

# Heuristic Column Generation Algorithm for Aircraft Dynamic Scheduling

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## Abstract

*Aircraft dynamic scheduling made a rational organization of surplus aircrafts to generate the optimized aircraft dispatch solution. The approach improved the operation efficiency and aviation enterprise benefits. To describe the complex relationship in aircraft dispatch, according to the characteristics of the airlines operation network, the research constructed an aircraft dynamic time-space network diagram, defined the surplus aircrafts' routing and constructed corresponding mathematical model. The proposed method found the routings set with heuristic methods in the column generation algorithm sub-problem process. Finally, the given example illustrated the method generate reasonable and practical aircraft solution for airlines in the effective time.*

**Keywords:** heuristic algorithm, aircraft dynamic scheduling, time-space network

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## 1. Introduction

In daily operations of airlines, natural disasters, accidents, and other public incidents always occurs uncertainty. Aircraft dynamic scheduling different from the concept of "irregular aircraft recovery" which caused by weather and other reasons, and more different from "dynamic scheduling" adjust flight times and aircraft according to changes in passenger demand which was put forward by some scholars. "Aircraft recovery" recover flight plan in fixed time-window through increasing aircraft quantity and delay or cancel flight. Otherwise, after satisfying the demands of aircraft capacity and time from mission, dynamic scheduling should dispatch surplus available aircraft in reasonable way, reduce flight cancel and airlines benefit losses. Because of flight control mechanism difference, airlines' aircraft dynamic scheduling in uncertainty has been mainly dependent on manual scheduling though SOC system from abroad including aircraft recovery module. The subjective adjustment always decided by operation controller's experience without considering from global properties. In the situation of time require, controllers always cancel all flight related to the transferred aircrafts. The adjustment brings economic and credit losses to airlines. Aircrafts dynamic scheduling research will change enterprise's process capacity which has important realistic meaning to civil aviation's development.

Due to management and mechanism related difference, the research on aircraft dynamic scheduling is rarely witnessed. U.S. Department of Defense signed agreements with U.S local airlines. Protocols require airlines to keep corresponding grade aircraft scale in war period for dispatch order from transportation command. Airlines will get enough commercial transportation business in reward, to make up the loss for aircraft dispatch [1]. In the technology, the scholars are more interesting in "aircraft recovery" research. In the early 1970s research into aircraft scheduling and aircraft recovery has be put forward. In 1996, a classical time-space network model including flight arc, grounding arc and overnight arc is proposed by Yan [2]. Flight cancellations, dispatch processing strategies are expressed in a single model. In the follow study [3], Yan constructs a multi-fleet network, and consider the problem as multi-commodity network flow problems. The problem is solved using the simplex method with Lagrange relaxation and flow decomposition algorithm. Benjamin introduces strategy including cancel, delay and switching in time-space network model [4]. Bard put forward an time-space model based on time-band network. The method calculates the actual cost of plan which is gradually generated though a linear integer programming optimization model [5]. Liu and others

proposed a MOEA (Multi-objective Evolutionary Algorithm) method to multi-fleet airlines. The method mainly focus on the take-ground, connecting flights, flight exchange, the total delay time and 30 minutes maximum delay time of the optimization objective function [6]. Eggenberg and others first proposed model the price problem to establish corresponding flight network for each plane or each fleet. The price problem was solved by column generation algorithm of the shortest path with resource constraints [7]. Andersson used a heuristic improved tabu search and simulated annealing algorithm [8]. Liu analysis three improved models(Resource allocation model, commodity network flow model and time-line model). The solution has be generated though column generation algorithm combined with improved model [9]. Yao target minimizing flight cost and uses time-space scheduling model to recover the flight plan [10]. It proposed heuristic algorithm based on path adjustment, making use of the Hungarian method to deal with the boundary constraint in the time-space network model, using phase processing delays list and mutation iterative method to obtain various solutions.

According to the above content and method, the research focused on the theory and algorithms about aircraft dynamic scheduling. Firstly, it generate relevant feasible aircraft routing set in the structured scheduling flight network though heuristic search algorithm. Then, according to the time-limited characteristic in the practical operation demand, it create aircraft scheduling solution using simplex method with restart technology and improved simplex method. The method will get final integer solution though branch and bound strategy. Finally, an instance is given to validate the feasibility and validity of the method.

## 2. Research Method

Aircraft dynamic scheduling manage appropriate fleet which carrying out the emergency task. For example, Libya evacuation ask for airbus A330-200, Boeing 747-400 with ETOPS and Yushu relief ask for airbus A319 and Boeing 757 in the high plateau task. It should be replaced using corresponding fleet when aircraft be dispatching. The technical status in aircrafts guarantee the flight performance be taken smoothly, and arrive in specified airport. For security reasons, each aircrafts satisfy periodic maintenance requirement, including A,B,C check constraints, so it is required to return base airport in the maintenance interval. Each aircraft routing's cumulative flight time should be no more than maintenance requirement.

In the solution, each aircraft only find one routing in the set. The flights including in the routing couldn't exist in other routings in the solution. Aircraft dynamic solution must reduce enterprise's loss and each aircraft's available flight time should be make good use. So, reasonable arrangement for surplus aircrafts' flight time and make full use of surplus flight plan is the objective function of the dynamic scheduling.

Different from "flight recovery" decision process in the fixed time window, operation controller couldn't predict the end time in the uncertainty. Dynamic scheduling adjusts the number of flight node and aircraft node in dynamic time window. It makes the solutions more scientific and reasonable and provide more choices for decision maker. Aircraft routing is defined as assign each aircraft flight task though adding arcs between nodes in the network. The combinatorial optimization for each routing is nodes segmentation process in the dashed frame. The dynamic scheduling plan must be able to guarantee to cover all surplus flights and each flight just be included once. Aircraft dynamic scheduling can be reflected in the dynamic time-space network as Figure 1 shows.

The research improved Bard's time-space network [5], constructed aircraft dynamic scheduling network with optimized technology. For each aircraft's location in time and space, it generated relevant aircraft routing with clearly track. Finally, it generated final solution though the routings in the set.

Aircraft node:  $K$  is the set involved by aircrafts corresponding fleet. Each aircraft node  $k$  means response airport  $k_{port}$  and available time  $k_{available}$  when receive instructions.

Flight node: all surplus flights set  $F$  in aircraft dynamic scheduling response time including flight node  $f$  and cancel node  $f_c$ . Each node has depart-airport  $f_{ouport}$ , arrive-airport  $f_{inport}$ , depart-time  $f_{outtime}$ , arrive-time  $f_{intime}$ .

Termination node: all routings generated from heuristic search which satisfy the dispatch constraints. Routing connect with termination node means staying and repair time.

Start arc: if aircraft node  $k_{port}$  has same depart airport  $f_{outport}$  with some flight nodes, and  $k_{available}$  is before the flight node's depart-time  $f_{outtime}$ , the nodes can be connect with start arc.

Connection arc: flight node has attributes  $f_{outport}$  and  $f_{inport}$ ,  $T_{bufftime}$  is aircraft's turnaround time, if  $f_{outtime} + T_{bufftime} \leq f_{intime}$ , it can be connected with connection arc between  $f$  and  $f'$ , means aircrafts could be perform  $f'$  after finishing  $f$ .

Termination arc: when flight node's  $f_{inport}$  is base airport or turnaround point, connect flight arc and termination node with termination arc, means one routing's generated.

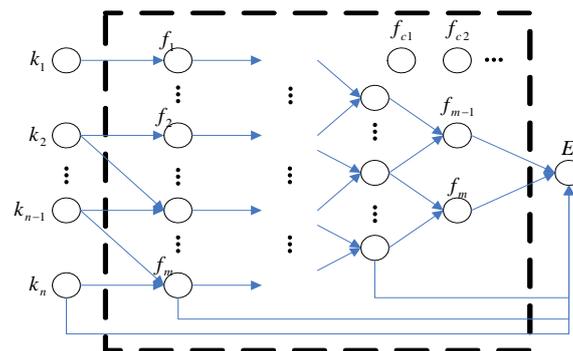


Figure 1. Aircraft Dynamic Time-space Network

When the aircraft  $k$  conform to the conditions of dispatching, and execute the corresponding flight task, the surplus flight plan has been dispatched with the rest of the aircraft capacity resource. Firstly, the research adopts heuristic depth-first search algorithm, starting from each plane node and ending in the node of termination, generating all flight routings. Though routing identification in the chosen method, the columns could join to the main problems in a quick way.

To describe main process of aircraft dynamic scheduling, the following steps are used:

Step1: The dispatching aircrafts will execute the corresponding flight plan, then determine the surplus aircraft capacity match with the rest of the flight plan, if so, turn step3; else turn step2.

Step2: delete the corresponding flight plan with dispatching aircrafts, turn step3.

Step3: According to the rules of the aircraft routing searching and heuristic search method to generate all feasible routings, turn step4.

Step4: Adopt the reset technology of modified simplex method, calculate the optimal objective function value, and finally turn into the surplus capacity scheduling plan, export the corresponding specification table.

The objective of the research is minimize the cost of the solution. Each aircraft routing must follow these rules:

- Aircraft source must return to base airport for maintenance after flight task.
- Routing's time-table must have time connection, and satisfy regulations in the turnaround.
- The routing constraints' accumulative flight time couldn't beyond the maintenance regulation.

The model can be written as Equation (1) to (5).

$$\min Z = \sum_k \sum_r c_r^k x_r^k + \sum_f c_f' y_f \quad (1)$$

$$s.t. \sum_k \sum_r a_r^f x_r^k + y_f = 1; \quad f \in F \quad (2)$$

$$\sum_r x_r^k \leq 1; \quad k \in K \quad (3)$$

$$x_r^k = 0, 1; \quad k \in K, r \in R \quad (4)$$

$$y_f = 0, 1; \quad f \in F \quad (5)$$

In the equations, objective function (1) minimizes the total costs in the new solution, where the first term is the sum of cost for each routing and the second term is the costs of flight cancel. Constraints (2) and (3) and (4) ensure each flight can be covered by routings or cancels and each aircraft only chose one routing or stop in the solution. Constraints (5) ensure flights can be taken or cancelled.

The huge number of routings in set  $R$  produce a large solution space. Linear programming algorithm can't get the answer in limited time. Heuristic column generation algorithm is the key study of the research. According to each aircraft's location and maintenance status, the algorithm generate all routings back to base airport. The huge number of routings in  $R$  produce a large solution space. In sub-problem, each routing minus corresponding flights and aircrafts dual variable values directly, generate corresponding contracted cost. The method omit every column generation iteration must adopt the shortest path algorithm, and can greatly improve the column generation algorithm operation time and efficiency.

To describe main process of heuristic column generation algorithm, the following steps are used:

Step1: Add some routings to limited main-problem, routing's uniqueness constrain (3) introduce the artificial variable  $x_k$ , and flight constrain (2) variable  $y_f$  could generated as initial basic variables.

Step2: Using improved simplex method to solve limits main-problem

Step3: (Sub-problem solution) According to the inverse of the basis  $C$  and corresponding basis cost  $C_B$  of flights and aircrafts, generated responding dual variable  $(\pi_f, \pi_k)$ , calculate the cost of each routing:

$$c_r^{k'} = c_r^k - \sum_{f \in F} a_r^f \pi_f - \pi_k$$

Step4: Assume  $\sigma = \min_{\forall r \in R} (c_r^{k'})$ , if  $\sigma < 0$ , add minimum routing to limited main-problem, turn step5, else  $\sigma \geq 0$ , algorithm end and turn step6.

Step5: Calculation the simplex table values  $B^{-1} P_r$ , and assume the insert variable,  $S = B^{-1} b / B^{-1} P_r$ , calculate out variable according the principle of minimum ratio, turn step2.

Step6: Judge scores, if the solution is score, using branch and bound strategy, else get the optimal solution.

### 3. Results and Analysis

On the basis of domestic airlines' flight plan and corresponding aircraft plan, the study verify the effectiveness of the algorithm though simulations. In the process of perform the flight plan, the experiment simulate the situation of dynamic scheduling mission, which dispatch 10% aircraft to take the uncertainty task. The surplus aircraft should be used to take the rest flight plan. the experiment require each aircraft back to base airport or original-plan aircraft to insure the next day operation. The surplus flight plan must be implemented as much as possible to reduce the operation cost of airline. Experiments compare the simplex, column generation algorithm and heuristic column generation algorithm, in the background of different scale of aircrafts and flights. These algorithms are optimization algorithms on the premise of optimal

scheduling solution generated. Table 1 (sample) show the finally generated aircraft schedule. The contrast results of these algorithms show in Table 2.

Based on the analysis of experimental results, when the aircraft and flight is tiny, three algorithms are able to reach a satisfactory plan in a short time. But with the expansion of the problem scale, the solution space grew exponentially, heuristic column generation algorithm of computational time significantly overcome the simplex algorithm response time disadvantages, and can achieve the requirements of practical application. The contrast of column generation algorithm time have been improved significantly also. Under the condition of different parameters, heuristic column generation algorithm is better than the simplex and the column generation algorithm, and get satisfactory results.

Table 1. Aircraft Dynamic Scheduling Plan

Aircraft number	Fleet	Flightnumber1	Departure time 1	Departure airport1	Landing time 1	Landing airport 1	Flightnumber 2	Departure time 2	Departure airport2	...
B6253	320	567	1040	PVG	1545	SIN	568	1800	SIN	...
B6207	320	703	1125	PVG	1355	HKG	704	1500	HKG	...
.....										...
B6158	320	5507	1900	PVG	NGB	1955	5508	2030	NGB	...

Table 2. Algorithm Time Results

Algorithm	Aircraft Flight	5	12	30	52	62	74	84
Simplex	Time (s)	0.241	1.119	52.477	119.225	192.399	365.793	>600
Column generated	Time (s)	0.274	2.1	35.388	81.307	135.674	313.99	489.749
Heuristic column generation	Time (s)	0.268	2.141	24.869	69.179	113.474	226.332	382.739

#### 4. Conclusion

On the basis of the characteristics of the flight management system, the research study aircraft dynamic scheduling problem in uncertainty. It constructed aircraft dynamic scheduling time-space network, modeling flow model base aircraft dynamic routing. Moreover, the method using heuristic path generating method to improve the processing efficiency, generated the best solution of aircraft scheduling. The results show that the algorithm could solve the dynamic scheduling problem, provides guarantee safety, high efficiency, stable operation and satisfy the real-time requirements in the daily operation.

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