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Internal Model Theory

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Abstract

Internal model control (IMC) method enables the system to be of good dynamic and steady performance, which is simple, and is easy to be implemented. In allusion to the cascade STATCOM feature of high order, instability, multi-variable, non-linearity and tight coupling, the mathematical model of cascade STATCOM in d-q-0 coordinates was deduced. Decoupling model of cascade STATCOM was given by Internal Model Control principle, computer simulation and experiment results were also given. Results show that with IMC, 3-phase currents control method of cascade STATCOM has good tracking performance and control precision both in a-b-c coordinates and in d-q-0 coordinates, and also achieves excellent current compensation results.

Keywords: internal model control, cascade STATCOM, current control strategy

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1. Introduction

Compared with the traditional SVC, the static synchronous compensator (STATCOM) has become the development trends, for its faster response, wider adjustable range and better wave quality of compensating current [1-2]. Besides, capacitors and reactors used in cascade STATCOM are smaller than those used in traditional SVC, which greatly reduce device's volume and cost. Article [3-5] pointed out that cascade STATCOM had broad application prospects, such as electric power, metallurgy fields and mining industry.

As to the decoupling method of cascade STATCOM mentioned in released publications, there are two models: one is the state-feedback decoupling; the other is the time decoupling which is an approximate method. Article [6-7] pointed out that the application of time decoupling was limited in practice, because its decoupling characteristics were related with the parameters of power network and the main circuit. Articles [8-10, 14] are typical documents about state-feedback decoupling. In article [8], the reactive power control strategy based on inverse system control and decoupling control of active power and reactive power was designed and realizes the dynamic control of physical prototype, but the response time was not quick. Article [9] presented a feedback linearization theory to realize the design, which was proved to be effective. Article [14] derived the current control strategy in d-q-0 coordinates by methods of nonlinear, and validated the method by simulation, but the actual use effects need to be tested by projects.

Based on article [11-16], this paper deduces the mathematical model of cascade STATCOM in d-q-0 coordinates, and builds simulation model by computer simulation software MATLAB/SIMULINK, at last verifies the design by computer simulation and test.

2. Mathematical Models

2.1. Main Circuit Topology

Figure 1 is the main circuit topology of cascade STATCOM. *L* represents the inductance. *R* represents power losses. $u_c(t)$ and $i_c(t)$ represent voltage and current outputted by cascade STATCOM. $i_s(t)$ represents system's current. $i_l(t)$ represents load current.



Figure 1. Structure Diagram of Cascade STATCOM

2.2. Mathematical Model in a-b-c Coordinates

Before constructing the mathematical model of cascade STATCOM, following assumptions are presented [9-10]:

(1) The power system is treated as balanced.

(2) R is used to express the power loss of filter capacitor and filter reactor.

(3) L is used to express the inductance of filter reactor.

(4) Output voltage of cascade STATCOM is directly proportional to the sum of dc capacitor voltage.

(5) Since cascade STATCOM is composed of seventeen-level cascaded inverters using carrier phase-shifting method, thus only the fundamental component of cascade STATCOM is considered, ignoring the harmonic components.

According to the above assumptions and the expression of output voltage generated by single phase H-bridge inverter, output voltage expression of cascade STATCOM can be obtained by:

$$\begin{pmatrix} u_{ca} \\ u_{cb} \\ u_{cc} \end{pmatrix} = K u_{dc} \begin{pmatrix} \sin(\omega t - \delta) \\ \sin(\omega t - 2\pi/3 - \delta) \\ \sin(\omega t + 2\pi/3 - \delta) \end{pmatrix}$$
(1)

K is the proportional coefficient. δ is the phase angle difference between the voltage outputted by cascade STATCOM and the system voltage.

The system voltage is symmetrical three phase voltage, and the real phase voltage is expressed by:

$$\begin{pmatrix} u_{sa} \\ u_{sb} \\ u_{sc} \end{pmatrix} = \sqrt{2}U_s \begin{pmatrix} \sin(\omega t) \\ \sin(\omega t - 2\pi/3) \\ \sin(\omega t + 2\pi/3) \end{pmatrix}$$
(2)

According to Kirchhoff's law, the mathematical equation of cascade STATCOM is expressed by:

$$L\frac{d}{dt}\begin{pmatrix}i_{ca}\\i_{cb}\\i_{cc}\end{pmatrix} = \begin{pmatrix}u_{sa}\\u_{sb}\\u_{cc}\end{pmatrix} - \begin{pmatrix}u_{ca}\\u_{cb}\\u_{cc}\end{pmatrix} - R\begin{pmatrix}i_{ca}\\i_{cb}\\i_{cc}\end{pmatrix}$$
(3)

With the energy balance theory, dc bus capacitor voltage equations can be obtained as follows:

$$\frac{d}{dt}\left(\frac{1}{2}Cu_{dc}^{2}(t)\right) = u_{ca}(t)i_{ca}(t) + u_{cb}(t)i_{cb}(t) + u_{cc}(t)i_{cc}(t)$$

$$\Rightarrow \frac{du_{dc}(t)}{dt} = \frac{K}{C}[i_{ca}(t)\sin(\omega t - \delta) + i_{cb}(t)\sin(\omega t - \frac{2\pi}{3} - \delta) + i_{cc}(t)\sin(\omega t + \frac{2\pi}{3} - \delta)]$$

$$Q i_{ca}(t) + i_{cb}(t) + i_{cc}(t) = 0$$

$$\Rightarrow \frac{du_{dc}(t)}{dt} = -\frac{\sqrt{3}K}{C}[i_{ca}(t)\cos(\omega t - \delta + \frac{\pi}{3}) + i_{cb}(t)\cos(\omega t - \delta)]$$

The mathematical model of cascade STATCOM is obtained by:

$$\frac{d}{dt} \begin{pmatrix} i_{ca} \\ i_{cb} \\ u_{dc} \end{pmatrix} = A \begin{pmatrix} i_{ca} \\ i_{cb} \\ u_{dc} \end{pmatrix} + \frac{\sqrt{2}U_s}{L} \begin{pmatrix} \sin \omega t \\ \sin(\omega t - \frac{2\pi}{3}) \\ 0 \end{pmatrix}$$

$$A = \begin{pmatrix} -\frac{R}{L} & 0 & -\frac{K}{L}\sin(\omega t - \delta) \\ 0 & -\frac{R}{L} & -\frac{K}{L}\sin(\omega t - \delta) \\ \frac{\sqrt{3}K}{C}\sin(\omega t - \delta - \frac{\pi}{6}) & \frac{\sqrt{3}K}{C}\sin(\omega t - \delta - \frac{\pi}{2}) & 0 \end{pmatrix}$$

$$(4)$$

Equation (4) is the mathematical model of cascade STATCOM in a-b-c coordinates

2.3. Mathematical Model in d-q Coordinates

Equation (4) is the mathematical model of cascade STATCOM in a-b-c coordinates, which is the differential equation with time-varying parameters (coefficient). For its hard theoretical analysis, Equation (4) is transferred by d-q coordinates transform as follows:

$$\frac{d}{dt} \begin{pmatrix} i_d \\ i_q \end{pmatrix} = T \cdot \frac{d}{dt} \begin{pmatrix} i_{ca} \\ i_{cb} \\ i_{cc} \end{pmatrix} + \frac{dT}{dt} \cdot \begin{pmatrix} i_{ca} \\ i_{cb} \\ i_{cc} \end{pmatrix}$$
(5)

In equation (5):

$$T = \sqrt{\frac{2}{3}} \begin{pmatrix} \cos(\omega t) & \cos(\omega t - \frac{2}{3}\pi) & \cos(\omega t + \frac{2}{3}\pi) \\ -\sin(\omega t) & -\sin(\omega t - \frac{2}{3}\pi) & -\sin(\omega t + \frac{2}{3}\pi) \end{pmatrix}$$

Take Equation (3) to Equation (5):

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$$\frac{d}{dt} \begin{pmatrix} i_d \\ i_q \end{pmatrix} = \begin{pmatrix} -\frac{R}{L} & \omega \\ -\omega & -\frac{R}{L} \end{pmatrix} \begin{pmatrix} i_d \\ i_q \end{pmatrix} + \frac{1}{L} \begin{pmatrix} u_{sd} - u_{cd} \\ u_{sq} - u_{cq} \end{pmatrix}$$
(6)

$$\Rightarrow Ls \begin{pmatrix} I_d(s) \\ I_q(s) \end{pmatrix} = \begin{pmatrix} -R & \omega L \\ -\omega L & -R \end{pmatrix} \begin{pmatrix} I_d(s) \\ I_q(s) \end{pmatrix} + \begin{pmatrix} U_{sd}(s) \\ U_{sq}(s) \end{pmatrix} - \begin{pmatrix} U_{cd}(s) \\ U_{cq}(s) \end{pmatrix}$$
(7)

Equation (7) is the mathematical model of cascade STATCOM in d-q coordinates. It is easy to see that currents coupling problem is not solved, therefore the method of currents decoupling is needed to realize the control strategy.

3. IMC Strategy in d-q Coordinates

3.1. IMC Principle

The structure diagram of IMC is shown in Figure 2, in which the internal model is expressed as $\hat{G}(s)$, the controlled object is expressed as G(s), and the internal model controller is expressed as $C_{_{MC}}(s)$.





Figure 2. Diagram of Internal Model Control

Figure 3. Equivalent Feedback Control Block Diagram

Figure 3 is the equivalent feedback control block diagram of Figure 2. In Figure 3, the equivalent controller F(s) is given by:

 $F(s) = [1 - C_{IMC}(s)\hat{G}(s)]^{-1}C_{IMS}(s)$

Although there is no accumulation function in the internal model controller, it can eliminate the system stead-state error by the structure. U(s) and I(s) represent voltage and current outputted by cascade STATCOM.

In addition to having similar virtues, IMC also has many outstanding characteristics such as follows:

(1) IMC can regulate the output errors caused by mismatch between $\hat{G}(s)$ and G(s).

(2) It can realize tracking without steady-state error by introducing the corresponding filter expressed as L(s), and it can also improve the robustness of the system.

The low-pass filter L(s) is given by:

$$L(s) = \frac{\lambda}{s+\lambda}$$

The identity matrix is expressed as I, λ is the reciprocal of cascade STATCOM system time constant.

3.2. Internal Model Decoupling Control

The following set of voltage and current equations can be obtained from Equation (7).

$$\begin{pmatrix} u_{sd} - u_{cd} \\ u_{sq} - u_{cq} \end{pmatrix} = \begin{pmatrix} Ls + R & -L\omega \\ L\omega & Ls + R \end{pmatrix} \begin{pmatrix} i_d \\ i_q \end{pmatrix}$$
(8)

$$\Rightarrow \begin{pmatrix} u_d \\ u_q \end{pmatrix} = \begin{pmatrix} Ls + R & -L\omega \\ L\omega & Ls + R \end{pmatrix} \begin{pmatrix} i_d \\ i_q \end{pmatrix}$$
(9)

 $\hat{G}^{-1}(s)$ is constructed as follows:

$$\hat{G}^{-1}(s) = \begin{pmatrix} Ls + R & -L\omega \\ L\omega & Ls + R \end{pmatrix}$$
(10)

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The IMC current controller, $C_{IMC}(s)$ is given by:

$$C_{IMC}(s) = \hat{G}^{-1}(s)L(s) = \lambda \begin{pmatrix} Ls + R & -L\omega \\ L\omega & Ls + R \end{pmatrix} \begin{pmatrix} \lambda & 0 \\ s + \lambda & 0 \\ 0 & \frac{\lambda}{s + \lambda} \end{pmatrix} = \lambda \begin{pmatrix} \frac{Ls + R}{s + \lambda} & \frac{-L\omega}{s + \lambda} \\ \frac{L\omega}{s + \lambda} & \frac{Ls + R}{s + \lambda} \end{pmatrix}$$
(11)

The transfer function can be obtained from Figure 3.

$$\frac{I(s)}{I^*(s)} = \frac{F(s)G(s)}{1 + F(s)G(s)} = \frac{C_{IMC}(s)G(s)}{1 + C_{IMC}(s)[G(s) - \hat{G}(s)]}$$
(12)

By combining equation (10), the controlled object G(s) is given by:

$$G(s) = \hat{G}(s) = \begin{pmatrix} Ls + R & -L\omega \\ L\omega & Ls + R \end{pmatrix}^{-1}$$
(13)

The equivalent controller F(s) can be obtained from Equation (11) and Equation (13).

$$F(S) = [1 - C_{IMC}(S)\hat{G}(s)]^{-1}C_{IMC}(S) = \lambda \begin{pmatrix} \frac{Ls + R}{s} & \frac{-L\omega}{s} \\ \frac{L\omega}{s} & \frac{Ls + R}{s} \end{pmatrix}$$
(14)

I(s) can be obtained from Equation (11), Equation (12) and Equation (13).

$$I(s) = C_{\mu\nu}(s)G(s)I^{*}(s) = \begin{pmatrix} \lambda & 0\\ s+\lambda & 0\\ 0 & \frac{\lambda}{s+\lambda} \end{pmatrix} I^{*}(s)$$
(15)

The d-axis current reference of $I^*(s)$ is expressed as i_d^* , and the q-axis current $I^*(s)$ is expressed as i_a^* . I(s) can be obtained from Equation (15).

$$\begin{pmatrix} i_d \\ i_q \end{pmatrix} = \begin{pmatrix} \frac{\lambda}{s+\lambda} & 0 \\ 0 & \frac{\lambda}{s+\lambda} \end{pmatrix} \begin{pmatrix} i_d^* \\ i_q^* \end{pmatrix}$$
(16)

Equation (16) shows that the d-q-axis currents realize a dynamic thorough decouple. The response of I to I^* can be equivalent to the first-order inertia link.

The d-q-axis currents can be obtained from Equation (7) as follows:

$$i_d = \frac{L\omega}{Ls+R}i_q + \frac{u_d}{Ls+R}$$
(17)

$$i_q = \frac{-L\omega}{Ls+R}i_d + \frac{u_q}{Ls+R}$$
(18)

The voltage outputted by cascade STATCOM expressed as U(s) can be obtained from Figure 3.

$$U(s) = F(s)[I^*(s) - I(s)] = \lambda \begin{pmatrix} \frac{Ls + R}{s} & \frac{-L\omega}{s} \\ \frac{L\omega}{s} & \frac{Ls + R}{s} \end{pmatrix} \begin{pmatrix} i_d^* - i_d \\ i_q^* - i_q \end{pmatrix}$$
(19)

The equivalent model of cascade STATCOM decoupled by Equation (17), Equation (18) and Equation (19) is shown in Figure 4. Internal model control is also a classical PI type control, but the regulating parameters are only related to minimal open-loop time constant. Article [10] have already made a detailed research on the relationship between the open-loop time constant of cascade STATCOM and the main circuit parameters, the relationship of parameters adjustment and the properties of the system is definite. The system acquires the reference of reactive current expressed as i_q^* from the control object. The reference of active current i_d^* is acquired from the controller for DC bus voltage regulation.



Figure 4. Internal Model Control of STATCOM

4. Computer Simulation and Prototype Trial

4.1. Simulation

In order to validate the correctness of decoupling internal model control (IMC) design and mathematical equations presented in this paper, the simulation tested of 6-KV, 2.8-MVar, star-configured STATCOM with eight H-Bridge PWM converters in each phase is implemented by Matlab/Simulink. The internal model control method is adopted in cascade STATCOM in d-q coordinates, and the load is three phase symmetric load. About 0.1 second before starting the device, the inductance loads with the capacity of 1.44M Var are put into operation. About 1.5 seconds after starting the device, the inductance loads with the capacity of 0.96MVar are put into operation, and then the inductance loads with the capacity of 0.72MVar are cut off in 2 seconds. Simulation waveforms are shown in Figure 5 and Figure 6. For the convenience of comparative analysis, voltage value reproduces at 40 times.

Simulation waveforms before and after compensation are shown in Figure 5 under steady state, such as system voltage u_{sa} , system current i_{sa} , load current i_{la} , output current of cascade STATCOM i_{ca} .



Figure 5. Waveforms of Grid Voltage, Current and Load Current at Inductive Load

Simulation waveforms at sudden loading operation are shown in Figure 6. When the load is changing, the complete compensation can be realized in two power frequency periods. The results of computer simulation shown in Figure 5 and Figure 6 prove that internal model control strategy brings good dynamic and static performance.



Figure 6. Waveforms of Grid Voltage, Load Current and STATCOM Current when Load Changes

4.2. Prototype Trial

Based on the computer simulation and the IMC strategy, 6-KV, 2.8-MVar, starconfigured STATCOM is developed as shown in Figure 7.



Figure 7. Structure Diagram of Medium Voltage H-bridge Cascade STATCOM



Figure 8. Dynamic Performance of STATCOM



Figure 9. Output Voltage and Current of STATCOM

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The dynamic performance is the important indicator of cascade STATCOM. Figure 8 shows the dynamic process from the start to the end of compensation. It also shows that STATCOM can realize fast detection of currents and dynamic compensation within 1.5 power frequency periods, which is consistent with above computer simulations.

The waveforms of output voltage and current of STATCOM after compensation in Figure 9 shows that the system has perfect output waveform and good harmonic characteristics.

5. Conclusion

In this paper, the mathematical model of STATCOM in both a-b-c coordinates and d-q coordinates is developed, and then the computer simulation model of cascade STATCOM is developed based on IMC theory. The computer simulations and prototype trial prove that the current decoupled control strategy established in this paper can realize complete decoupling, and bring good dynamic and static performance.

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References

- D Harikrishna, NV Srikanth. Dynamic Stability Enhancement of Power Systems Using Neural-Network Controlled Static-Compensator. *TELKOMNIKA Indonesia Journal of Electrical Engineering*. 2012; 10(3): 9-16.
- [2] Ali Mohammadi, Sajjad Farajianpoer, Saeed Tavakoli, SMasoud Barakati. Fluctuations Mitigation of Variable Speed Wind Turbine through Optimized Centralized Controller. *TELKOMNIKA Indonesia Journal of Electrical Engineering*. 2012; 10(5): 18-21.
- [3] Wang Zhaoan, Yang Jun, Liu Jinjun. Harmonic suppression and reactive power compensation. Beijing: China Machine Press. 1998: 164-205.
- [4] Hanson DJ, Horwill C, Gemmell BD, et al. A STATCOM-based relocatable SVC project in the UK for national grid. IEEE Power Engineering Society Winter Meeting. New York, USA. 2002: 532-537.
- [5] Yu Qiangguang, Li Pei, Liu Wenhua, et al. Overview of STATCOM technologies. IEEE International Conference on Electric Utility Deregulation, Restructuring and Power Technologies. Hong Kong, China. 2004: 647-652.
- [6] Chiasson JR, Tolbert LM, McKenzie KJ. et al. A unified approach to solving the harmonic elimination equations in multilevel converters. *IEEE Transactions on Power Electronics*. 2004; 19(2): 478-490.
- [7] Poh Chiang Loh, Holmes DGT. Implementation and control of distributed PWM cascaded multilevel inverters with minimal harmonic distortion and common-mode voltage. *IEEE Transactions on Power Electronics.* 2005; 20(1): 90-100.
- [8] Wei Wenhui, Liu Wenhua, Song Qiang, et al. Research on fast dynamic control of static synchronous compensator using cascade multilevel inverters. *Proceedings of the CSEE*. 2005; 25(2): 23-28.
- [9] Zha Xiaoming, Zhang Maosong, Sun Jianjun. Modeling of cascade D-STATCOM with decoupled state variable feedback linearization control. *Proceedings of the CSEE*. 2010; 30(28): 107-113.
- [10] Seinke JK. Switching Frequency Optimal PWM Control of A Three-level Inverter. IEEE Transactions on Power Electronics. 2006; 7(3): 487- 496.
- [11] Geng Juncheng, Liu Wenhua, Yu Xufeng, et al. Modeling of cascade statcom. *Proceedings of the CSEE*. 2003; 23(6): 66-70.
- [12] Sirisukprasert S, Huang AQ, Lai JS. Modeling, analysis and control of cascaded-multilevel converterbased STATCOM.Proc. IEEE-PES general meeting. Toronto, Canada. 2003: 2561-2568.
- [13] Sirisukprasert S. The modeling and control of a cascaded-multilevel converter-based STATCOM. Virginia, the United States & Virginia Polytechnic Institute and State University. 2004.
- [14] Wang Xuan, Lin Jiayang, Teng Letian, Wang Ke, Yuan Meng, et al. *Current Control Strategy of Chain Circuit STATCOM in d-q-0 Coordinates.* Proceeding of the CSEE. 2012; 32(15): 48-54.
- [15] Wang Xuan, Xiong Chaoying, Fu Jian, et al. Dynamic modeling of chain circuit STATCOM. *Proceedings of the CSEE.* 2012; 32(9): 1-6.
- [16] Wang Xuan, Fu Jian, Teng Letian, et al. *Study on current control strategy of chain circuit STATCOM.* Proceedings of the CSEE. 2012; 32(12): 1-6.