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Model Reference Adaptive Control Based on Lyapunov Stability Theory

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Abstract

According to the obvious characteristics of time-varying nonlinear controlled object, in order to improve the control performance, can use appropriate adaptive control strategy. Based on the research object system of a certain amount of input to control variables, according to Lyapunov stability theory, the research object system input/output variable a model reference adaptive control system was design. And after analysis the method to improve the system starting stage characteristics, dynamic process performance and system anti-interference and resistance to process parameters is presented. Simulation and research results show that the proposed control method is effective.

Keywords*: intelligence control, model reference adaptive control, Lyapunov, stability theory*

1. Introduction

MRAC (Model reference adaptive control) is a kind of adaptive control method based on the reference model, which is conductive for the control object to control its time-varying nonlinear characteristics [1, 2]. This method has the flexibility, adaptability and robustness, and is capable of automatically adjusting the controller gain and stabilizing the system [3]. In the MRAC system, a reference model can be designed based on the performance of the desirable device and the output of the reference model is regarded as the expected output signal of the device. Therefore, the controlling performance of the device could be improved by modifying the reference model parameters which makes the output of the system match the expected response to the input.

The stability theory of Lyapunov introduced the Lyapunov function with generalized energy properties and analyzed its definiteness to establish the corresponding conclusions to judge the stability of the system [4-6]. The servo system has strong nonlinearity and time-varying characteristics [7]. which is well suited to the adaptive control method**.** Some scholars used artificial intelligence algorithm, and speed control of the servo system is studied, such as literature [8-10]. Research finding the servo system which is well suited to the adaptive control method [11]**.** In this paper, the rotating speed control strategy of servo system is studied. The mathematical model of the servo system is obtained using the systemic identification method, and on this basis, according to the nonlinear characteristics of the servo system, coupled with the Lyapunov stability theory, this paper studies the MRAC system by controlling the input-output variables and analyzes the controlling characteristics of the system at the start-up phase, its performance in the dynamic process and its anti-jamming capability and anti-process parameters. The experimental results show that the proposed control method has better dynamic performance when the stability of the system is ensured.

2. Design of MRAC Rotating Speed Controller of Servo Motor

As part of the state variables of the servo motor control system are not easy to be determined or accurately observed, the Lyapunov method based on the input-output variable is

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used to study the MRAC rotating speed controller of servo motor, whose structure is as shown in Figure 1.

As shown in Figure 1, y_r refers to the system input, $W_m(s)$ is the reference model, k_0 is the adjustable feed forward gain controller of the adaptive controller, F_1 and F_2 are two auxiliary signal generators. In the normal operation state, by comparing the actual output y_p and the output y_m of the reference model $W_m(s)$, the tracking error e_1 of the reference model is obtained. Conduct dynamic adjustment for the adjustable parameters of the adaptive controller according to e_1 to compensate for the loss caused by the non-linear characteristics of the servo system and its parameter changes on the control performance.

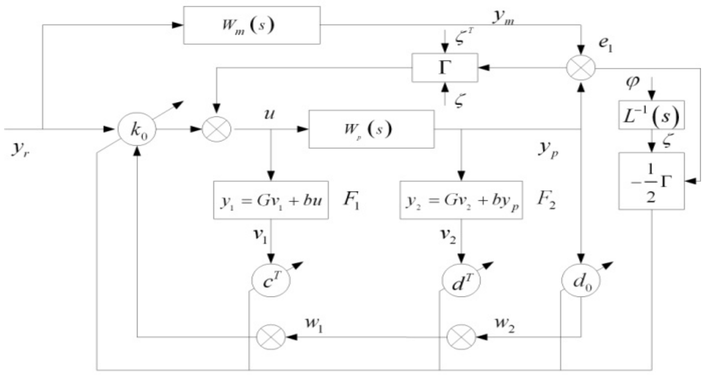

Figure 1. Structure of the Control System

2.1. Reference Model Design

The mathematical model of the controlled object is described as formula (1).

$$\frac{Y_p(s)}{U(s)} = W_p(s) = \frac{k(T_3 s+1)}{s(T_1 s+1)(T_3 s+1)} = \frac{K_p(s+b_0)}{s^3 + a_{p2}s^2 + a_{p1}s} \tag{1}$$

Among them k, T_1, T_2, T_3 refer to the process parameters, which are unknown quantities changing with the working environment and conditions. By the changes algebra, the follows could be obtained $a_{p1} = \frac{1}{T_1T_2}$, $a_{p2} = \frac{1}{T_1} + \frac{1}{T_2}$, $b_0 = \frac{1}{T_3}$, $k_p = \frac{kT_3}{T_1T_2}$.

According to the system shown in Figure 1, it is necessary to design the reference model based on the characteristics and control requirements of the servo system. In accordance with the transfer function mathematical model in formula (1), the following reference model is designed, as shown in formula (2).

$$\frac{Y_m(s)}{Y_r(s)} = W_m(s) = \frac{k_m(s+d)}{(s+a)(s+b)(s+c)} = \frac{b_{m1}s + b_{m0}}{s^3 + a_{m2}s^2 + a_{m1}s + a_{m0}} \tag{2}$$

Among them $a_{m0} = abc, a_{m1} = ab + bc + ac, a_{m2} = a + b + c, b_{m0} = k_m d, b_{m1} = k_m$.

In this case, the reference model should be selected to endow its output the desired characteristics, and the numerator and denominator have the same order as well as the same relative degree to the process $W_p(s)$.

The relative degree of $W_p(s)$ is 2, and the relative degree of the reference model $W_m(s)$ should also be 2. According to the definition of positive real function, $W_m(s)$ does not have positive real nature and the adaptive law of parameters require the additional polynomial $L(s)$.

Thereby, the state equation and output equation of the reference model are as shown in formula (3).

$$\dot{x}_m = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_{m0} & -a_{m1} & -a_{m2} \end{pmatrix} x_m + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} y_r \quad , \quad y_m = \begin{pmatrix} b_{m0} & b_{m1} & 0 \end{pmatrix} x_m \tag{3}$$

In the literature [2], the author selected $a = 4, b = 5, c = 6, d = 3, k_m = 40$ to meet the system's requirements for dynamic performance and stability, so $a_{m0} = 120$, $a_{m1} = 74$, $a_{m2} = 15$, $b_{m0} = 120$, $b_{m1} = 40$ is calculated.

Take the above results into formula (3) , and obtain formula (4).

$$\dot{x}_m = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -120 & -74 & -15 \end{pmatrix} x_m + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} y_r \quad , \quad y_m = \left(120, 40, 0\right) x_m \tag{4}$$

2.2. System Identification

According to formula (1), the system identification method is utilized and the process output $y_p(t)$ is obtained by measurement. The longest linear shift register sequence whose statistical properties are similar to the white noise is regarded as the input signal, and the least square method is used to obtain the control model of the system, as shown in Equation (5).

$$\dot{x}_p = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -a_{p1} & -a_{p2} \end{pmatrix} \dot{x}_p + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u \quad , \quad y_p = (k_p b_0, k_p, 0) x_p \tag{5}$$

The process parameters $T_1 = 1$, $T_2 = 2$, $T_3 = 4$, $k = 0.4$ are obtained by calculation, and $a_{p1} = 0.5$, $a_{p2} = 1.5$, $b_0 = 0.25$, $k_p = 0.8$, and the state space representation can be expressed as formula (6).

$$\dot{x}_p = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -0.5 & -1.5 \end{pmatrix} \dot{x}_p + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u \quad , \quad y_p = (0.2, 0.8, 0) x_p \tag{6}$$

The output error is:

$$e_1(t) = y_p(t) - y_m(t) \tag{7}$$

2.3. Add the Polynomial to Determine the State Equation and Filtering Equation of the Auxiliary Signal Generator

When choosing $L(s)$, to make the relative order of $W_m(s)L(s)$ is equal to 1, select $L(s) = s + a_0$. As $a_0 > 0$, make $a_0 = 5$ and $L(s) = s + 5$.

Select the filter $F(s) = N_m(s)L(s)$ to construct the state equation of the auxiliary signal, as shown in Equation (8).

$$\begin{aligned} \dot{v}_1 &= Gv_1 + bu \\ \dot{v}_2 &= Gv_2 + by_r \end{aligned} \quad (8)$$

In Equation (8), G and b are the coefficient matrix, and its structure and specific parameters are closely related with the control results, and make $\det(sI - G) = F(s) = N_m(s)L(s)$.

$F(s) = N_m(s)L(s) = (s+d)(s+a_0) = s^2 + (a_0 + d)s + da_0 = s^2 + g_1 s + g_0$ could be selected as the state equation of the auxiliary signal generator, wherein $g_0 = da_0, g_1 = a_0 + d$.

As the available information is insufficient, two auxiliary signal generators F_1 and F_2 are introduced. The input signal of F_1 is the input u of the controlled object and there are $(n-1)$ adjustable parameters $c_i (i = 1, 2, ..., n-1)$; the input of F_2 is the output y_p of the controlled object with n adjustable parameters $d_i (i = 0, 1, 2, ..., n-1)$, coupled with the feed forward gain k_0 , and there are a total of $2n$ adjustable parameters.

Wherein,

$$\begin{aligned} w_1 &= c^T v_1 \\ W_1(s) &= c^T (sI - G)^{-1} b = \frac{C(s)}{F(s)} \end{aligned} \quad (9)$$

$$\begin{aligned} w_2 &= d^T v_2 + d_0 \\ W_2(s) &= d_0 + d^T (sI - G)^{-1} b = \frac{D(s)}{F(s)} + d_0 \end{aligned} \quad (10)$$

In Equation (9) and (10), $c = (c_1, c_2, ..., c_{n-1})^T$ and $d = (d_1, d_2, ..., d_{n-1})^T$ are adjustable parameter vectors. $F(s)$ is the to stable monic polynomial of $n-1$ order. $C(s)$ and $D(s)$ are the $n-1$ order polynomial. as $G = \begin{pmatrix} 0 & 1 \\ -g_0 & -g_1 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, formula (11) is deduced.

$$\begin{aligned} \dot{v}_1 &= \begin{pmatrix} \dot{v}_{1,1} \\ \dot{v}_{1,2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -g_0 & -g_1 \end{pmatrix} v_1 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u \\ \dot{v}_2 &= \begin{pmatrix} \dot{v}_{2,1} \\ \dot{v}_{2,2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -g_0 & -g_1 \end{pmatrix} v_2 + \begin{pmatrix} 0 \\ 1 \end{pmatrix} y_p \end{aligned} \quad (11)$$

2.4. Determine the Signal Vector and the Adjustable Parameter Vector

Make $\varphi^T = \begin{pmatrix} y_r & v_{1.1} & v_{1.2} & y_p & v_{2.1} & v_{2.2} \end{pmatrix}^T, \theta^T = \begin{pmatrix} k_0 & c_1 & c_2 & d_0 & d_1 & d_2 \end{pmatrix}^T$, $\zeta = L^{-1}(s)\varphi$. In the following simulation, take $\theta(0) = \begin{pmatrix} 3.5 & -0.5 & 3.1 & -3.5 & 0.05 & -0.63 \end{pmatrix}$ as the initial value.

The filtering vector $\zeta = L^{-1}(s)\varphi = \frac{1}{s+5}\begin{pmatrix} y_r & v_{1.1} & v_{1.2} & y_p & v_{2.1} & v_{2.2} \end{pmatrix}, \dot{\zeta} = -a_0\zeta + \varphi = -5\zeta + \varphi$.

To achieve the adaptive parameter adjustment law, select the Lyapunov function as formula (12).

$$V = \frac{1}{2}\left(e^{T} P e + \tilde{\theta}^{T} \Gamma^{-1} \tilde{\theta}\right) \tag{12}$$

In formula (12), P and Γ are the positive definite symmetric matrixes. In the following simulation calculation, take the positive definite symmetric matrix $\Gamma = diag\left(0.1 \quad 4 \quad 4 \quad 4 \quad 4 \quad 4\right)$.

The parameter adjustment law is:

$$\dot{\theta} = -e_1 \Gamma \zeta \tag{13}$$

The control law is:

$$u(t) = \theta^{T} \varphi + \dot{\theta}^{T} \zeta = \theta^{T} \varphi - e_1 \zeta^{T} \Gamma \zeta \tag{14}$$

3. Implementation of System Control Algorithm

Algorithms are implemented by using the numerical integration method. Specific algorithm steps are shown as follows:

Step 1. Initializing related variables.

Step 2. According to formula (4), $y_m(t)$ is obtained; based on formula (6), $y_p(t)$ is obtained; then by formula (7), $e_1(t)$ is worked out.

Step 3. According to formula (11) , $v_1(t)$ and $v_2(t)$ are obtained.

Step 4. Based on $\dot{\zeta} = -a_0 \zeta + \varphi = -5\zeta + \varphi$, $\zeta(t)$ is obtained.

Step 5. Based on $\dot{\theta} = -e_1 \Gamma \zeta$, $\theta(t)$ is obtained.

Step 6. Based on $u(t) = \theta^{T} \dot{\varphi} - e_1 \zeta^{T} \Gamma \zeta$, the control variable $u(t)$ is obtained.

Step 7. Enter the cycle with $i = N$.

Step 8. Output the graphics.

4. Simulation Experiment

To verify the validity of the method, to as a simulation study, numerical integration method is used to carry out the test on the aforementioned servo system model under the environment of Matlab-based simulation tool.

According to the study objectives, two sets of simulation tests are designed to verify the validity of the method.

4.1. Simulation Test I

This set of test is a qualitative simulation of the system. According to the aforementioned MRAC controller, the servo motor model formula (1) and (2) are used to simulate the study system described in Figure 1. The tracking response of the system to the input and the output error are analyzed. Square wave signal with the input amplitude of 1 is used as the actual system input $y_r(t)$, and the output response and the output error are shown in Figure 2 ,and $u(t)$ is shown in Figure 3.

The results show that at the initial stage of the control, the system generates shock with undesirable tracking due to the large deviation between the initial value of the control system parameters and the expected value. However, as time goes on, e_1 declines due to the adjustment of the adaptive rate and the controller parameters is increasingly close to the expected value. The output of the controlled object quickly tracks the variation of reference model output, indicating the system has good tracking ability, which shows that the proposed MRAC controllers are valid. The controller parameters $d_0(t)$, $d_1(t)$ and $d_2(t)$ changing cure are shown in Figure 4, and controller parameters $k_0(t), c_1(t)$ and $c_2(t)$ changing cure are shown in Figure 5.

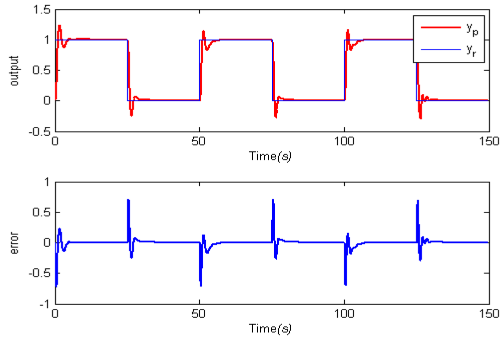

Figure 2. MRAC Control Response Curves and Output Error

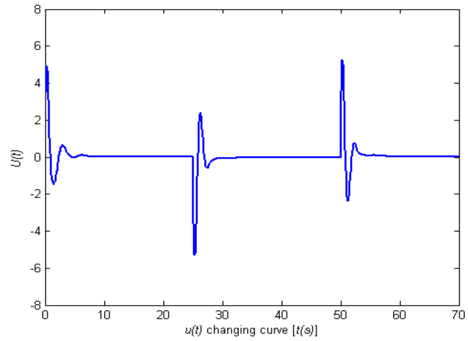

Figure 3. Variation Curve of $u(t)$

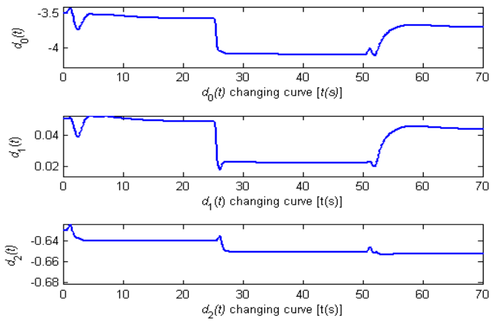

Figure 4. Variation Curves of $d_0(t)$, $d_1(t)$ and $d_2(t)$

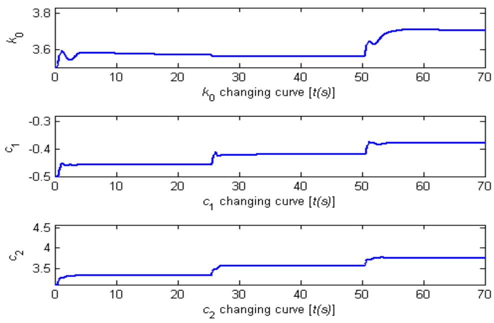

Figure 5. Variation Curves of $k_0(t)$, $c_1(t)$ and $c_2(t)$

4.2. Simulation Test II

This simulation test mainly studied the anti-interference ability and process parameters variation. For the unit step signal with the given reference input, when $t = 25$ and $t = 50$ time, respectively add constant disturbance with the amplitude of 0.2 and 0.6 at the process output end. The changing curves of $y_p(t)$ and $e_1(t)$ are as shown in Figure 6, the changing curve of $u(t)$ is as shown in Figure 7, the changing curve of $y_m(t)$ is as shown in Figure 8.

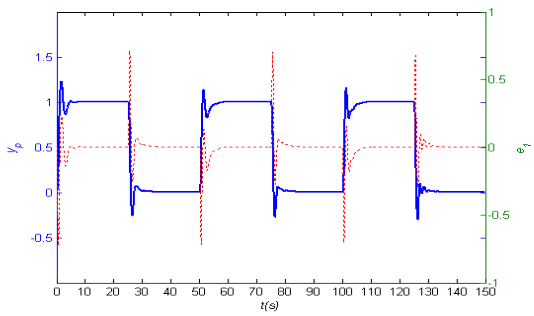

Figure 6. Variation Curves of $y_p(t)$ and $e_1(t)$

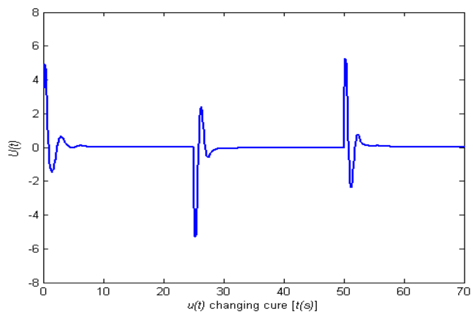

Figure 7. Variation Curve of $u(t)$

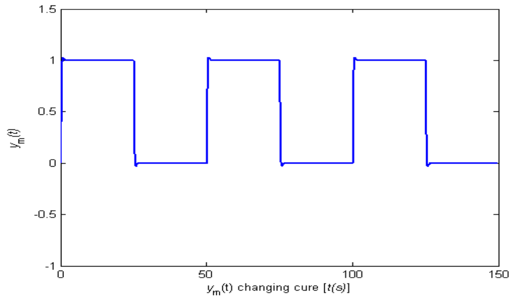

Figure 8. Variation Curve of $y_m(t)$

The simulation results, as shown in Figure 6-8, indicate that the system has strong anti-interference ability.

Change the process parameters suddenly when the interference occurs, and their variation is shown in Table 1 when $t = 25$ and $t = 50$.

Table 1. the variation of process parameters when $t = 25$ and $t = 50$

	a_{p1}	a_{p2}	k_p	b_0
$t = 25$	$a_{p1} \to 0.8a_{p1}$	$a_{p2} \to 0.65a_{p2}$	$k_p = 1.2k_p$	$b_0 \to 1.4b_0$
$t = 50$	$a_{p1} \to 0.9a_{p1}$	$a_{p2} \to 0.85a_{p2}$	$k_p = 1.3k_p$	$b_0 \to 1.2b_0$

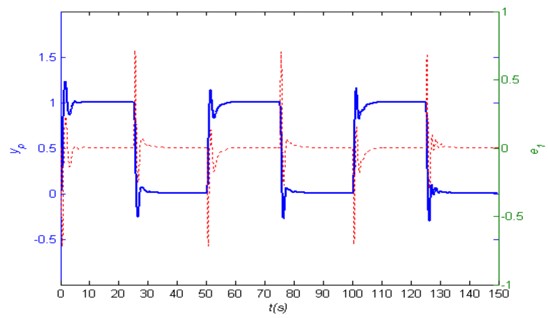

Figure 9. Variation Curves of $y_p(t)$ and $e_1(t)$

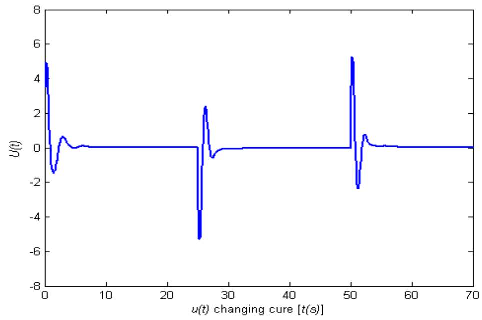

Figure 10. Variation Curve of $u(t)$

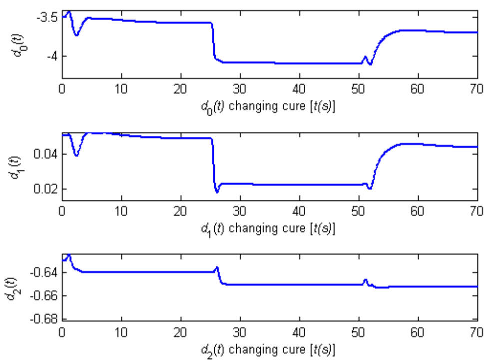

Figure 11. Variation Curves of $d_0(t)$, $d_1(t)$ and $d_2(t)$

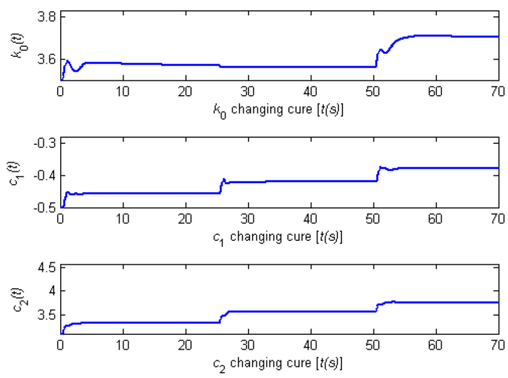

Figure 12. Variation Curves of $k_0(t)$, $c_1(t)$ and $c_2(t)$

According to the changing conditions of Table 1, the variation of process output $y_p(t)$ and the output error $e_1(t)$ with time is as shown in Figure 9. The curve of $u(t)$ is as shown in Figure 10, The controller parameters $d_0(t), d_1(t)$ and $d_2(t)$ cure are shown in Figure 11, and controller parameters $k_0(t), c_1(t)$ and $c_2(t)$ changing cure are shown in Figure 12.

The simulation results, as shown in Figure 9-12, indicate that the system has strong abilities to resist changes in process parameters.

5. Conclusion

This paper designed the specific MRAC rotating speed controller of servo motor based on Lyapunov stability theory. The results of the simulation tests show that MRAC is an effective way to control the rotating speed of this kind of servo motor, which gives the system good tracking ability. Meanwhile, the system has strong capacities to resist interference and changes in process parameters. It also could overcome the influence of the time-varying nonlinearity on the control performance. Based on the input and output information of the system, the controlling effect of MRAC system designed according to the Lyapunov stability theory are closely related to the parameters settings, so appropriate selection and optimization of the parameters is conductive to achieve the desired aims and meet the controlling requirements. In addition, this kind of controlling thought also has a certain reference value for the control problems of other complex systems.

Acknowledgements

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