

Algorithms for Lorenz System Manifold Computation

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Abstract

A new algorithm is presented to compute both one dimensional stable and unstable manifolds of planar maps. It is proved that the gradient of the global manifold can be predicted by the known points on the manifold with a gradient prediction scheme and it can be used to locate the image or preimage of the new point quickly. The performance of the algorithm is demonstrated with hyper chaotic Lorenz system.

Keywords: discrete dynamical system, stable manifold, unstable manifold, hyperbolic fixed point, gradient prediction

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1. Introduction

2D manifold is a collection of 1D sub-manifolds. So the first step is to compute enough 1D sub-manifolds to cover the 2D manifold. During the computation, the corresponding for messages of mesh points on the 1D sub-manifold is recorded [1-2].

As mentioned above, 2D manifold is a collection of 1D sub-manifolds. So the first step is to compute enough 1D sub-manifolds to cover the 2D manifold. During the computation, the Foliation arc-length of mesh points on the 1D sub-manifold is labeled [3-5].

2. The Procedure of 2D Manifold Computation

Take a round circle centered at hyperbolic fixed point x_0 on the 2D local manifold, then select N mesh points on the circle uniformly. The Foliation arc-length of the 1D sub-manifold is ARC . Label the 1D sub-manifold as L_1 and take it as a reference line. Then compute another 1D sub-manifold L_2 through the next point on the circle up to Foliation arc-length ARC and check the distance between L_2 and the reference line. The distance is measured by the greatest distance between two mesh points of the same Foliation arc-length with one point taken from L_1 and the other taken from L_2 . If the distance is greater than $SIZE_{max}$ (the maximum size of the mesh), a new 1D sub-manifold need to be inserted between them. The new 1D sub-manifold is through the midpoint of the two mesh points corresponding to L_1 and L_2 on the circle. Then evaluate the distance between the new 1D sub-manifold and the reference line, if the distance is still greater than $SIZE_{max}$, go on to insert new 1D sub-manifold with the method mentioned above. Otherwise, take the new 1D sub-manifold as the reference line and compute the next 1D sub-manifold through the next point on the circle [4-9].

After the mentioned process is completed, we need to check the distance between neighboring 1D sub-manifolds again to remove those who lie too close to each other. For three adjacent 1D sub-manifolds L_i , L_{i+1} and L_{i+2} , if the distance between L_i and L_{i+1} is smaller than $SIZE_{min}$ (the minimum size of the mesh) and the distance between L_i and L_{i+2} is less than $SIZE_{max}$, L_{i+1} is deleted.

In the next step, the result is visualized. For every 1D sub-manifold that has been computed, pick out the points whose Foliation arc-length is $k * step$ ($k = 1, 2, \dots$) to represent the

original 1D sub-manifold. Mesh size is defined by the value of $step$. Because the Foliation arc-length of mesh points of the original 1D sub-manifold is not exactly an integer multiple of $step$, linear interpolation is required to get the expected points. Connect the mesh points who have the same Foliation arc-length on all the reconstructed 1D sub-manifolds successively with line segments to visualize the 2D manifold as circles, that is to say, the foliation arc-length of the mesh points on the same circle is identical. We can also represent the 2D manifold as a surface by covering it with triangular grids. The triangulation between two neighboring circles is depicted in Figure 1.

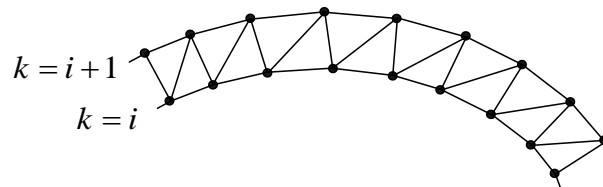


Figure 1. The Triangulation between Two Neighboring Circles

If all the sub-manifolds are computed from the initial circle, it will be seemingly unnecessary to use the “intricate” procedures presented in the previous subsection. But the potential risk is that too many sub-manifolds are accumulated in the weak direction of the 2D manifold, and if the distance between adjacent points on the initial circle approaches the computational precision limits, no more sub-manifolds could be inserted even if the distance is still too great. An alternative is to compute the 2D manifold with higher computation precision, but the computation expense will be too great. In this paper, we apply an integrated method: if the distance between the counterparts on the initial circle of two adjacent sub-manifolds is greater than threshold $precision$, then add a new point on the initial circle between them and compute the inserted sub-manifold through it; otherwise, the recursive procedure is employed.

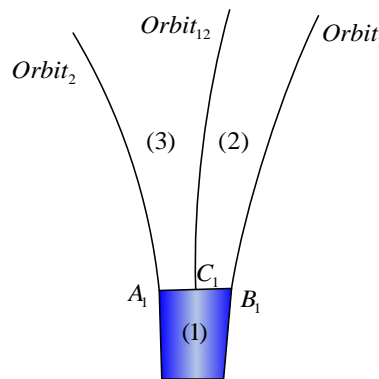


Figure 2. Visualization of 2D Manifold

In the next step, the result is visualized by triangulation. And this step can be incorporated with the aforementioned recursive algorithm.

3. Simulation

In this paper Lorenz system is used as an example for simulation. Lorenz system is a model describing the dynamics of atmospheric convection, and it is well known for its butterfly shaped chaotic attractor [10-11]. The model is written as:

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = \rho x - y - xz \\ \dot{z} = xy - \beta z \end{cases} \quad (1)$$

When $\sigma = 10$, $\rho = 28$ and $\beta = 8/3$, the attractor is chaotic, as shown in Figure 2(a). The model is continuous and in the form of an ordinary differential equation. By using difference, the system is discretized.

$$\begin{cases} \frac{x_{n+1} - x_n}{T} = \sigma(y_n - x_n) \\ \frac{y_{n+1} - y_n}{T} = x_n(\rho - z_n) - y_n \\ \frac{z_{n+1} - z_n}{T} = x_n y_n - \beta z_n \end{cases} \quad (2)$$

The previous is simplified as:

$$\begin{cases} x_{n+1} = T\sigma(y_n - x_n) + x_n \\ y_{n+1} = Tx_n(\rho - z_n) - Ty_n + y_n \\ z_{n+1} = T(x_n y_n - \beta z_n) + z_n \end{cases} \quad (3)$$

In order to maintain the property of the continuous Lorenz system, the value of T need to be appropriate. If T is too great, the approximation is too coarse and the discrete system is not chaotic anymore; on the other hand, if T is too small, the evolution speed of the system is too slow. We find that when $T = 0.01$, the discrete Lorenz system has a chaotic attractor similar to that of the continuous Lorenz system, at the same time, the system evolves at a moderate speed.

The origin is a hyperbolic fixed point of the discrete Lorenz system, and the Jacobian matrix at it is:

$$A = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.28 & 0.99 & 0 \\ 0 & 0 & 0.9733 \end{bmatrix} \quad (4)$$

Jacobian matrix A has 3 real eigenvalues: $\lambda_1 = 0.7717$, $\lambda_2 = 1.1183$ and $\lambda_3 = 0.9733$. It is interesting to notice that the discrete Lorenz has 2D stable manifold, which is also similar to that of the continuous Lorenz system.

In Figure 3(a) and Figure 3(b), the 2D stable manifold is represented by 1D sub-manifolds. Figure 3(a) and Figure 3(b) showed the same manifold seen from different direction. In order to show more details, the upper part and lower part of the manifold are plotted separately. The minimum distance between adjacent 1D sub-manifold is $precision = 0.001$. However, if all the sub-manifolds are computed with starting points on the initial circle, the 2D manifold can only be computed up to arc-length 80 with the same accuracy parameters because the minimum distance is approximately 10^{-24} and is approaching the accuracy limits. Totally 1263 sub-manifolds are computed with starting points on the initial circle to cover the 2D manifold. In contrast, when $ARC = 80$, only 1020 sub-manifolds are computed with the proposed algorithm and only 25 of these sub-manifolds are computed with starting points on the initial circle. The minimum distance is $precision = 0.001$. We can see that the proposed algorithm not only reduces the total number of sub-manifolds but also avoids generating too

many points near the initial circle. So our algorithm has obvious advantages especially when computing a large piece of 2D manifold.

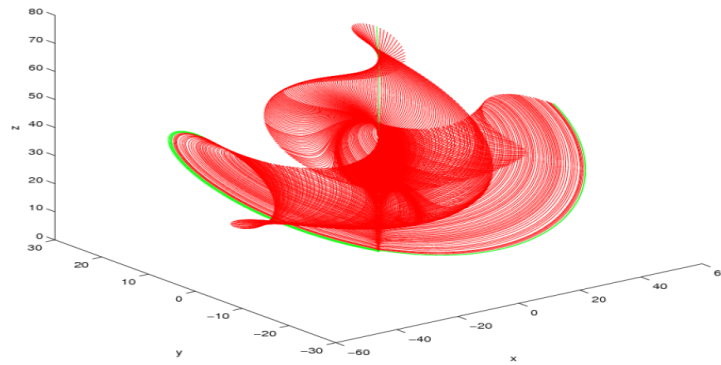


Figure 3(a). Part of the Stable Manifold, Represented by 1D Sub-manifolds

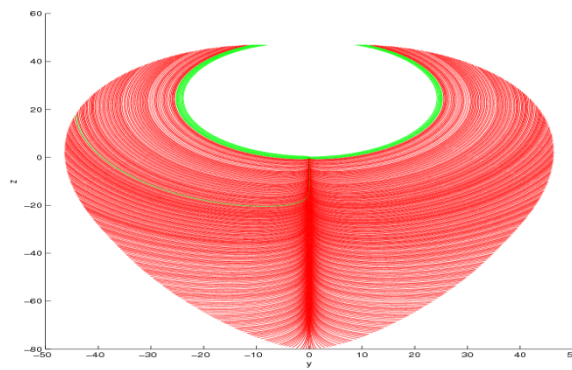


Figure 3(b). Part of the Stable Manifolds, Represented by 1D Sub-manifolds

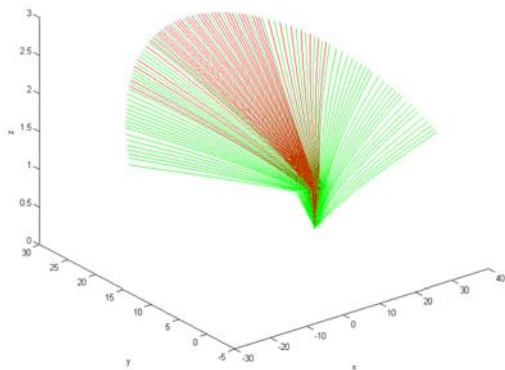


Figure 4. Part of the Stable Manifold, Represented by 1D Sub-manifolds

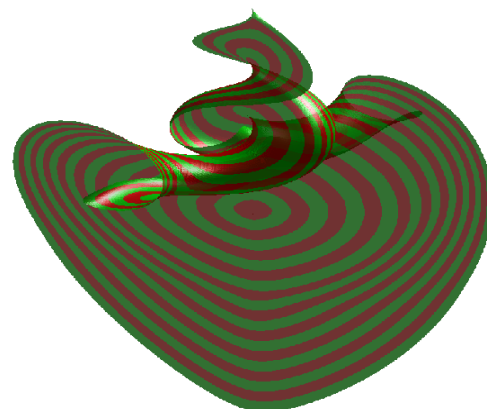


Figure 5. The 2D Stable Manifold of Discrete Lorenz System, using both Foliation Arc-length and Euclid Arc-length

In Figure 4, part of the stable manifold is plotted and represented by a group of 1D sub-manifolds. The green ones are starting from the initial circle, while the red ones are computed with the algorithm. The minimum distance between adjacent 1D sub-manifolds is

$precision = 0.001$, and there are totally 369 sub-manifolds in the 2D manifold of Figure 4 and 246 of them are computed with starting points on the initial circle. However, if all the sub-manifolds are computed with starting points on the initial circle, the minimum distance is approximately 10^{-5} and the number of 1D sub-manifolds is 452. The number of sub-manifolds and the density of points near the initial circle are both reduced by applying the proposed algorithm.

In Figure 5, we use both Foliation arc-length and Euclid arc-length to control the growth. Compared to Figure 5, the growth of lower part of the manifold is getting a little worse, but the growth of the upper part (which has a complicated structure) is improving. So, the overall performance is improved.

4. Conclusion

Compared to the algorithm in reference [3], it is clear that our algorithm does better in controlling the growth of the 2D manifold. What's more, the algorithm in reference [3] only computes 2D unstable manifold of a map while our algorithm is capable of computing both 2D stable and unstable manifold.

The weak point of our algorithm is too much mesh points are generated at the inner part of the 2D manifold, and it is a promising key point where the algorithm can be revised in the future.

Acknowledgements

The work is supported by Tackle Key Problems in Science and Technology of He Nan province in China (Grant No. 112102210014), Tackle Key Problems in Science and Technology of Xinxiang city in China (Grant No. ZG11009).

References

- [1] Z You, EJ Kostelich, JA Yorke. Calculating stable and unstable manifolds. *Int. J. Bifurc. Chaos Appl. Sci. Eng.* 1991; 1: 605-623.
- [2] TS Parker, LO Chua. *Practical Numerical Algorithms for Chaotic Systems*. Springer, Berlin, 1989.
- [3] D Hobson. An efficient method for computing invariant manifolds. *J. Comput. Phys.* 1991; 104: 14-22.
- [4] B Krauskopf, HM Osinga. Growing unstable manifolds of planar maps, 1517, 1997, <http://www.ima.umn.edu/preprints/OCT97/1517.ps.gz>.
- [5] B Krauskopf, HM Osinga. Growing 1D and quasi-2D unstable manifolds of maps. *J. Comput. Phys.* 1998; 146: 404-419.
- [6] JP England, B Krauskopf, HM Osinga, Computing One-Dimensional Stable Manifolds and Stable Sets of Planar Maps without the Inverse. *SIAM J. Appl. Dyn. Syst.* 2004; 3: 161-190.
- [7] M Dellnitz, A Hohmann. A subdivision algorithm for the computation of unstable manifolds and global attractors. *Numer. Math.* 1997; 75: 293-299.
- [8] J Palis, WD Melo. *Geometric Theory of Dynamical Systems*. Springer-Verlag, New York, 1982.
- [9] W Govaerts, Khoshshiar R Ghaziani, Yu A Kuznetsov, HGE Meijer. CI MatContM: A toolbox for continuation and bifurcation of cycles of maps, 2006, <http://www.matcont.UGent.be>.
- [10] Khairudin. RBFNN Control of A Two-Link Flexible Manipulator Incorporating Payload. *TELKOMNIKA Telecommunication Computing Electronics and Control*. 2010; 08: 57-164.
- [11] P Srikanth, Ashwani Kumar Chandel. Inverse S-Transform Based Decision Tree for Power System Faults Identification. *TELKOMNIKA Telecommunication Computing Electronics and Control*. 2011; 09: 99-106.