

Optimal Decision-making on Charging of Electric Vehicles

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Abstract

Large-scale development of electric vehicles (EVs) will have a significant impact on the power grid with their great demand for electric power consumption. It is necessary to design the optimal charging scheme for EVs. In this paper, we propose a centralized and a decentralized scheme of optimal decision-making on charging of EVs based on a dynamic pricing model. The objective of both schemes is to minimize the costs of EVs by establishing EV charging optimal schedules that fills the overnight load valley. The centralized scheme, whose objective is to minimize the total charging costs of all EVs, determines the charging schedules of EVs through solving a global optimization problem by the utility. The decentralized scheme is based on congestion game theory, under which the charging schedules are determined locally and directly by EVs to minimize their own costs. Detailed mathematical models and solution procedures are presented for both the centralized and decentralized schemes. Results show that both schemes have good performance on valley-filling.

Keywords: electric vehicles, minimum cost, optimal charging, valley-filling

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1. Introduction

The charging of a large population of EVs has a significant impact on the power grid [1-18]. A number of studies have been undertaken to explore the potential impacts of high penetrations of EVs on the power grid and many centralized optimization approaches for EV charging have been proposed [2-12]. Different centralized optimization approaches have different objectives, such as to minimize generation costs, power losses and load variance, or maximize load factor and supportable EV penetration level. All of them have not considered the costs of EVs.

So far, few decentralized optimal schemes for EV charging have been presented [15-17]. Ma [15] proposes a decentralized charging strategy, which is only effective in the case where all EVs consume the same amount of energy at the same charging power. Gan [16] gives two decentralized algorithms, one synchronous and one asynchronous, of which the latter is more practical, but its convergence rate is lower and its performance is likely to be affected by communication delays and failures. Vaya [17] proposes a decentralized charging strategy based on tariff, where different prices can be defined at different nodes, but the amount of calculation of the scheme increases with the number of nodes in grid.

In order to encourage EVs to charge during load valleys, we model the charging price, seen by all EVs, as a monotone increasing function of the total load on the grid. Based on this pricing model, we propose a centralized and a decentralized scheme of optimal decision-making on charging of EVs.

However, under the decentralized scheme, with this dynamic charging price, if EVs predict a low price at a certain time, they will all charge, which then results in large peak in demand at that time. In the end, there can be unpredictable and large peaks in demand and prices, which, in turn, result in higher costs for individual EV. Thus, EVs should consider the others' charging strategies when making a decision on its own charging strategy. In this scenario, game theory provides a framework to evaluate and design EVs charging strategy, since it naturally models interactions in distributed decision making processes [18-20]. At present, the applications of the game theory in power system mainly focus on topics of

electricity market. In this paper, we utilize game theory to solve EVs decentralized optimal charging problem. Our contributions are summarized as follows.

We formulate a centralized optimal charging scheme whose objective is to minimize the total cost of EVs, which is different with other centralized approaches [4-15]. The global optimal solution fills the load valley effectively.

We formulate a decentralized optimal charging scheme, which is also effective in the case where EVs consume different amount of energy at different charging power. We build a congestion game model of EVs charging. We prove the EV charging congestion game model is a potential game, which is sure to have a Nash equilibrium solution. Then a distributed solution is proposed. The equilibrium solution can fill the load valley effectively as the centralized method.

2. Centralized Scheme

Centralized scheme requires the utility to collect much private information from all the EVs, such as state of charge, charging capacity and the initial of departure time, and on this basis, the utility solves the global optimal problem whose objective is to minimize the total cost of all EVs, attaining the charging profiles of all EVs and then centrally control EVs' charging.

We assume the charging price is dynamic, which is modeled as a monotone increasing linear function of the total load including non-EVs base load and EVs charging load on the grid. The charging price model is given as follows:

$$\begin{cases} Pr_t = kq_t \\ q_t = s_t + l_t \end{cases} \quad (1)$$

Where t is charging interval, k is price coefficient, q_t is the total load at interval t , s_t is the total charging load at interval t , l_t is the base load at interval t .

Based on (1), the total cost of EVs charging at t is:

$$\begin{aligned} c_t &= \int_{l_t}^{q_t} kq_m dq_m \\ &= \frac{k}{2} (q_t^2 - l_t^2) = \frac{k}{2} [(s_t + l_t)^2 - l_t^2] \\ &= \frac{k}{2} (s_t^2 + 2s_t l_t) \end{aligned} \quad (2)$$

Thus, the centralized scheme to minimize the total cost of EVs charging is described as the following optimal problem:

$$\min_{s_t, s_{i,t}} \sum_{t=1}^T c_t = \sum_{t=1}^T \frac{k}{2} (s_t^2 + 2s_t l_t) \quad (3)$$

Subject to:

$$s_t = \sum_{i=1}^N s_{i,t} \omega_i ; \quad (4)$$

$$s_{i,t} = 0 \text{ or } 1 ; \quad (5)$$

$$\omega_i \sum_{t=1}^T s_{i,t} = C_{i,D} ; \quad (6)$$

Where T is the number of intervals during the charging period in a day, $s_{i,t}$ is the charging strategy of Ev i at t , which only has two values that "0" means no charging and "1" means charging, N is the number of EVs, ω_i is the charging power of Ev i , $C_{i,D}$ is the charging capacity of Ev i . (6) guarantees the battery of every EV to be full at the end of the charging.

The above global optimal problem is a nonlinear integer programming with linear constraints, which can be solved by branch and bound method [22]. However, the size of this centralized optimization increases with the increasing number of EVs. If only the total charging load at every interval are viewed as optimization variables, the optimization problem can be easily solved by interior point method [22], but it is a very elaborate problem to define the charging strategy of every EV.

3. Decentralized Scheme

As can be seen in section 2, it is difficult to work out the optimum solution of the centralized scheme when the number of EVs is huge. What's more, the centralized approach may be unrealizable due to a reluctance among EV owners to allow third parties to directly control their charging behaviors [7]. Hence, a decentralized scheme is more suitable for the case with a high penetration level of EVs. In this section, we propose a decentralized scheme based on congestion game theory.

3.1. Basics of Congestion Game

3.1.1. The Definition of Congestion Game

A congestion model [23-24] can be defined as a tuple $(N, E, (S_i)_{i \in N}, (c_e)_{e \in E})$ where:

$N = \{1, 2, \dots, n\}$ denotes the set of player.

$E = \{1, 2, \dots, r\}$ denotes the set of resource.

Each player i has a strategy space S_i , in which each specific strategy $s_i \in S_i$ is the set of resource, that is $S_i \subseteq 2^E$.

The congestion cost of resource $e \in E$ is determined by a function $c_e(\cdot)$ that depends on the congestion level.

Based on this congestion model, a congestion game is defined as a tuple $(N, (S_i)_{i \in N}, (c_i)_{i \in N})$, where the cost of player i under the strategy combination of $s = \{s_1, \dots, s_n\}$ is:

$$c_i(s) = \sum_{e \in S_i} c_e(n_e(s)) \quad (7)$$

Where $n_e(s)$ denotes the number of players using resource e under the strategy combination of s , and is called the congestion level of resource e .

3.1.2. Existence of the Nash equilibrium Solution of Congestion Game

If a congestion game admits a real function $\Phi: S \rightarrow R$ ($S = \times_{i \in N} S_i$) with argument of strategy combinations, when any player i change its strategy from s_i to s'_i , always satisfies the following:

$$c_i(s_i, s_{-i}) - c_i(s'_i, s_{-i}) = \Phi(s_i, s_{-i}) - \Phi(s'_i, s_{-i}) \quad (8)$$

then the congestion game is called a potential game, function Φ is potential function [23-24]. Where s_{-i} denotes the others' strategies besides player i .

Theorem: Every potential game has at least one pure Nash equilibrium [23-24].

3.2. Game Model of EVs Charging

We assume that EVs charge with the constant power, EVs' decentralized optimal decision-making on charging can be described as the following congestion game:

The players are the set of n EVs, $N=\{1, 2, \dots, n\}$;

The resources E are the charging intervals t during charging period, $t=\{1, 2, \dots, T\}$;

The strategy of EV i is the charging vector $s_i = \{s_{i,t}\}$, $t \in \{1, T\}$;

Under the strategy combination of $s = \{s_1, \dots, s_n\}$, the charging cost of EV i is:

$$c_i(s) = \sum_{t=1}^T s_{i,t} k q_t(s) \omega_i = \sum_{t=1}^T s_{i,t} k \left(\sum_{i \in J_t(s)} \omega_i + l_t \right) \omega_i \quad (9)$$

Where $q_t(s)$ is the congestion level of the congestion game, $J_t(s)$ is the set of EVs charging at t under $s = \{s_1, \dots, s_n\}$, ω_i is constant charging power of EV i .

In the game, each EV defines its optimal charging strategy to minimize its charging cost.

3.3. Existence of the Nash Equilibrium Solution of EV Charging Game

We formulate the potential function of the EV charging game model as follows:

$$\Phi(s) = \sum_{t \in \bigcup_{i=1}^n s_i} k \left(\sum_{i \in J_t(s)} \omega_i^2 + \sum_{i < j; i, j \in J_t(s)} \omega_i \omega_j + l_t \sum_{i \in J_t(s)} \omega_i \right) \quad (10)$$

The proof of the potential function is given in Appendix. It indicates the EV charging game is a potential game. We can then use Theorem in 3.1.2 to guarantee the convergence of the game to a pure Nash equilibrium.

3.4. Distributed Solution of EV Charging Game

The equilibrium solution of the game is attained by iterations according to the following steps:

1) Initializations. Each EV proposes initialized charging strategy stochastically when iteration $m=0$.

2) The utility broadcasts the base load to all EVs and initialized aggregate EV demand to EV i .

3) EV i solves the following optimization problem to attain the optimum charging strategy and reports it to the utility, if $c_i(s_i^{m-1}, s_{-i}^m) - c_i(s_i^m, s_{-i}^m) \leq \varepsilon$, let $s_i^m = s_i^{m-1}$.

$$\min_{s_{i,t}} c_i = \sum_{t=1}^T s_{i,t} \Pr_t \omega_i \quad (11)$$

Subject to:

$$\Pr_t = k \left(\sum_{-i \in J_t(s)} \omega_{-i} + s_{i,t} \omega_i + l_t \right); \quad (12)$$

$$s_{i,t} = 0 \text{ or } 1; \quad (13)$$

$$\omega_i \sum_{t=1}^T s_{i,t} = C_{i,D} \quad (14)$$

Where ω_{-i} is charging power of EVs other than EV i at t .

4) Update next EV's aggregate EV demand, the EV calculates its optimum charging strategy according to 3).

5) Repeating 4) until all EVs' optimum charging strategies calculation are finished.

6) Next iteration begins, repeating 4)~5), until all EVs' strategies won't change.

From the above steps, we can see that each EV make a decision on the optimal strategy and update it based on the other Evs' strategies, until an equilibrium is reached. The local computational complexity of the distributed solution has no relevance to the EV population size.

4. Results and Analysis

We examine EVs charging starting from 20:00 PM to 05:00 AM. The charging period is evenly divided into 10 intervals. Each interval has a length of 1 h. The base load at each interval is simulated by scaling the typical day load in Guangdong grid of china in winter of 2006 by a factor of 1/5700. The total number of the EVs is set to 100 by default. We set $k=2 \times 10^{-4}$ yuan/kWh/kW.

In order to show the schemes proposed in this paper are effective for EVs with different charging capacity and different charging power, we assume there are three types of EVs: charging power 6kW with charging capacity of 24kWh, charging power 5kW with battery capacity of 20kWh, charging power 3kW with battery capacity of 12kWh, respectively occupy 50%, 30% and 20% of the total number of EVs. To simplify the analysis, we assume the charging capacity equals the battery capacity.

We set $\varepsilon = 10^{-4}$ yuan, the distributed solution of the decentralized scheme in section 3.4 converges after 5 iterations.

We compare the centralized and decentralized schemes proposed in this paper with the free charging scheme, in which the charging strategy of an EV at an interval is allocated based on the electricity price on the previous day.

The variation of the charging load in each interval of three schemes is shown in Figure 1. We can see from Figure 1 that under three schemes, EVs charge at the intervals with a lower base load to achieve a low cost.

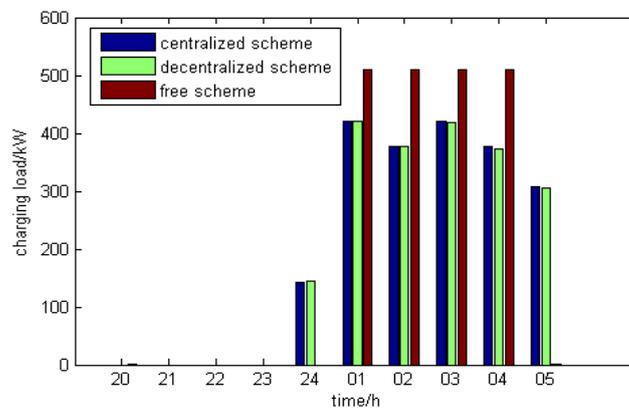


Figure 1. Charging Load of EVs

The variation of the total load in each interval of three schemes with 100 EVs and 200 EVs are shown in Figure 2 and Figure 3. We can see from Figure 2 that all the schemes achieve "valley-filling", however, under the centralized and decentralized schemes the total load profile are much flatter, while the load fluctuation brought by the free charging scheme is larger. Figure 3 shows that with the number of EVs increases, the load fluctuation brought by the free charging scheme is more obvious, and there will be new load peaks.

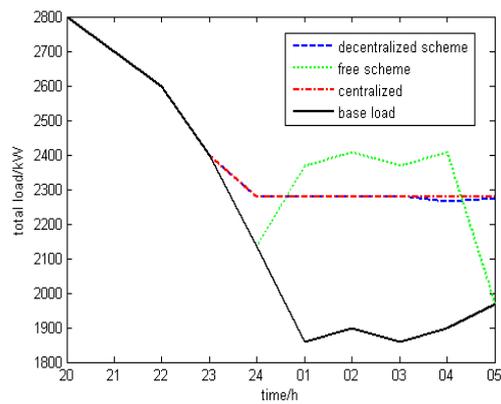


Figure 2. Load Profiles in Three Charging Scheme with 100 EVs

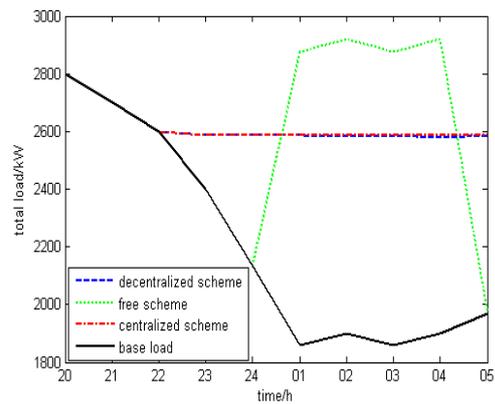


Figure 3. Load Profiles in Three Charging Scheme with 200 EVs

The comparison of the total charging costs of all EVs with different EV numbers of three schemes is given in Figure 4. We can see from Figure 4 that under the centralized and decentralized schemes, EVs will pay the utility almost the same cost, which is less than that under the free charging scheme. Furthermore, compared with the free charging scheme, the centralized and decentralized scheme will get more cost saving when the number of EV increases. When the number of EV is 100, 200, 300 and 400, the corresponding saving is 4.7%, 10.8%, 17.18% and 22.75%.

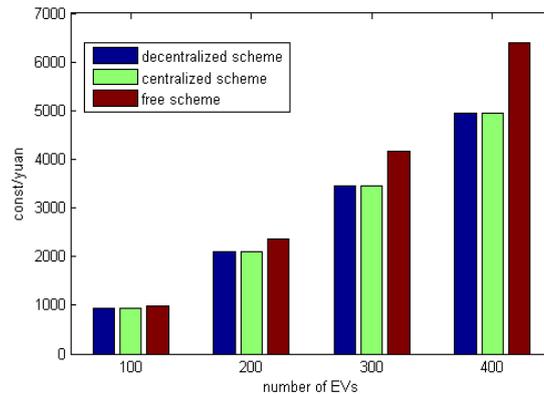


Figure 4. Cost of EVs in Three Charging Schemes

5. Conclusion

In this paper, we present a centralized and a decentralized optimal decision-making on charging of electric vehicles to fill load valley, by means of dynamic pricing model. We first formulate the global optimal problem of centralized scheme, in which the charging strategies of EVs are optimized to minimize the total cost. The globally optimal solution provides the globally minimal total cost. However, the centralized optimal scheme poses the problem of EV acceptance and requires the utility to have strong calculation capability when the number of EVs is large. To develop a more practical scheme, we formulate the decentralized scheme based on the game theory, under which each EV defines optimum charging strategy to minimize its charging cost based on the other EVs' strategies. Under the scheme, the decision-making is made by EVs, so the scheme is more willing to be accepted, what's more, the number of EV will not affect the amount of calculation, therefore, the decentralized scheme is a more practical scheme. Simulation results show that the decentralized scheme can achieve a close

performance on flattening the day load profile compared to the centralized scheme. Future works will focus on the optimal scheme of EVs charging and discharging.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (No. 51177111).

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Appendix-Proof of Potential Function

Proof: When the charging strategy of Ev i change form s_i to s'_i , and the other Evs' strategies keep s_{-i} , there exists two situations:

(1) Ev i charged at t before, after it change its strategy it didn't charge at t , thus the decreased cost of Ev i is:

$$\Delta c_{i,t}(s) = k \left(\sum_{i \in J_t(s)} \omega_i + l_t \right) \omega_i,$$

The decrement of potential function of resource t is:

$$\Delta \Phi_t(s) = k \left(\omega_i^2 + \sum_{j \neq i, j \in J_t(s)} \omega_i \omega_j + l_t \omega_i \right) = k \omega_i \left(\omega_i + \sum_{j \neq i, j \in J_t(s)} \omega_j + l_t \right) = k \omega_i \left(\sum_{i \in J_t(s)} \omega_i + l_t \right),$$

$$\text{Thus } \Delta \Phi_t(s) = \Delta c_{i,t}(s).$$

(2) Ev i didn't charge at t before, after it change its strategy it charge at t , thus the increased cost of Ev i is:

$$\Delta c_{i,t}(s) = k \left(\sum_{j \in J_t(s)} \omega_j + \omega_i + l_t \right) \omega_i,$$

The increment of potential function of resource t is:

$$\Delta \Phi_t(s) = k \left(\omega_i^2 + \sum_{j \in J_t(s)} \omega_i \omega_j + l_t \omega_i \right) = k \omega_i \left(\omega_i + \sum_{j \in J_t(s)} \omega_j + l_t \right),$$

$$\text{Thus } \Delta \Phi_t(s) = \Delta c_{i,t}(s).$$

$$\text{All this leads up to: } \Delta \Phi(s) = \sum_t \Delta \Phi_t(s) = \sum_t \Delta c_{i,t}(s) = \Delta c_i(s).$$

$$\text{That is } c_i(s_i, s_{-i}) - c_i(s'_i, s_{-i}) = \Phi(s_i, s_{-i}) - \Phi(s'_i, s_{-i}).$$