TELKOMNIKA Indonesian Journal of Electrical Engineering
Vol.12, No.4, April 2014, pp. 2439 ~ 2447
DOI: http://dx.doi.org/10.11591/telkomnika.v12i4.4739

Switching Surface Design for Nonlinear Systems: the Ship Dynamic Positioning

Diallo Thierno Mamadou Pathe*[1], Li Hongsheng[2], Bian Guangrong[3]
[1,2]School of Mechanical and Electrical Engineering, Wuhan University of Technology,
No. 122, Luoshi Road, Hongshan District, Wuhan, Hubei, P.R. China, Postcode: 430070
[3]Air Force Service College, Jiangsu Province, P.R. China
*Corresponding author, e-mail: diallo.lessi@hotmail.com[1], lihswh@gmail.com[2], 44973901@qq.com[3]

Abstract
In this paper a design of the switching-surface for the nonlinear system is studied. The aim was to prove that with the linear matrix inequality the coefficients of the sliding surface can be determined optimally for the control law structure. The advantages of the use of the linear matrix inequality reside in the accurate determination of the coefficients of the sliding surface. The sliding mode control for dynamic positioning of the ship with our proposed switching-surface is done. The objective of this control was to make sure that the ship follows a predetermined track. The good trackings are observed from the simulation results which confirm the robustness of the control law obtained by our proposed switching-surface.

Keywords: *control, sliding mode, switching-surface, LMI, dynamic positioning*

1. Introduction

Dynamic positioning system (DPS) has been applied on vessel since the 1960s, and today DPS is equipped on many new vessels used for freight transport, offshore exploration and exploitation [1]. The objective of dynamic positioning systems in ship is to maintain the marine vessel in a fixed position and heading in the horizontal plane or to follow a predetermined track by means of the ship propulsion system [2]. In this perspective; we have designed a control law for the ship to achieve the desired behaviors. In the aspect of control methods, fuzzy control and sliding mode control are different from conventional control theory, and each of them has its advantages and disadvantages. Fuzzy control needs not an accurate mathematical model of object creation and has a good robustness. However, once control rule and coefficient are fixed, fuzzy control cannot adapt condition change well. Sliding mode control has the advantage of fast response characteristic, and it is not sensitive to parameter variation and fast load changes [3]. Too many fuzzy rules make the network structure become complex and have the poor generalization capability and over fitting [4]. The disadvantage of the sliding mode control is the presence of the chattering in the controller (most frequently in the first order sliding mode) which can be mitigate or reduced by the use of the higher order sliding mode control. The designed control law for the ship in this paper is a sliding mode or switching control law. This control law was obtained from a switching-surface based on the linear matrix inequality approaches. Sliding mode controller is an influential nonlinear controller to certain and uncertain systems which it is based on system's dynamic model [5].

The purpose of the switching control law is to drive the plant's state trajectory onto a prespecified (user-chosen) surface in the state space and to maintain the plant's state trajectory on this surface for all subsequent time. This surface is called a switching-surface and the resulting motion of the state trajectory a sliding mode [6]. The sliding mode control or variable structure control design generally breaks down into two phases. The first phase is to design or choose a sliding manifold/switching surface, so that the plant state restricted to the surface has desired dynamics. The second phase is to design a switched control that will drive the plant state to the switching surface and maintain it on the surface upon interception [6, 7].

In this study we use the linear matrix inequality known as LMI in the design of the sliding manifold/switching surface. After the determination of the optimized sliding surface we design a

Received August 20, 2013; Revised October 12, 2013; Accepted November 5, 2013

sliding mode controller which will be used in the tracking control for dynamic positioning of the ship. The simulations result shows the good tracking of the positions, and velocities of the ship. This paper is organized in 7 sections. The LMI formulation is presented in Section 2. In Section 3, the non-linear model is drawn which is intended for the switching-surface design by the LMI in Section 4. Sliding mode controller is determined in Section 5. In Section 6 an application to the tracking control for dynamic positioning of the ship is presented. This Section is divided in two subsections. At first the system model of the Ship is presented while the second part shows the simulation results obtained by MATLAB. Finally a conclusion is given in the Section 7.

2. The LMI formulation

The history of LMIs in the analysis of dynamical systems goes back more than 100 years. The story begins in about 1890, when Lyapunov published his seminal work introducing what we now call Lyapunov theory [8]. He showed that the differential equation $\dot{X}(t) = AX(t)$ is stable if and only if there exists a positive definite matrix P such that $A^T P + PA < 0$.

2.1. Definition

The typical linear matrix inequality or LMI problem has the form: $\min f(X_1, X_2,..., X_n)$ subject to: $L(X_1, X_2,..., X_n) - R(X_1, X_2,..., X_n) < 0$ where $X_1, X_2,..., X_n$ are matrix variables with some prescribed structure, $L(.)$, $R(.)$ are affine combinations of $X_1, X_2,..., X_n$ and their transpose, and $\min f(X_1, X_2,..., X_n)$ is a linear function of the entries of $X_1, X_2,..., X_n$, finally “ < 0 ” stands for “semi-definite” [8, 9]. Many control problems such as the standard Lyapunov can be formulated as LMI minimization or feasibility problem.

2.2 Theorem

Lyapunov theorem

$$\begin{cases} \dot{X}(t) = f(X(t), t) = AX(t) \\ X(t_0) = X_0 \end{cases} \tag{1}$$

The equilibrium point $X_e = 0$ is stable in the sense of Lyapunov if: There is a Lyapunov function $V(X(t)) > 0$ continue in $X(t)$ such that $V(0) = 0$, and $\dot{V}(X(t)) \leq 0$.

By choosing $V(X(t)) = X^T(t)PX(t)$, where P is a symmetric positive definite matrix; the system (Equation 1) is asymptotically stable if there exist $P > 0$, $Q > 0$ such that:

$$A^T P + PA = -Q \tag{2}$$

The matrix equation (Equation 2) is the algebraic Lyapunov equation.
Note that: for $Q > 0$ the matrix equality (Equation 2) can be written as:

$$A^T P + PA < 0 . \tag{3}$$

The inequality matrix (Equation 3) is called a Lyapunov inequality on P and it is a special form of an LMI. The feasibility solution of this LMI can be finding by Matlab.

3. The Non-linear Model

Consider the systems that have a state model nonlinear in the state vector $X(.)$ and linear in the control vector $U(.)$ of the form [6, 10]:

$$\dot{X}(t) = F(X, t, u) = f(X, t) + B(X, t)U(X, t) \tag{4}$$

Where $X(t) \in R^n$,$U(t) \in R^m$ and $B(X,t) \in R^{n \times m}$; further, each entry in $f(X)$ and $B(X,t)$ is assumed continuous with the bounded continuous derivative with respect to X .

The dynamics of equation (Equation 4) can be write as:

$$\begin{cases} \dot{X}_1 = f_1(X,t) \\ \dot{X}_2 = f_2(X,t) + B_2(X,t)U(X,t) \end{cases} \tag{5}$$

Where $X_1 \in R^{n-m}$, $X_2 \in R^m$ and $X = [X_1, X_2]^T \in R^n$

In this part, we consider the non-linear systems (Equation 5) which have f_1 linear; so $\dot{X}_1$ can be written as:

$$\dot{X}_1 = [A_{11}\ A_{12}] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = A_{11}X_1 + A_{12}X_2 \tag{6}$$

For the equation (Equation 6) consider X_2 as a new control law; the objective is to find a stabilizing state-feedback law $X_2 = KX_1$ where K is an unknown matrix which will be determined by the linear matrix inequality (LMI).

In equation (Equation 6) we replace X_2 by KX_1 and we obtain:

$$\dot{X}_1 = A_{11}X_1 + A_{12}KX_1 = (A_{11} + A_{12}K)X_1 \tag{7}$$

By analogy with (Equation 1); the linear system (Equation 7) is stable if and only if a symmetric positive definite matrix P exist such that the inequality (Equation 8) or equivalently (Equation 9) where $H > 0$ yields [8].

$$[A_{11} + A_{12}K]^T P + P[A_{11} + A_{12}K] < 0 \tag{8}$$

$$H[A_{11} + A_{12}K]^T + [A_{11} + A_{12}K]H < 0 \tag{9}$$

From (Equation 9) let define $Y = KH$, so that for $H > 0$ we have $K = YH^{-1}$ and substituting K into (Equation 9) we obtain the LMI (Equation 10). The feasibility solution of this LMI can be find through Matlab.

$$A_{11}H + HA^T{}_{11} + A_{12}Y + Y^T A^T{}_{12} < 0 \tag{10}$$

4. Switching-surface design by the LMI

Consider the regular form (Equation 5) for the design of the switching-surface.

4.1. Proposition

For the system dynamic (Equation 5) which has f_1 linear, our proposed switching-surface is defined by:

$$\sigma(X,t) = [S_1\ S_2] \begin{bmatrix} X_1 - X_{1r} \\ X_2 - X_{2r} \end{bmatrix} = S_1[X_1 - X_{1r}] + S_2[X_2 - X_{2r}]. \tag{11}$$

Where X_{1r} and X_{2r} are the desired functions; $[S_1\ S_2]$ is the matrix gain which we want find by the LMI with the condition S_2 non singular.

The system (Equation 5) is in a sliding mode, that is, for some t_1, $\sigma(X,t) = 0$ for all $t > t_1$.

$$\sigma(X,t)=0 \Rightarrow X_2 = -S_2^{-1}S_1X_1 + S_2^{-1}S_1X_{1r} + X_{2r} \quad (12)$$

The goal is to determine S_1 and S_2 to achieve a desired behavior of the linear system (Equation 6); replacing X_2 by KX_1 in equation (Eq. 12) we find out:

$$X_2 = -S_2^{-1}S_1X_1 + S_2^{-1}S_1X_{1r} + X_{2r} = KX_1 \Rightarrow \begin{cases} K = -S_2^{-1}S_1 \\ S_2^{-1}S_1X_{1r} + X_{2r} = 0 \end{cases}$$

Without loss of generality we can take $S_2 = I$ (matrix identity), finally we get $K = -S_1$. The switching-surface becomes:

$$\sigma(X,t) = -K[X_1 - X_{1r}] + I[X_2 - X_{2r}]. \quad (13)$$

5. Sliding Mode Controller

In the precedent section, the sliding surface (Equation 13) for the system dynamic (Equation 5) which has f_1 linear has been determined by the LMI.

5.1. Theorem

For the system model defined in (Equation 5) which has f_1 linear, the sliding mode controller which makes the tracking errors tend asymptotically to zeros in finite time can be written as:

$$U(X,t) = U_{eq}(X,t) + U_r(X,t) \quad (14)$$

Where $U_{eq}(X,t) = -B_2^{-1}(X,t)\beta(X)$ with $\beta(X) = f_2(X,t) - IK([A_{11} + A_{12}K]X_1 - \dot{X}_{1r}) - \dot{X}_{2r}$ is the equivalent control, and $U_r(X,t) = -B_2^{-1}(X,t)sign(\sigma(X,t))$ is the robust control term.

Proof: Let consider the candidate Lyapunov function V :

$$V = \frac{1}{2}\sigma^T\sigma \Rightarrow \dot{V} = \frac{1}{2}(\dot{\sigma}^T\sigma + \sigma^T\dot{\sigma}) = \sigma^T\dot{\sigma}$$

The first derivative with respect to t of σ (Equation 13) is done: $\dot{\sigma}(X,t) = -K(\dot{X}_1 - \dot{X}_{1r}) + I(\dot{X}_2 - \dot{X}_{2r})$. Replacing $\dot{X}_2$ from the system (Equation 5) and $\dot{X}_1$ from the equation (Equation 7) we determine:

$$\dot{\sigma}(X,t) = -K[[A_{11} + A_{12}K]X_1 - \dot{X}_{1r}] + I[[f_2(X,t) + B_2(X,t)U(X,t)] - \dot{X}_{2r}] \quad (15)$$

The equivalent control law: constitutes a control input which, when exiting the system, produces the motion of the system on the sliding surface whenever the initial state is on the surface [6]. It is determined by assuming $\dot{\sigma}(X,t) = 0$, from where we obtain:

$$U_{eq}(X,t) = -B_2^{-1}(X,t)\beta(X) \quad (16)$$

With $\beta(X) = f_2(X,t) - IK([A_{11} + A_{12}K]X_1 - \dot{X}_{1r}) - \dot{X}_{2r}$.

The robust control law: In equation (Equation 15), we have to replace $U(X,t)$ by the equation (Equation 14) and $U_{eq}(X,t)$ by the equation (Equation 16). Finally, $\dot{\sigma}(X,t) = IB_2(X,t)U_r(X,t)$ and $\dot{V} = \sigma^T[IB_2(X,t)U_r(X,t)]$.

By assuming $U_r(X,t) = -B^{-1}{}_2(X,t)sign(\sigma)$, the derivative of the Lyapunov function V is negative ($\dot{V} = -|\sigma| \le 0$) which mean that the system is stable.

6. Application to the Tracking Control for Dynamic Positioning of the Ship

In this section, we want show through the simulations that the proposed technique leads to a good tracking trajectory in Ship dynamic positioning. In this perspective, using the dynamic model of the Ship; we have determined the state representation of the system which is used in the Matlab simulation.

6.1. System Model of the Ship

The reduced equations of motion of dynamic positioning (DP) ship in surge, sway and yaw can be expressed as follows [2]:

$$\begin{cases} M\dot{\upsilon} + D\upsilon = \tau \\ \dot{\eta} = J(\eta)\upsilon \end{cases} \quad (17)$$

Where $\upsilon = [u, v, r]^T \in R^3$ denotes the low-frequency velocity vector, τ is a vector of control forces and moments, $\eta = [x, y, \psi]^T$ denotes the position and orientation vector with coordinates in the earth-fixed frame, $J(\eta)$ is a velocity transformation matrix that transforms velocities of the ship-fixed to the earth-fixed reference frame. The inertia matrix M is assumed to be positive definite, and $D > 0$ is a matrix representing linear hydrodynamic damping [2, 11].

The successive derivative of η from the system (Equation 17) is done:

$$\begin{cases} \dot{\eta} = J(\eta)\upsilon \\ \ddot{\eta} = [\dot{J}(\eta) - J(\eta)M^{-1}D]J^{-1}(\eta)\dot{\eta} + J(\eta)M^{-1}\tau = A(\eta)\dot{\eta} + B(\eta)\tau \\ \dddot{\eta} = [A(\eta) + \dot{B}(\eta)B^{-1}(\eta)]\ddot{\eta} + [\dot{A}(\eta) - \dot{B}(\eta)B^{-1}(\eta)A(\eta)]\dot{\eta} + B(\eta)\dot{\tau} \end{cases} \quad (18)$$

with $A(\eta) = [\dot{J}(\eta) - J(\eta)M^{-1}D]J^{-1}(\eta)$, and $B(\eta) = J(\eta)M^{-1}$.

The system (Equation 18) can be represented in state space as (Equation 5) with the variables: $x_1 = \eta$, $x_2 = \dot{x}_1$ and $x_3 = \dot{x}_2$.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = [A + \dot{B}B^{-1}]x_3 + [\dot{A} - \dot{B}B^{-1}A]x_2 + B\dot{\tau} \end{cases} \Leftrightarrow \begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} O & I \\ O & O \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} O \\ I \end{bmatrix} x_3 = f_1(X,t) \\ \dot{x}_3 = f_2(X,t) + B_2(X,t)U(X,t) \end{cases} \quad (19)$$

with $f_1(X,t) = \begin{bmatrix} O & I \\ O & O \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} O \\ I \end{bmatrix} x_3$, $f_2(X,t) = [A + \dot{B}B^{-1}]x_3 + [\dot{A} - \dot{B}B^{-1}A]x_2$, $U(X,t) = \dot{\tau}$, $X = [x_1 \quad x_2 \quad x_3]^T$, and $B_2(X,t) = B$; where I and O are respectively the identity and null matrix with appropriate dimensions. By taking: $X_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$; $X_2 = x_3$; $A_{11} = \begin{bmatrix} O & I \\ O & O \end{bmatrix}$; $A_{12} = \begin{bmatrix} O \\ I \end{bmatrix}$ we get the linear form: $\dot{X}_1 = A_{11}X_1 + A_{12}X_2$.

Let denote the desired functions as: $X_{1r} = [x_{1r}, x_{2r}]^T$, $X_{2r} = x_{3r}$ with $x_{1r} = \eta_{1r} = [x_{1r}, y_{1r}, \psi_{1r}]^T$; $x_{2r} = \dot{\eta}_{1r} = \eta_{2r} = [x_{2r}, y_{2r}, \psi_{2r}]^T$, and $x_{3r} = \dot{\eta}_{2r} = \eta_{3r} = [x_{3r}, y_3, \psi_{3r}]^T$. From the system equation (Equation 18), we have the relation $\dot{\eta} = J(\eta)\upsilon$, similarly for the desired trajectory we can take $\dot{\eta}_{1r} = J(\eta_{1r})\upsilon_{1r}$ where: $\eta_{1r} = [x_{1r}, y_{1r}, \psi_{1r}]^T = [x_r, y_r, \psi_r]^T$ and $\upsilon_{1r} = [u_r, v_r, r_r]^T$.

Note that x_r, y_r, ψ_r represents the desired positions, and u_r, v_r, r_r the desired velocities of the ship.

The proposed switching-surface is done by the equation (Equation 13) and the sliding mode control law by the equation (Equation 14).

6.2. Simulation Results

The desired positions are x_r, y_r (chooses to be squares), and ψ_r (choose to be sinusoidal); the desired linear velocities are u_r , and v_r ; the desired angular velocity is r_r .

The numerical parameters of the model is done [11]:

$$M = \begin{bmatrix} 1.1274 & 0 & 0 \\ 0 & 1.8902 & -0.074 \\ 0 & -0.074 & 0.1278 \end{bmatrix}, D = \begin{bmatrix} 0.0358 & 0 & 0 \\ 0 & 0.1183 & -0.0124 \\ 0 & -0.0041 & 0.0308 \end{bmatrix}, \text{ and}$$

$$J(\eta) = J(x_1) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

According to the above parameters, the LMI (Equation 10) is feasible and the solutions obtained by Matlab done the values of H and K :

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}, \text{ and } K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$$

Where $H_{11} = 1.7339 I_3$, $H_{12} = H_{21} = -0.8510 I_3$, $H_{22} = 1.3614 I_3$, $K_1 = -1.5773 I_3$, $K_2 = -1.6865 I_3$, and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

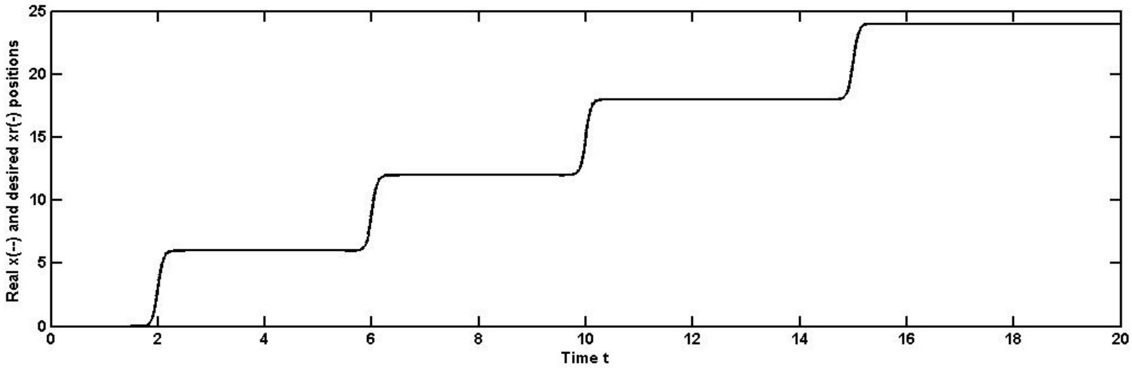

Figure 1. Positions x(—) and x_r (–)

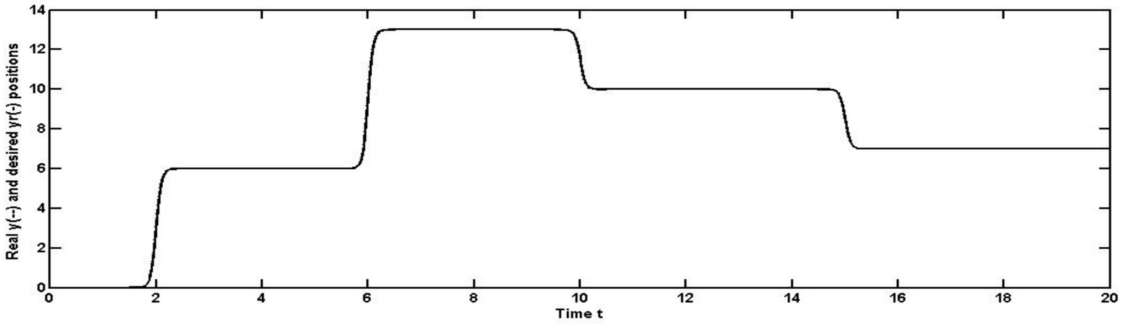

Figure 2. Positions $y(--)$ and $y_r(-)$

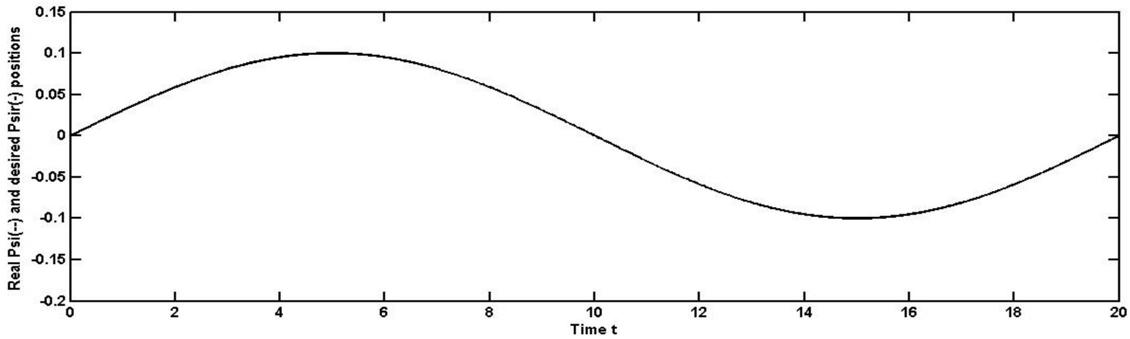

Figure 3. Yaw Angle $\psi(—)$ and $\psi_r(-)$

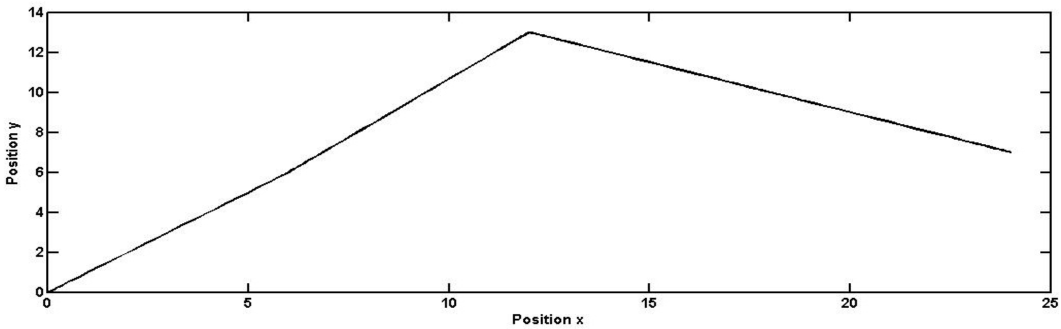

Figure 4. The Position y in Function of x

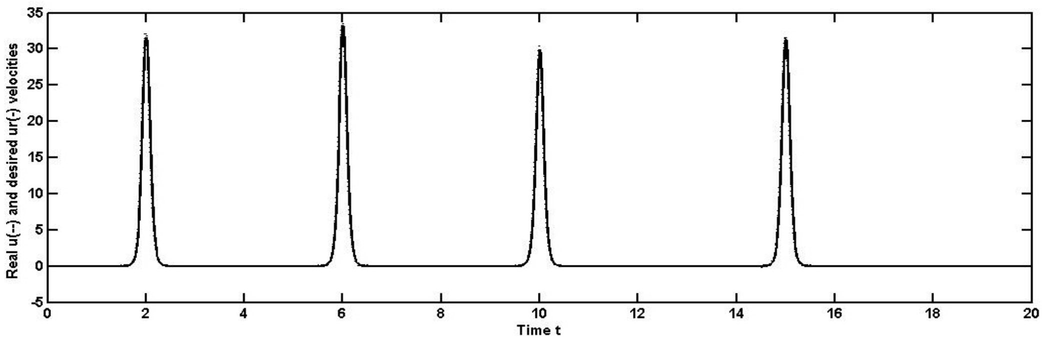

Figure 5. Velocities $u(—)$ and $u_r(-)$

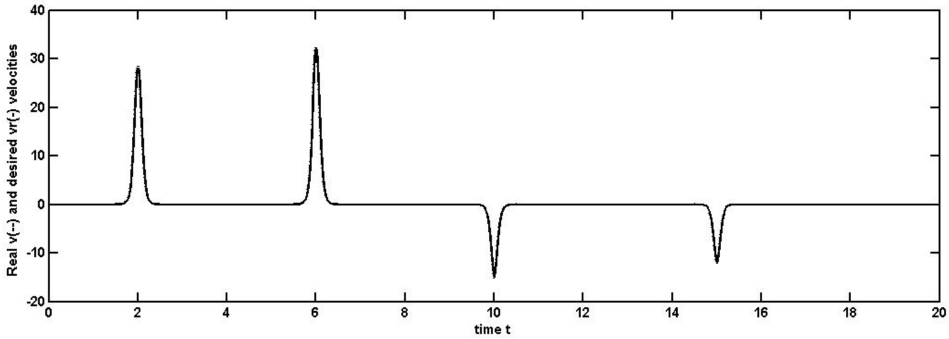

Figure 6. Velocities $v(—)$ and $v_r\,(-)$

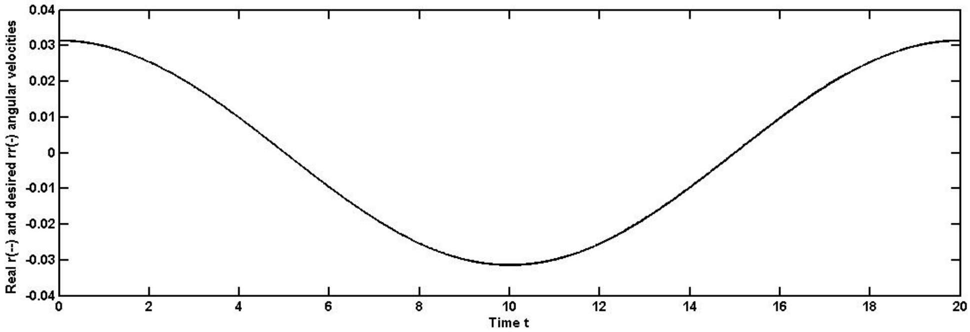

Figure 7. Yaw Velocities $r(—)$ and $r_r\,(-)$

We can observe respectively the real $x(—)$, $y(—)$ and the desired $x_r\,(-)$, $y_r\,(-)$ positions (Figure 1, Figure 2), the real $\psi(—)$ and the desired $\psi_r(-)$ yaw angles (Figure 3), the position y in function of x (Figure 4); respectively the real $u(—)$, $v(—)$ and the desired $u_r(-)$, $v_r\,(-)$ velocities (Figure 5, Figure 6), the real $r(—)$ and the desired $r_r\,(-)$ yaw velocities (Figure 7).

7. Conclusion

In this work, the switching-surface is designed using the LMI optimization technique for the non-linear systems defined in (Equation 5) satisfying the linear condition as defined previously. With the designed switching-surface; a sliding mode controller is proposed. As an application, the system model of the ship is used for the tracking trajectory in dynamic positioning. The simulations result shows the good performance of the used technique.

References

[1] Van Phuoc Bui, Sang Won Ji, Kwang Hwan Choi, Young Bok Kim. *Nonlinear Observer and Sliding Mode Control Design for Dynamic Positioning of a Surface Vessel.* International Conference on Control, Automation and Systems (ICCAS). Jeju Island. 2012: 1900-1904.
[2] Myung-Hyun Kim. Nonlinear Control and Robust Observer Design for Marine Vehicles. Ph.D Thesis. Virginia Polytechnic Institute and State University; 2000.
[3] Liping Fan, Dong Huang, Minxiu Yan. Fuzzy Sliding Mode Control for a Fuel Cell System. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2013; 11(5): 2800-2809.
[4] Jing Zhao, Ming Li, Zhihong Wang. Fuzzy Neural Networks Learning by Variable-dimensional Quantum-behaved Particle Swarm Optimization. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2013; 11(10): 6216-6223.

[5] Farzin Piltan, Shahnaz Tayebi Haghighi. Design Gradient Descent Optimal Sliding Mode Control of Continuum Robots. *IAES International Journal of Robotics and Automation (IJRA)*. 2012; 1(4): 175-189.
[6] William S. Levine. Editor. Control System Advanced Methods. New York: CRC Press Taylor and Francis Group, LLC. 2011.
[7] TC Kuo, YJ Huang, BW Hong. Design of Adaptive Sliding Mode Controller for Robotic Manipulators Tracking Control. *World Academy of Science, Engineering and Technology*. 2011; 0053: 190-194.
[8] Stephen Boyd, Laurent El Ghaoui, Eric Feron, Venkataramanan Balakrishnan. *Editors*. Linear Matrix Inequalities in System and Control Theory. Philadelphia: SIAM Society for Industrial and Applied Mathematics. 1994.
[9] Mario Innocenti, Gianpiero Campa. Robust *control of Underwater Vehicles: Sliding Mode VS. LMI Synthesis*. Proceedings of the 1999 American Control Conference (ACC99). San Diego. 1999; 5: 3422-3426.
[10] Sam Maurus. *Matlab Simulation of a Robot Control Scheme*. HES5250 Robot System Design. Teaching Period. 2009: 5.
[11] Liu Furong, Chen Hui, Gao Haibo. Application of Moving Horizon Filter for Dynamic Positioning Ship. *Journal of Wuhan University of Technology*. 2010; 32(12): 117-120.