Application of L-series of Formation in Fuzzy Pattern Recognition

Jinhong Li*[1], Kangpei Zhao[2]
[1]Faculty of Science/Qilu University of Technology, Jinan China, 250353
[2]College of Mechanical Engineering/Shandong Jiaotong University, Jinan China, 250353
*Corresponding author, e-mail: lijinhong77@126.com[1], kpzhao@sdjtu.edu.cn[2]

Abstract

In this paper, the model base and objects to be identified are classed as information granules by the information granulation idea. A proper system of polar coordinates is established. The fuzzy pattern recognition problem is studied by the L-series of formation in the abstract analytic number theory. A method is given to evaluate the model base type of the information granules, which consist of massive data. Additionally，it is used to analyze the relation between process parameters and weld appearance in the welding process. At the same time, the relations between weld appearance and process parameters are studies by the rough set and the disposal of data discretization based on attribute-priority algorithm. The results with these two methods are coincident. The method proposed in this paper is proved to be true.

Keywords: fuzzy pattern recognition, L-series of formation, information granules

1. Introduction

In daily life, the people can recognize the graphics, characters and languages by the senses. But in many fields, such as meteorological science, engineering reconnaissance, environmental engineering, medicine and criminal investigation, there is a common characteristic. We need to judge and identify what type the objective with fuzzy belong to, using the known knowledge. This is a pattern recognition problem.

As a new subdiscipline in artificial intelligence in recent 30 years, the classical recognition methods are focused on the principle of maximum degree of membership and principle of proximity. Subsequently, some scholars proposed some recognition model aiming at different problems. In reference [1], a model and method of fuzzy recognition of sustainable development system is put forward. It includes the fuzzy decision theory and method of evaluating index weight vector. Zhang Shoufeng denoted the fuzzy conception by triangular fuzzy number in reference [2] and presented a new multilevel fuzzy pattern recognition model, using this model to comprehensively evaluate and recognize an enterprise competence. All these recognition methods mentioned above are with the help of fuzzy reasoning. The sample and model base are measured by utilizing membership function. In fact, the fuzzy rules are usually determined according to experience. It is difficult to establish an accurate and reasonable membership function, which will limit their applications.

In this paper, we will discuss the fuzzy pattern recognition problems by the idea of abstract analytic number theory, established by John Knofmacher. We think of the model base set as arithmetical semigroup, and consider the objects to be identified as an equivalence class. Finally the relation between the objects and the known model base is given by L-series. By this method, we can abstract some special domains as arbitrary semigroups and dispose of more types of recognition problems. Additionally, we can avoid the unnecessary trouble by choosing different membership functions in different environment and reduce the influences of subjective factors.

2. Preliminaries

In 1970, John Knofmacher established the abstract analytic number theory and

introduced the arithmetic of commutative semigroup G with an identity element 1. We use P to denote the subset of G consisting of all free generators, namely the prime elements in G. We also assume that there is a norm $|\cdot|$ defined on G. We call $(G; |\cdot|)$ an arithmetic semigroup if the following conditions are satisfied:

(i) The unique factorization principle. Every element $a \neq 1$ in G has a unique factorization of the form $a = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_s^{\alpha_s}$, where p_i are distinct elements of P and α_i are positive integers. The uniqueness is understood to be only up to the order of the factors indicated.

(ii) $|1| = 1$; $|p| > 1$ for $p \in P$.

(iii) $|a \cdot b| = |a| \cdot |b|$ for all $a, b \in G$.

(iv) the total number $N_G(x)$ of elements $a \in G$ of norm $|a| \leq x$ is finite, for each real $x > 0$.

Let X be a finite Abelian group formed with identity-preserving homeomorphisms $\chi : G \to C^\times$. Usually χ is called a character. Define an equivalence relation $\sim \chi$ on G by letting $a \sim_X b$ if and only if $\chi(a) = \chi(b)$, for all $\chi \in X$.

Let Γ_X (or simply Γ) be the set of all different equivalence classes $[a] (a \in G)$ under this relation. The pair (G, Γ_X) is called an arithmetical formation and is abbreviated as (G, Γ). The order $h = \text{card } \Gamma$ of Γ will be called the class number of (G, Γ).

3. Assumption

(i) Assume that discourse domain G is an arithmetic semigroup.

(ii) Denote the basic information in the domain as the element a in G.

(iii) Under the equivalence relation $\sim \chi$, the existing information can form n information granules. We denote that $[a]_i = \{b \mid b \sim_X a\}$, $i = 1, \cdots, n$, which is the model base in G.

4. Establishment of Mathematics Model

For the information granule to be identified $[c] = \{d \mid d \sim_X c, c, d \in G\}$, we will return $[c]$ to an existing model base according to the characteristic of the element in $[c]$.

(i) To establish polar coordinate system

For the information granule made up of several data, we view it as the dot in polar coordinate system. Let $\rho = \|[a]\|$, where ρ is the polar radius of $[a]$, $[a]$ is the equivalence class in Γ and $\|\cdot\|$ is the norm defined in Γ. $\theta = 2\pi \dfrac{d([a])}{\max d[a]}$ is the polar angle of point $[a]$, where $d([a])$ is the diameter of equivalence class $[a]$, that is $\max\limits_{a', a'' \in [a]} |a' - a''|$, which reflects the scatter degree of the elements in $[a]$.

We let $s = \sigma + it = \rho \cos\theta + \rho \sin\theta$, where s is a point on complex plane R^2.

(ii) To define L-series in (G, Γ)

Let $L_G(s, \chi) = \sum\limits_{a \in G} \dfrac{\chi(a)}{|a|^s}$ is the L-series in (G, Γ). For an arbitrary information granule $[a]$, we have $L_{[a]}(s, \chi) = \sum\limits_{a \in [a]} \dfrac{\chi(a)}{|a|^s}$. Due to that $L_G(s, \chi) = \sum\limits_{a \in G} \dfrac{\chi(a)}{|a|^s}$ is analytic for $\text{Re } s > \eta$, where η is a positive number, it is also continuous [3]. If the information of the two objects are

very close, then the difference between their corresponding L-series is very small. Therefore, for some positive integer δ, we arbitrarily choose the domain with the diameter smaller than or equal to δ on the complex plane R^2. In this domain, we have $\left|L_{[a]}(s,\chi) - L_{[c]}(s,\chi)\right| < \varepsilon$ except for finite point s, where ε is an arbitrary small positive number. Then it means $[a]$ and $[c]$ are close enough. We define the minimum approach set of $[c]$ as

$$[c]_{\min} = \left\{ [a] \Big| \min\left\{ \left\| L_{[a]}(s,\chi) - L_{[c]}(s,\chi) \right\| \right\}, [a] \neq [c] \right\}.$$

(iii) Decision rule

(a) If $[c]_{\min}$ only have an element, then this element is the model base of $[c]$;

(b) If the number of the elements in $[c]_{\min}$ is greater than or equal to 2, then we choose a proper $[a]$, such that $\|[a] - [c]\|$ is the smallest, as its alternative model. It is easy to know that these $[a]$ are on the circle with center point $[c]$.

(c) If the $[a]$ selected in (2) only have one, then this equivalence class is the model base of $[c]$. Otherwise, we choose $[a]$ in the same line with $[c]$ as its model base.

(d) If the $[a]$ in the same line with $[c]$ only have one, then this equivalence class is the model base of $[c]$. Otherwise, there are two equivalence classes $[a_1]$ and $[a_2]$. They have the same close degree with $[c]$. And $[a_1]$, $[a_2]$ and $[c]$ are collinear. We choose the equivalence class corresponding with the smallest between $\|[a_1]\|$ and $\|[a_2]\|$ as its model base.

5. Application in Relation Model Analysis between Process Parameters and Weld Appearance

In the welding process, there is a close relation between the process parameters and weld appearance. Therefore the optimizations for welding process parameters are attached importance to improve the welding quality. But due to the complexity of the welding process, it is hard to obtain its system model by the classical or traditional methods[4]. In view of the above, the influences of multiprocess parameters on the weld appearance will be discussed by pattern recognition in this paper.

In this paper, the selected material is ZL114A (aluminum with δ =8mm), which are welded on the conditions of trim=0.65, μ=17.6, L(m^3 / h)=1.5. Twelve groups of experimental data with condition attributions that {C=laser power P(a_1), WPS(a_2), V (a_3), I (a_4)} and decision attribution {D= weld width (d_1) weld depth (d_2)}.

Table 1. Welding Process Parameters and Weld Dimension

No.	Power (Kw)	WFS (in/min)	V (mm/min)	I (A)	Weld Width	Weld Depth
e1	4.0	120	1000	98	7.5	6.1
e2	3.5	120	1000	98	8.5	5.0
e3	3.0	120	1000	98	8.0	4.6
e4	2.5	120	1000	98	7.0	3.2
e5	4.0	100	1000	81	8.9	8.2
e6	4.0	140	1000	117	8.5	8.1
e7	4.0	160	1000	135	9.2	6.5
e8	4.0	200	1000	175	10.2	7.1
e9	4.0	120	800	98	9.0	8.0
e10	4.0	120	900	98	8.2	8.0
e11	4.0	120	1100	98	7.5	7.6
e12	4.0	120	1200	98	7.8	6.5

Let the domain $U = \{e_1, \cdots, e_{11}\}$, conditional attribution $C = \{a_1^1, \cdots, a_4^1\}$ and the decision attribution $D = \{d_1, d_2\}$. According to the rough set theory, we can compute that the equivalence relation partition $U/D = \{Y_1, \cdots, Y_6\}$, where $Y_1 = \{e_1, e_4, e_{11}\}$, $Y_2 = \{e_2, e_3\}$, $Y_3 = \{e_5, e_6\}$, $Y_4 = \{e_7, e_8\}$, $Y_5 = \{e_9\}$, $Y_6 = \{e_{10}\}$ [5]. We assume that $|\cdot|$ is Euclid norm and $\|\cdot\|$ is Euclid norm of the central vector in equivalence class elements. Define that the characteristic $\chi(e_i)$ of e_i is the reciprocal of the element number. By the model we established, the e_i in Table1 should belong to model base Y_1.

To prove the result to the true, we next extract the data by rough set theory and the disposal of data discretization based on attribute-priority algorithm In recent years rough set method occurred and was applied in many fields. As a newer soft computing tool, it can analyze and dispose the uncertainty of the experimental data effectively. And it is widely used in model establishment and data mining.

6. Disposal of Data Discretization
The aim of data discretization is to transform the continuous data into dispersed data for the convenience of rough set disposal. According to the material thickness and its welding requirements, we first carry on the discretization disposal for decision attribution as follows:

$$d_1 : (1 - d_1 < 8; \ 2 - 8 \le d_1 < 9 : 3 - 9 \le d_1); \ d_2 : (1 - d_1 < 7.5; \ 2 - 7.5 \le d_2 < 8; \ 3 - 8 \le d_2).$$

For decision attribution condition, we realize the disposal of data discretization with attribute-priority algorithm.

Definition 1 [6]: Let the decision system $S = <U, A=C \bigcup D, V, f>$, where U is a nonempty domain, C is nonempty conditional attribution set and $B \subseteq C$. For every subset $X \subseteq U$, the upper approximation set of X can be defined as follows by the definition of basic set B:

$$B\_(X) = \bigcup \{Y_i \,|\, (Y_i \in U \,|\, Ind(B) \wedge Y_i \subseteq X\} \cdot$$

Definition 2 [6]: Assume that the set ariety $F = \{X_1, x_2, \cdots, X_n\}$ $(U = \bigcup X_i)$ is the defined knowledge in domain U, and B is an attribute subset. The approximate class mass of B for F is defined as follows: $rB(F) = \sum |B\_(X_i)| / |U|$.

Definition 3 [7]: F is the classification derived from the attribute set D. The importance of attribute subset B' in attribute set B is defined as $rB(F) - rB \setminus B'(F)$.

Definition 4 [6]: The decision system $S = <U, A=C \bigcup D, V, f>$. Assume that U is nonempty domain, C is nonempty conditional attribute set, $B \subseteq C$ and $d \subseteq D$, then the relative positive region of decision attribute d for B is $pos_B(d) = \bigcup \{B\_(X) \,|\, X \in U / ind(d)\}$.

The discretization algorithm based on the attribute importance is as follows.The first step is to compute the importance for every conditional attribution. The importance of a_i is:

$$card(pos_C(D)) / card(U) - card(pos_{C \setminus \{a_i\}}(D)) / card(U) \,.$$

The second step is to sort the conditional attribute in order according to their importance. If their importance is the same, we sort them in number sequence of the attribute break-point.

The third step is to modify the smaller value in two attribute values close to C_j in information system by considering the existence for every break-point C_j in attribute a_i. Here a_i is the attribute whose importance is nonzero. If there not existed any conflict in information system, then $Ca_i = Ca_i \setminus \{C_j\}$. Otherwise, we need to recover the modified attribute values.

The fourth step is to round the data in the processed information table. If there are different values, we deal with the corresponding data by adding one. By means of the above algorithm, the final information table is obtained (see Table 2).

7. Equivalent Classification

Let the domain $U = \{e_1, \cdots, e_{12}\}$, conditional attribution $C = \{a'_1, a'_3, a'_4\}$ and the decision attribution $D = \{d_1, d_2\}$. According to the rough set theory, we can compute that the equivalence relation partition $U/C = \{X_1, \cdots, X_8\}$, where $X_1 = \{e_1, e_{11}, e_{12}\}$, $X_2 = \{e_2, e_7, e_8\}$, $X_3 = \{e_3\}$, $X_4 = \{e_4\}$, $X_5 = \{e_5\}$, $X_6 = \{e_6\}$, $X_7 = \{e_9\}$, $X_8 = \{e_{10}\}$. The equivalence relation partition $U/D = \{Y_1, \cdots, Y_6\}$, where $Y_1 = \{e_1, e_4, e_{11}, e_{12}\}$, $Y_2 = \{e_2, e_3\}$, $Y_3 = \{e_5, e_6\}$, $Y_4 = \{e_7, e_8\}$, $Y_5 = \{e_9\}$, $Y_6 = \{e_{10}\}$. From this we can see that the results with these two methods are coincident. The weld appearances of e_{12}, e_4 and e_{11} are very close from the macroscopic morphology in the weld.

Table 2. The Final Information Table

No.	α'_1 Power V (Kw)	α'_2 I (mm/min)	α'_3 (A)	Weld width d1 (mm)	Weld Depth d2(mm)
e1	3	3	2	1	1
e2	3	3	4	2	1
e3	2	3	4	2	1
e4	1	3	4	1	1
e5	3	3	1	2	3
e6	3	3	3	2	3
e7	3	3	4	3	1
e8	3	3	4	3	1
e9	3	1	4	3	3
e10	3	2	4	2	2
e11	3	3	2	1	1
e12	3	3	2	1	1

8. Summary

In this paper, we view the information granule to be recognized as a point on the complex plane. A simple and operable recognition method is given by the analytic property of L-series. Some discourse domains we discussed can all be abstracted as an arithmetical semigroup. Therefore, this method has the characteristics of generality and wide ranges.

Acknowledgements

References

[1] Shouyu Chen, Daojun Zhang, Guangtao Fu. Study on fuzzy recognition model of center city in area layout. *Journal of Liaoning Technical University.* 2002; 21: 814-817 (in Chinese).

[2] Shoufeng Zhang, Rongjia Luo, Dongxiang Li. Application of multi-hierarchy and multi-pole fuzzy pattern recognition to enterprise competence. *Journal of Wuhan university of technology.* 2003;25: 83-86 (in Chinese).

[3] J Knofmacher. Abstract analytic number therem, North-Holland. 1975.

[4] Wenhang Li, Shanben Chen, Jiayou Wang: Model of Pulsed GTAW Process based on Variable Precision Rough Set. *Transactions of the China Welding Instituttion.* 2008; 29: 57-59 (in Chinese).

[5] Jinhong Li, Kangpei Zhao. *Relation model between process parameters and weld appeareance based on rough set.* 2012 IET InternationalConference. Ei. 2012.

[6] Wang Guoyin. Routh Set Theory and Knowledge Accquiring. Xi'an: Xian Jiaotong University Press. 2001.

[7] Fan Juan, Wang Hong-yan. Disposal of Data Discretization in KnowledgeDiscovering. *Journal of Baoding teachers college.* 2006; 19(2): 40-41.