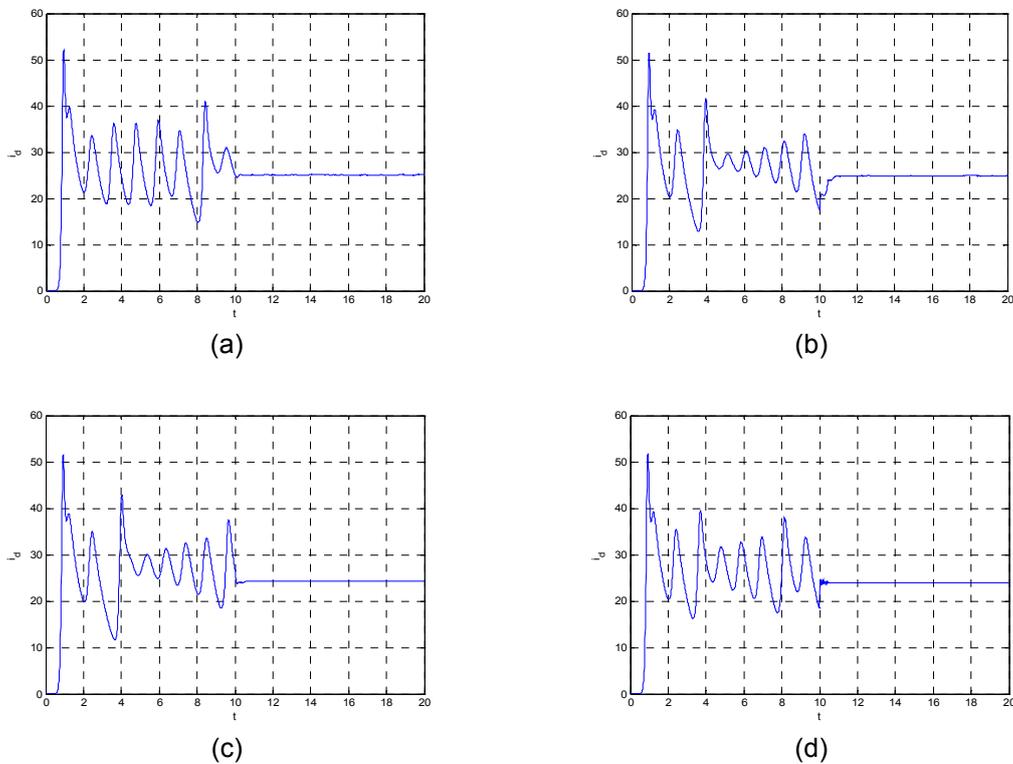
Figure 6. State Trajectories of PMSM Chaotic System with Different Parameter μ: a) μ=0.5, b) μ=2, c) μ=4 d) μ=10

5. Conclusion

We develop a novel nonlinear feedback control scheme that accounts for parameter uncertainties in a PMSM chaotic system. This controller is designed based on CLF theory. The advantages of the proposed controller are as follows:

1) It has a unified form for PMSM chaotic system with or without parametric uncertainties. So the exact mathematic model of the system is not required. If there are no uncertain parameters, the parameter δ meets $\delta=0$ in the control law (14);

2) The response speed of the closed-loop are tunable. It has been verified that an appropriate increase of the gain μ can effectively improve the responsiveness of the system;

3) Its structure is easy to design and implement. The presented method is equal to only add the control voltages to the state equation of PSMS. So it can solve the problem existing in [8] and [11] that there is no controllable variable to control in the speed equation of the system.

Future research should investigate the implementation of the proposed control scheme by using an experimental setup. The scheme can also be extended to synchronize PMSM chaotic systems with uncertain parameters.

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