TELKOMNIKA Indonesian Journal of Electrical Engineering
Vol.12, No.2, February 2014, pp. 1625 ~ 1633
DOI: http://dx.doi.org/10.11591/telkomnika.v12i2.4379

FFT Analysis on Coupling Effect of Axial and Torsional Vibrations in Circular Cross Section Beam of Steam Turbine Generators

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Abstract

This paper presents a novel method to nonlinearly investigate the dynamics of the coupled axial and torsional vibrations in the circular cross section beam of the steam turbine generator using the FFT analysis. Firstly, the coupled axial and torsional vibrations of a beam are proved by equivalent law of shearing stress and different boundary conditions. Then, a nonlinear mathematical model of the coupled axial and torsional vibrations is established by the Galerkin method. Lastly, the fast Fourier transform (FFT) is employed to investigate the coupled effect of the beam vibration. A practical calculation example is calculated numerically and the coupled mechanism of the beam's axial and torsional vibrations is analyzed in detail. The analysis results show that the frequencies of the coupled response would be existed in some special orders and the coupled response frequencies are smaller than the single vibration. Since for the first time the coupled mechanism of the beam's axial and torsional vibrations is theoretically analyzed, the findings in this work may provide directive reference for practical engineering problems in design of steam turbine generators.

***Keywords**: steam turbine generators, non-linearity, FFT, coupled mechanism, vibration*

1. Introduction

Beam structure has been widely used in the field of aviation, shipbuilding, construction and other engineering, and the circular cross section beam is important component for the steam turbine generators. The vibration analysis and dynamic behavior of beams have been investigated from a single form vibration to coupled vibration, from a single cross section to varying section, from a single material to composite material, and from the linear theory to nonlinear theory [1, 2]. Among the FFT has been proven to be an indispensable tool for the dynamic analysis of beam vibration.

With the increasing of beam span and deformation, the elastic coupling between various vibration forms of the beam is evident, and the coupled vibration analysis of the beams has been an important research area in recent years. Nayfeh [3] established the coupled equation of the beam's longitudinal and transverse vibrations with the analytical method. Xia [4] analyzed the harmonic responses of beams with longitudinal and transverse coupling vibration by the Incremeatal Harmonic Balance method and discovered the phenomenon of inverted peak in amplitude-frequency response curve. Han [5, 6] simplified a flexible tower model in the marine environment as a beam's longitudinal-transverse coupled vibration, and derived the coupled movement equation in the boundary conditions and analyzed the equation in free vibration and forced vibration. Banerjee [7] has got the conclusion that the coupling between the bending and torsional displacements in the free vibratory modes of a beam occurs when the mass centre (centroid) and the shear centre (centre of twist) of the beam cross-section are non-coincident. He researched the cross section shape of beam element, including the thin-walled open-section beam, T beam, wing beam, etc. He used Wittrick-Williams methods for solving equations after establishing accurate dynamic stiffness matrix [8] of the space continuous beam and used opening box girder model as a example to study the effect of the warping stiffness for the natural frequency and got the conclusion that there was calculation error without considering warping. At the same time, the accurate dynamic stiffness matrix, the frequency equation and

Received August 4, 2013; Revised September 14, 2013; Accepted October 15, 2013

the vibration model formula [9] are applied respectively to solving the bending - torsional coupled vibration of a composite material Timoshenko beam. Other Scholars also used wing beam as the research model to research the free vibration of beam element [10] and bending under certain load and random load, as well as the coupled vibration response [11]. The explicit formula has been established to investigate the cantilever Euler Bernoulli beam' bending and torsional coupled vibration [12], the general quality is used to explain the dynamic response of the beam's flexural-torsional coupled vibration [13] and bending torsional coupled vibration [14, 15]. However, there is little work for the investigation of the beam's axial and torsional coupled vibration due to that in many conditions the shearing stress was ignored. For the circular cross section beams, the shearing stress could not be ignored. Hence, it is imperative to analyze the coupled axial and torsional vibration for circular cross section beams.

In order to address the above mentioned issue, this work present the investigation of the nonlinear dynamics of the coupled axial and torsional vibrations in the circular cross section beams of steam turbine generators using the FFT analysis. The Galerkin method was adopted to establish the coupled model of the beam's axial and torsional vibrations and the frequency response of the system's impulse test was analyzed by FFT. Numerical analysis was carried out to evaluate the dynamics of the proposed model.

2. Mathematical Model

2.1. Coupling Analysis

As shown in Figure 1, put a circular cross section beam in the plane rectangular coordinate system, its axis and *X* axle are in the same line, and there is a $\hat{x}$ torque on the beam. So the $\hat{x}$ direction of the shear strain and shear stress can be calculated as:

$$\Delta\gamma = \frac{\Delta\theta \cdot r}{dx} = \frac{\partial\theta}{\partial x} r\,, \quad \tau = G \cdot \gamma \tag{1}$$

Where θ is the angle deformation, r is the radius, G is the shear elasticity modulus, γ is the shear strain and τ is the shear stress.

According to the law of shear stress reciprocal of the elastic mechanics, there exists $\tau' = \tau$. Hence, there is a additional axial shear stress parallel the *X* axle, the force will make beam element deformed along the axial. The axial force can be expressed as:

$$F = G\int_A \gamma \cdot dA = G\int_0^r \frac{\partial\theta}{\partial x} r \cdot d(\pi r^2) = 2G\frac{\partial\theta}{\partial x}\pi\int_0^r r^2 \cdot dr = \frac{2}{3}\pi r^3 G\frac{\partial\theta}{\partial x} \tag{2}$$

Without considering the circle beam axial vibration, the boundary conditions of torsional vibration equation as follows:

The fixed end: $\theta(0,t) = 0\,, \theta(l,t) = 0$ (3)

The free end: $\left.\frac{\partial\theta(x,t)}{\partial x}\right|_{x=0} = 0\,, \left.\frac{\partial\theta(x,t)}{\partial x}\right|_{x=l} = 0$ (4)

Where l is the length of the beam.

Considering the circle beam axial vibration, the length of the beam will change to l_1, and the boundary conditions also change as follows:

The fixed end: $\theta(0,t) = 0\,, \theta(l_1,t) = 0$ (5)

The free end: $\left.\frac{\partial\theta(x,t)}{\partial x}\right|_{x=0} = 0\,, \left.\frac{\partial\theta(x,t)}{\partial x}\right|_{x=u} = 0$ (6)

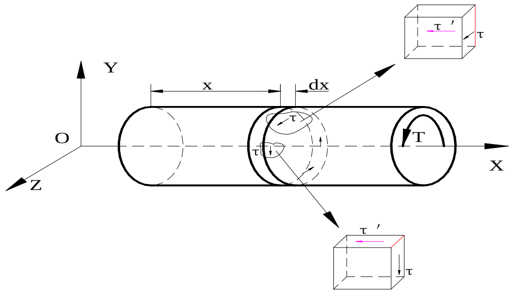

Figure 1. Circular Cross Section Beam Axial Torsional Coupled Vibration Analysis

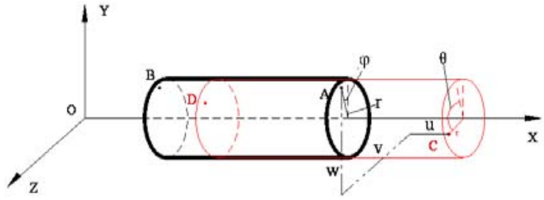

Figure 2. Circular Cross Section Beam Axial Torsional Deformation

2.2. Coupling Equation

In circular cross section beam with length dx of infinitesimal section, set up the rectangular coordinate system as shown in Figure 1. point ***A*** and point ***B*** are the point in the end face without the elastic deformation; *r* is the distance between the circle beam center and point ***A***; φ is the angle between ***Y*** axle and ***OA***; while point ***C*** and point ***D*** are the positions of the point ***A*** and point ***B*** after the elastic deformation. So the displacements of the point ***A*** and point ***B*** can be expressed as:

$$\Delta\vec{A} = u(x,t)\vec{i} + r\cos\varphi\vec{j} + r\sin\varphi\vec{k} \tag{7}$$

$$\Delta\vec{B} = (u + u_x dx)\vec{i} + r\cos(\varphi + \theta_x dx)\vec{j} + r\sin(\varphi + \theta_x dx)\vec{k} \tag{8}$$

According to the Figure 2, there is:

$$\Delta\vec{A} + \Delta\vec{C}\vec{D} = \Delta\vec{B} + dxi \tag{9}$$

Where $\Delta\vec{C}\vec{D}$ is the vector from point ***C*** to point ***D***. So the deformation length of the infinitesimal section is:

$$|\Delta CD| = ds = \sqrt{[(1+u')dx]^2 + r^2[\cos(\varphi+\theta' dx) - \cos\varphi]^2 + r^2[\sin(\varphi+\theta' dx) - \sin\varphi]^2} \tag{10}$$

After reduction, that is $|\Delta CD| = ds = [(1+u')^2 + \theta'^2 r^2]^{1/2} dx$, that is:

$$dx/ds = [(1+u')^2 + \theta'^2 r^2]^{-1/2} \tag{11}$$

The unit vector for parallel infinitesimal section is:

$$\Delta CD/|\Delta CD| = \delta = \{(1+u')dx\vec{i} + r[\cos(\varphi+\theta' dx) - \cos\varphi]\vec{j} + r[\sin(\varphi+\theta' dx) - \sin\varphi]\vec{k}\}/ds \tag{12}$$

After reduction, that is:

$$\delta = [(1+u')\vec{i} - r\cdot\theta'\sin(\varphi + \frac{\theta'}{2}dx)\vec{j} + r\cdot\theta_x\cos(\varphi + \frac{\theta'}{2}dx)\vec{k}]\frac{dx}{ds} \tag{13}$$

If there is initial axial force N_0 in the circular cross section beam, so the beam deformation during its tension changes, then the instantaneous value is:

$$N = N_0 + \frac{EA(ds - dx)}{dx} = N_0 - EA + EA\frac{ds}{dx} \tag{14}$$

Where ***A*** is the cross-sectional area for circular cross section beam.

According to the momentum equation, there is:

$$\partial(N\delta)/\partial x = \rho A(\ddot{u}i + \ddot{v}j + \ddot{w}k) \tag{15}$$

Substitute (8), (9) and (10) into (15), according to equal vector, we can get:

$$\frac{\partial}{\partial x}[(\frac{N_0 - EA}{\sqrt{(1+u')^2 + \theta'^2 r^2}} + EA)(1+u')] = \rho A\ddot{u} \tag{16a}$$

$$\frac{\partial}{\partial x}[(\frac{N_0 - EA}{\sqrt{(1+u')^2 + \theta'^2 r^2}} + EA)\cdot(-r)\cdot\theta'\sin(\varphi + \frac{\theta'}{2}dx)] = \rho A\ddot{v} \tag{16b}$$

$$\frac{\partial}{\partial x}[(\frac{N_0 - EA}{\sqrt{(1+u')^2 + \theta'^2 r^2}} + EA)\cdot r\cdot\theta'\cos(\varphi + \frac{\theta'}{2}dx)] = \rho A\ddot{w} \tag{16c}$$

Noted that $(\rho A\ddot{v})^2 + (\rho A\ddot{w})^2 = (\rho Ar\ddot{\theta})^2$, for circular the cross section beam, there is $J_P = \pi r^4/2 = A\cdot r^2/2$, according to (10), ignored the initial axial force N_0 and consider the torsional deformation, we can get:

$$\begin{cases} \rho A\ddot{u} - EAu'' = EAr^2(\theta'\theta'' - u'\theta'^2 - 2u'\theta'\theta'') \\ \rho J_P\ddot{\theta} - GJ_P\theta'' = \dfrac{r^2}{2}EA(2u'\theta'' + u'^2\theta'' - u''\theta' - u'u''\theta') \end{cases} \tag{17}$$

Herein, (17) is the axial- torsional coupled vibration equation of the circular cross section beam. When $r = 0$, that means only considering the vibration in the beam's shaft, (17) would simplified as the single form vibration, which proved that there is no coupled vibration phenomenon on the axis of the beam with circular cross section.

3. Coupling Equation Analysis

Ignore the high-order items in (11), it yields:

$$\begin{cases} \ddot{u} - a^2u'' = a^2r^2\theta'\theta'', a = \sqrt{G/\rho} \\ \ddot{\theta} - b^2\theta'' = a^2(2u'\theta'' - u''\theta'), b = \sqrt{E/\rho} \end{cases} \tag{18}$$

For the first equation in (18), it contains the radius parameter r, which means the axial coupled force on the different point does not equal in a circular cross section; namely axial amplitude in the same cross section is not the same, and the deformation is a parabolic. Hence, the natural frequencies of the various points on the same cross section are not consistent. However, if the coupled item is considered as a small parameter, the amplitude can be the same on the same cross section and it is feasible to solve this equation.

Based on the Galerkin method, axial displacement u and torsion angle θ can be expressed as the linear superposition of vibration mode function, i.e.

$$u(x,t) = \sum_{i=1}^{m} U_i(x)T_{ui}(t)\text{, } \theta(x,t) = \sum_{i=1}^{n} \Theta_i(x)T_{\theta i}(t) \tag{19}$$

Where, $U_i(x)$ and $\Theta_j(x)$ are the axial and torsional vibration mode function, respectively, and they are determined by boundary conditions; $T_{ui}(t)$ and $T_{\theta j}(t)$ are the time function; m and n are the axial and torsional mode, respectively.

For the two ends free beam with circular cross section, according to the boundary conditions, the vibration mode function is defined as:

$$U_i(x) = \cos(i\pi x/l)\,,\;\; \Theta_i(x) = \cos(i\pi x/l) \quad (20)$$

According to (18), (19) and (20), we derive:

$$\sum_{i=1}^{m}[\ddot{T}_{ui}(t) + \omega_{ui}^2 T_{ui}(t)]\cos\frac{i\pi x}{l} = a^2 r^2 (\frac{\pi}{l})^3 \sum_{j=1}^{n}\sum_{k=1}^{n} jk^2 T_{\theta j}(t) T_{\theta k}(t)\sin\frac{j\pi x}{l}\cos\frac{k\pi x}{l} \quad (21a)$$

$$\sum_{j=1}^{n}[\ddot{T}_{\theta j}(t) + \omega_{\theta i}^2 T_{\theta j}(t)]\cos\frac{j\pi x}{l} = a^2 (\frac{\pi}{l})^3 \sum_{i=1}^{m}\sum_{j=1}^{n} ij T_{ui}(t) T_{\theta j}(t)(2j\sin\frac{i\pi x}{l}\cos\frac{j\pi x}{l} - i\cos\frac{j\pi x}{l}\sin\frac{i\pi x}{l}) \quad (21b)$$

Where $\omega_{ui}^2 = a^2(i\pi/l)^2$, $\omega_{\theta i}^2 = a^2(j\pi/l)^2$.

For (21), select $m = n = 1$, suppose $a^2(\pi/l)^2 = \omega^2$, $b^2(\pi/l)^2 = \lambda^2$, and multiply $\cos(\pi x/l)dx$ by both sides of (21), we can get:

$$\begin{cases} \ddot{T}_u(t) + \omega^2 T_u(t) = \dfrac{4}{3}\omega^2 r^2 T_\theta^2(t) \\ \ddot{T}_\theta(t) + \lambda^2 T_\theta(t) = \dfrac{4}{3}\omega^2 T_u(t) T_\theta(t) \end{cases} \quad (22)$$

Using the multi-scale method to solve (22), after a series of simplification it yields:

$$\begin{cases} T_{u2} = \dfrac{4}{3} r^2 B\overline{B} + \dfrac{4\omega^2 r^2 B^2 e^{2i\lambda T_0}}{3(\omega^2 - 4\lambda^2)} + cc \\ T_{\theta 2} = -\dfrac{4\omega}{3(\omega + 2\lambda)} ABe^{i(\omega+\lambda)T_0} - \dfrac{4\omega}{3(\omega - 2\lambda)} A\overline{B} e^{i(\omega-\lambda)T_0} + cc \end{cases} \quad (23)$$

For (23), $\omega/\lambda = \sqrt{a^2(\pi/l)^2 / b^2(\pi/l)^2} = \sqrt{E/G} = \sqrt{2(1+\nu)} \neq 2$,where ν is the poisson's ratio. Therefore, there is no internal resonance.

If considering the condition of the internal resonance, choose $\nu \approx 0.3$ and we can get:

$$\omega_m / \lambda_n = \sqrt{2(1+\nu)} \approx 1.6$$

When take the 4th order mode of the axial and take the 5th order mode of the reverse there may be internal resonance.

Take the mode number $m = n = 5$ into (21) we derive:

$$\ddot{T}_{u1} + \omega_{u1}^2 T_{u1} = a^2 r^2 \frac{\pi^2}{l^3}[\frac{4}{3}T_{\theta 1}^2 + \frac{64}{15}T_{\theta 2}^2 + \frac{324}{35}T_{\theta 3}^2 + \frac{1024}{63}T_{\theta 4}^2 + \frac{2500}{99}T_{\theta 5}^2 - \frac{24}{15}T_{\theta 1}T_{\theta 3} - \frac{40}{105}T_{\theta 1}T_{\theta 5} - \frac{512}{105}T_{\theta 2}T_{\theta 4} - \frac{1800}{189}T_{\theta 3}T_{\theta 5}] \quad (24a)$$

$$\ddot{T}_{u2} + \omega_{u2}^2 T_{u2} = a^2 r^2 \frac{\pi^2}{l^3}[\frac{128}{15}T_{\theta 1}T_{\theta 2} - \frac{512}{105}T_{\theta 1}T_{\theta 4} + \frac{128}{7}T_{\theta 2}T_{\theta 3} - \frac{128}{9}T_{\theta 2}T_{\theta 5} - \frac{4086}{135}T_{\theta 3}T_{\theta 4} + \frac{12800}{231}T_{\theta 4}T_{\theta 5}] \quad (24b)$$

$$\ddot{T}_{u3}+\omega_{u3}^2T_{u3}=a^2r^2\frac{\pi^2}{l^3}[-\frac{4}{5}T_{\theta1}^2+\frac{64}{7}T_{\theta2}^2+12T_{\theta3}^2+\frac{1024}{55}T_{\theta4}^2+\frac{2500}{91}T_{\theta5}^2 +\frac{648}{35}T_{\theta1}T_{\theta3}-\frac{1800}{189}T_{\theta1}T_{\theta5}-\frac{4608}{135}T_{\theta2}T_{\theta4}+\frac{735}{11}T_{\theta3}T_{\theta5}] \tag{24c}$$

$$\ddot{T}_{u4}+\omega_{u4}^2T_{u4}=a^2r^2\frac{\pi^2}{l^3}[-\frac{284}{105}T_{\theta1}T_{\theta2}+\frac{8192}{63}T_{\theta1}T_{\theta4}+\frac{512}{15}T_{\theta2}T_{\theta3} +\frac{12800}{231}T_{\theta2}T_{\theta5}+\frac{2048}{55}T_{\theta3}T_{\theta4}+\frac{2048}{39}T_{\theta4}T_{\theta5}] \tag{24d}$$

$$\ddot{T}_{u5}+\omega_{u5}^2T_{u5}=a^2r^2\frac{\pi^2}{l^3}[-\frac{4}{21}T_{\theta1}^2-\frac{64}{9}T_{\theta2}^2+\frac{324}{11}T_{\theta3}^2+\frac{1024}{39}T_{\theta4}^2+\frac{100}{3}T_{\theta5}^2 -\frac{200}{21}T_{\theta1}T_{\theta3}+\frac{5000}{99}T_{\theta1}T_{\theta5}+\frac{12800}{231}T_{\theta2}T_{\theta4}+\frac{5000}{91}T_{\theta3}T_{\theta5}] \tag{24e}$$

$$\ddot{T}_{\theta1}+\omega_{\theta1}^2T_{\theta1}=a^2\frac{\pi^2}{l^3}(\frac{4}{3}T_{u1}T_{\theta1}-16T_{u1}T_{\theta3}-\frac{292}{21}T_{u1}T_{\theta5}+\frac{64}{15}T_{u2}T_{\theta2}-\frac{9472}{105}T_{u2}T_{\theta4} +\frac{92}{5}T_{u3}T_{\theta1}+\frac{324}{35}T_{u3}T_{\theta3}-\frac{700}{3}T_{u3}T_{\theta5}+\frac{256}{3}T_{u4}T_{\theta2}+\frac{1024}{63}T_{u4}T_{\theta4} +\frac{284}{21}T_{u5}T_{\theta1}+\frac{4700}{21}T_{u5}T_{\theta3}+\frac{2500}{99}T_{u5}T_{\theta5}) \tag{24f}$$

$$\ddot{T}_{\theta2}+\omega_{\theta2}^2T_{\theta2}=a^2\frac{\pi^2}{l^3}(\frac{208}{15}T_{u1}T_{\theta2}-\frac{448}{15}T_{u1}T_{\theta4}-\frac{16}{3}T_{u2}T_{\theta1}+\frac{300}{7}T_{u2}T_{\theta3}-\frac{364}{3}T_{u2}T_{\theta5} -\frac{176}{7}T_{u3}T_{\theta2}+\frac{320}{3}T_{u3}T_{\theta4}+\frac{2624}{105}T_{u4}T_{\theta1}-\frac{1088}{15}T_{u4}T_{\theta3}+\frac{49600}{231}T_{u4}T_{\theta5} +\frac{944}{9}T_{u5}T_{\theta2}-\frac{36800}{231}T_{u5}T_{\theta4}) \tag{24g}$$

$$\ddot{T}_{\theta3}+\omega_{\theta3}^2T_{\theta3}=a^2\frac{\pi^2}{l^3}(-\frac{4}{5}T_{u1}T_{\theta1}+\frac{1188}{35}T_{u1}T_{\theta3}-\frac{300}{7}T_{u1}T_{\theta5}+\frac{64}{7}T_{u2}T_{\theta2} +\frac{256}{3}T_{u2}T_{\theta4}-\frac{108}{7}T_{u3}T_{\theta1}+12T_{u3}T_{\theta3}+\frac{2052}{11}T_{u3}T_{\theta5}-\frac{256}{3}T_{u4}T_{\theta2} +\frac{1024}{55}T_{u4}T_{\theta4}+\frac{100}{3}T_{u5}T_{\theta1}-\frac{1404}{11}T_{u5}T_{\theta3}+\frac{2500}{91}T_{u5}T_{\theta5}) \tag{24h}$$

$$\ddot{T}_{\theta4}+\omega_{\theta4}^2T_{\theta4}=a^2\frac{\pi^2}{l^3}(-\frac{80}{21}T_{u1}T_{\theta2}+\frac{3904}{63}T_{u1}T_{\theta4}-\frac{16}{15}T_{u2}T_{\theta1}+\frac{496}{15}T_{u2}T_{\theta3} +\frac{31600}{231}T_{u2}T_{\theta5}+\frac{16}{15}T_{u3}T_{\theta2}+\frac{2368}{55}T_{u3}T_{\theta4}-\frac{1856}{63}T_{u4}T_{\theta1} -\frac{64}{11}T_{u4}T_{\theta3}+\frac{2752}{39}T_{u4}T_{\theta5}-\frac{18800}{231}T_{u5}T_{\theta2}-\frac{704}{39}T_{u5}T_{\theta4}) \tag{24i}$$

$$\ddot{T}_{\theta5}+\omega_{\theta5}^2T_{\theta5}=a^2\frac{\pi^2}{l^3}(-\frac{4}{21}T_{u1}T_{\theta1}-\frac{28}{3}T_{u1}T_{\theta3}+\frac{9700}{99}T_{u1}T_{\theta5}-\frac{64}{9}T_{u2}T_{\theta2}+\frac{1984}{33}T_{u2}T_{\theta4} -\frac{4}{21}T_{u3}T_{\theta1}+\frac{324}{11}T_{u3}T_{\theta3}+\frac{7300}{91}T_{u3}T_{\theta5}-\frac{256}{21}T_{u4}T_{\theta2}+\frac{1024}{39}T_{u4}T_{\theta4} -\frac{4700}{99}T_{u5}T_{\theta1}+\frac{200}{91}T_{u5}T_{\theta3}+\frac{100}{3}T_{u5}T_{\theta5}) \tag{24j}$$

So (24) is the axial torsional coupled vibration equation in the first 5 order modes.

4. Numerical Analysis and Results

The data of the beam used for the analysis are as follows: elastic modulus $E = 2.1\times10^{11} N/m^2$, Poisson's ratio $\nu \approx 0.3$, length $l = 3m$, radius $r = 0.03m$.

Establish a system simulation model according to (24), the axial displacement and torsional displacement of the beam element were obtained in the first 5 modes. Figure 3 shows the analysis results. Because each order mode has the same energy, the vibration frequency and vibration peak in different modes satisfy certain multiple relationship in both two kinds of the axial and torsional vibrations.

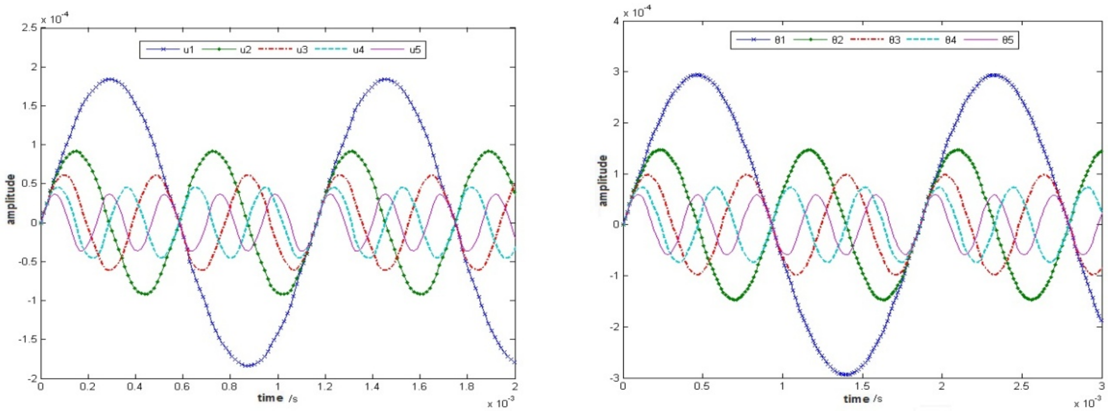

Figure 3. The First 5 Order Modes of the Beam with Circular Cross Section Considering the Axial Torsional Coupled Vibration: (a) torsional vibration, (b) axial vibration

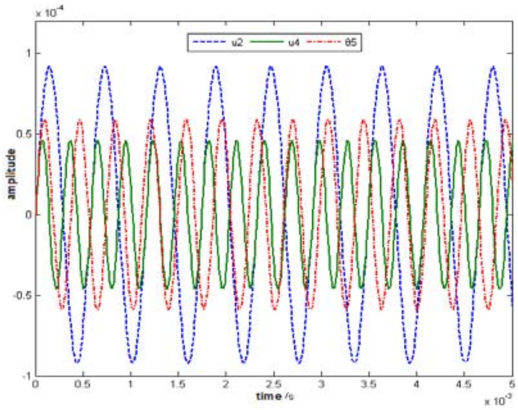

Figure 4. Axial the 2nd, 4th Order and Torsional the 5th Order Vibration (m = n = 5)

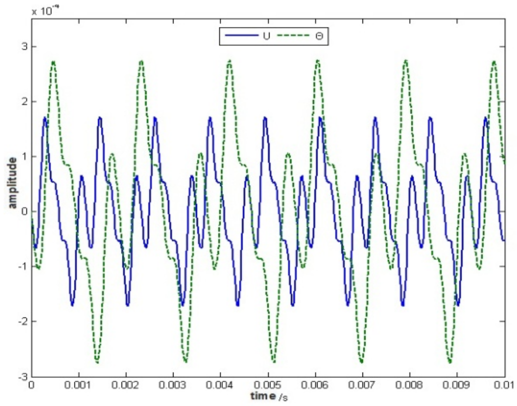

Figure 5. Axial-torsional Coupled Vibration Time-domain Diagram in a Cross Section

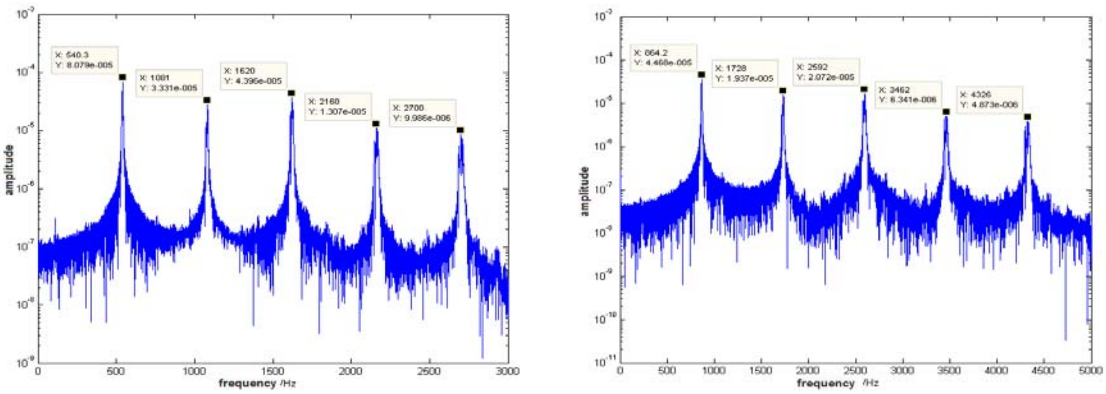

Figure 6. Axial-torsional Coupled Vibration Frequency Domain in a Cross Section (a) torsional vibration, (b) axial vibration

In order to understand the change of natural frequency of the beam element with circular cross section in coupled axial-torsional condition, the fast Fourier transform (FFT) is applied to investigating the coupling effect. According to the internal resonance occurring condition, the time-domain diagram considering the axial and torsional coupled vibrations with the second, fourth order axial vibration and the fifth torsional order is shown in Figure 4. From the figure it can be seen that the ratio of these three kinds of vibration frequencies is nearly rational number, but there are no displacement change in the classic internal resonance.

From (19), the axial-torsional coupled vibration equation from 1 m to an end face could be obtained as:

$$\begin{cases} u(x,t) = \cos(\frac{\pi}{3})T_{u1} + \cos(\frac{2\pi}{3})T_{u2} + \cos(\frac{3\pi}{3})T_{u3} + \cos(\frac{4\pi}{3})T_{u4} + \cos(\frac{5\pi}{3})T_{u5} \\ \theta(x.t) = \cos(\frac{\pi}{3})T_{\theta1} + \cos(\frac{2\pi}{3})T_{\theta2} + \cos(\frac{3\pi}{3})T_{\theta3} + \cos(\frac{4\pi}{3})T_{\theta4} + \cos(\frac{5\pi}{3})T_{\theta5} \end{cases} \quad (25)$$

According to the result of Figure 4 and (25), the vibration time-domain diagram of the cross section was obtained and shown in Figure 5.
Use FFT transform for Figure 5, the vibration frequency domain of the cross section is shown in Figure 6, and the natural frequency is listed in Table 1.

Table 1. Natural Frequency in the Beam (Hz)

Type	Coupled	1	2	3	4	5
Axial vibration	No	864.8	1729.6	2594.4	3459.2	4324.0
Axial vibration	Yes	864.2	1728	2592	3642	4326
Torsional vibration	No	540.4	1080.8	1621.2	2161.6	2702.0
Torsional vibration	Yes	540.3	1081	2592	3642	4326

5. Conclusion

In order to investigate the nonlinear dynamics of the coupled axial and torsional vibrations in the circular cross section beam of steam turbine generators, a novel coupled model has been established for the first time in this work. Numerical analysis was carried out to evaluate the dynamics of coupled model of the beam's axial and torsional vibrations. The *FFT analysis* results show that the coupled vibration natural frequencies are smaller than the single type vibration in the corresponding orders except the second order natural frequencies in the torsional vibration, which may be caused by calculation error. At the same time, there are some additional natural frequency by the coupling effect near the main natural frequency in the frequency domain, which could be ignored. Hence, the analysis results of the proposed coupled model can provide valuable theoretical reference for the design and application of circular cross section beam in the steam turbine generators.

Acknowledgements

This project is sponsored by the China Three Gorges University 2012 Talents Scientific Research Starting Fund (No: KJ2012B045). The author would like to thank the Editor-in-Chief, Tole Sutikno for his kind help and suggestions.

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