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Multidisciplinary Multi-objective Optimization Method for Vehicle Crashworthiness Design

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Abstract
This paper proposes a multidisciplinary multi-objective optimization method for vehicle Crashworthiness design. Multi-objective optimization methods are discussed. Considering the objective functions with difference dimensions, we improve α -method based on normalizes objective function. As the numerical example, the vehicle crashworthiness design problem is calculated, and we compare the results with SO method, interior point method and active-set method, where interior point method and active-set method are based the improved α -method. Examples indicate that this algorithm has less number of iteration than the others.

Keywords: *multidisciplinary, multiobjective, crashworthiness design*

1. Introduction

Due to the nature of the collision numerical analysis, simulation and optimization design of the automobile collision safety is a very tough problem, the instability and uncertainty of collision analysis has been concerned [1-3]. Experience shows that usually the simulation process will repeat for several times before reaching a satisfactory result, the instability hinders the integration of optimization procedures and analysis process. Meanwhile, due to the calculation consumption of the explicit finite element analysis, it is quite difficult to achieve secure integration optimization process. Chen used the global optimizing capability of genetic algorithm to develop a method for the collision structure and crashworthiness optimization design [4], Knap and Holnicki-Szulc optimizing designed the improved structure by the dynamic analysis and the concept of two-stage design based on the VDM [5], many other researchers have proposed a number of theories and methods.

2. Multi-objective Optimization Methods

The idea of multi-objective optimization was originally raised in 1896 by French economist V. Pareto. He converted many essentially incomparable goals into the optimization of a single target from the perspective of political economics, thus included the multi-objective programming problem and the concept of multi-objective. In 1947, von Neumann and Morgenstern presented the multi-objective problem with contradicting multiple decision-makers from the perspective of game theory.

In reference [6], Cuoyun Lin and Jiali Dong elaborated the theory and method of multi-objective optimization in details. Many papers discuss the application of the multi-object optimization [7-8].

Consider multi-objective programming (VP):

$$\min_{x\in R} f(x)=\left(f_1(x), f_2(x),\dots, f_p(x)\right)^T$$

Where,

$$R=\left\{x\in E^n \middle| g(x)=\left(g_1(x), g_2(x),\dots, g_m(x)\right)^T\le 0, h(x)=\left(h_1(x),\cdots, h_l(x)\right)^T=0\right\}.$$

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The following general definitions and theorems hold for the multi-objective programming:

Definition 1: Let $x^* \in R$, if for any $x \in R$, we can get $f\left(x^*\right) \leq f\left(x\right)$. Then for every $f\left(x^*\right) \leq f\left(x\right)$, we can get $f_j\left(x^*\right) \leq f_j\left(x\right)$. Namely $x*$ is the absolute optimum solution to (VP). It is noted as R_{ab}.

Definition 2: Let $x^* \in R$, if does not exist $x \in R$ to satisfy $f\left(x\right) \leq f\left(x^*\right)$, then at least exist a $1 \leq j_0 \leq p$, can make $f_{j_0}\left(x\right) < f_{j_0}\left(x^*\right)$. Namely x^* is the effective solution to (VP), also referred as Pareto optimum solution. It is noted as R_{pa}.

Definition 3: Let $x^* \in R$, if does not exist $x \in R$ to satisfy $f\left(x\right) \leq f\left(x^*\right)$, then x^* is the weakly effective solution for (VP). It is noted as R_{wp}.

Theorem 1: For problem (VP), $R_{ab} \subset R_{pa} \subset R_{wp} \subset R$.

Theorem 2: (1) if $R_{ab} \neq \Phi$, then $R_{ab} = R_{pa}$.

(2) If R is a convex set, $f\left(x\right)$ is a strictly convex vector function on R, then $R_{pa} = R_{wp}$.

3. Determination Methods for the Weighting Coefficients Based on Improved α - Method

Consider the multi-objective programming problem (VP). The premise of the α -method is that problem (VP) has no absolute optimal solution.

First use unconstrained nonlinear programming method to solve the following p single goal problems:

$$\min_{x \in R} f_i\left(x\right), i = 1, 2, \cdots, p$$

Assume x^j is the optimal solution, and note $f_i(x^j) = f_i^j \;\; j = 1, 2, ..., p; i = 1, 2, ..., p$. Now normalize the p values, let:

$$g_i^j = \frac{f_i^j}{\sqrt{\sum_{i=1}^{p} (f_i^j)^2}},$$

In the image space, draw a hyper plane over p points $(g_1^j, g_2^j, ..., g_p^j)^T \;\; (j = 1, 2, ..., p)$.

Assume the equation is $\sum_{i=1}^{p} \lambda_i g_i^j = \alpha$, where $\sum_{i=1}^{p} \lambda_i = 1$, note $\lambda = (\lambda_1, \lambda_2, ..., \lambda_p)^T$, so we can get:

$$\begin{cases} \sum_{i=1}^{p} \lambda_i = 1 \\ \sum_{i=1}^{p} g_i^j \lambda_i - \alpha = 0, \quad j = 1, 2, ..., p \end{cases} \tag{1}$$

This is a linear equations of $p+1$ equations of $p+1$ variables including $\lambda_1, \lambda_2, ..., \lambda_p, \alpha$.

Note matrix as $A=(g_i^j)$, it can be proved that: when problem (VP) has no absolute optimal solution, inverse matrix A^{-1} of matrix A exists, and the Equation (1) has a unique solution:

$$\lambda^T = \frac{e^T A^{-1}}{e^T A^{-1} e}, \alpha = \frac{1}{e^T A^{-1} e},$$

Where $e=(1,1,...,1)^T \in R^p$. When all $\lambda_i, i=1,2,...,p$ are greater than 0, set them as the weighting coefficients. Normalize the objective function:

$$g_i(x) = \frac{f_i(x)}{\sqrt{\sum_{i=1}^{p} f_i(x)^2}}$$

And solve the single goal nonlinear programming problem of linear weighted sums:

$$\min_{x \in R} \lambda_i g_i(x) \quad . \tag{2}$$

Then optimum solution of problem (2) must be the effective solution of formula (VP). When all $\lambda_i, i=1,2,...,p$ are greater than or equal to 0, set them as the weighting coefficient, the optimal solution of problem (2) must be the weakly efficient solution of problem (VP) [9].

α -method can also be simplified, when $j \neq i$, do not calculate $f_i(x^j) = f_i^j$. Only calculate $f_i(x^i) = f_i^i$, then the Equation (1) can be simplified as:

$$\begin{cases} \sum_{i=1}^{p} \lambda_i = 1 \\ g_i^i \lambda_i - \alpha = 0, \quad j = 1,2,...,p \end{cases} \tag{3}$$

Similarly, when problem (VP) has no absolute optimal solution, Equation (3) has only one solution:

$$\lambda_i = \frac{1/g_i^i}{\sum_{j=1}^{p} 1/g_j^j}, \alpha = \frac{1}{\sum_{j=1}^{p} 1/g_j^j}, i = 1,2,...,p,$$

Where, $\lambda \geq 0$ not necessarily exist. But when all $g_i^i \geq 0$, naturally all $\lambda_i, i=1,2,...,p$ are greater than or equal to 0, then take $\lambda_i, i=1,2,...,p$ as the weighting coefficient equation (3).

4. Actual Example Verification

4.1. Problems of Initial Design Optimization

The instance of Xingtao Liao is used [10], Dodge Warriors cars are his study objects, he establishes the finite element model of vehicle collision and finite element model of deformable obstacle avoidance, he also verifies the effectiveness of the model, uses stepwise regression agent model, NSGA - II multi-objective genetic algorithm, combines the LS-DYNA finite element software, studies the 100% head-on collision of that car and the multi-objective optimization problem of 40% head-on offset collision. As shown in Figure 1:

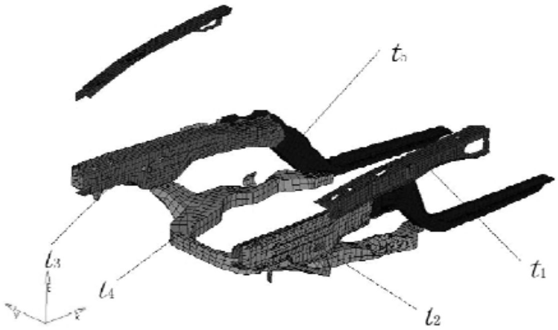

Figure 1. The Design Variables of Vehicle Model

The thickness t_1,t_2,t_3,t_4,t_5 (mm) of the 5 reinforcements of the established vehicle model are set as design variables, the vehicle mass $Mass$, the integral value A_{in} of the B-pillar acceleration of 0.04~ 0.08 seconds in 100% head-on collision and pedals intrusion of 40% head-on offset collision $Intrusion$ are set as objective functions to realize the multi-objective optimization. The automotive mass $Mass$ is selected as the optimization goal to get light cars, collision, the integration value of the B-pillar acceleration of 0.04~0.08 seconds of 100% head-on collision are selected as the target function for the higher acceleration during that period, the reduced integral can promote the reduction of the entire acceleration, and the pedals intrusion $Intrusion$ of 40% head-on offset collision is selected as the objective function for its large impact on the vehicle security [11].

The optimization problem can be represented mathematically by formula (4).

$$\begin{cases} \min & f(t)=\left(Mass, A_{in}, Intrusion\right)^T \\ s.t. & 1mm \le t_1,t_2,t_3,t_4,t_5 \le 3mm \end{cases} \tag{4}$$

Let $t=(t_1,t_2,t_3,t_4,t_5)$, the agent models of the three objective functions are respectively:

$$\begin{aligned} f_1(t) = Mass = {} & 1640.2823+2.357\ 328\ 5t_1+2.322\ 003\ 5t_2+4.568\ 876\ 8t_3 \\ & +7.721\ 363\ 3t_4+4.455\ 950\ t_5 \end{aligned} \tag{5}$$

$$\begin{aligned} f_2(t) = A_{in} = {} & 6.5856+1.15t_1-1.0427t_2+0.9738t_3+0.8364t_4-0.3695t_1t_4 \\ & +0.0861t_1t_5+0.3628t_2t_4-0.1106t_1^2-0.3437t_3^2+0.1764t_4^2 \end{aligned} \tag{6}$$

$$\begin{aligned} f_3(t) = Intrusion = {} & -0.0551+0.018t_1+0.1024t_2+0.0421t_3-0.0073t_1t_2+0.024t_2t_3 \\ & -0.0118t_2t_4-0.0204t_3t_4-0.008t_3t_5-0.0241t_2^2+0.0109t_4^2 \end{aligned} \tag{7}$$

4.2. Non-decomposition Method (SO Method)

As the difference of magnitude orders between the three objective functions are huge, literature [9] and [12] described the non-decomposition method (SO method), gives a certain weight according to the importance of the objective functions, it considers more of the integral value A_{in} of the B-pillar acceleration while considers less of the mass $Mass$ and pedals intrusion $Intrusion$, weight setting of $Mass$, A_{in} and $Intrusion$ are respectively 0.2,0.7 and 0.1. Structure the objective function: Let $f'_j(t)=\dfrac{f_j(t)-\alpha_j}{\beta_j-\alpha_j}$, where α_j , β_j are respectively the lower and upper limit of the j th function. Solve the problem:

$$\begin{cases} \min & 0.2Mass' + 0.7A_{in}' + 0.1Intrusion' \\ s.t. & 1mm \le t_1, t_2, t_3, t_4, t_5 \le 3mm \end{cases}$$

4.3. Comparison of the Optimization Results

SO method based improved α - method (denoted α -ISO method) is used, first solve the following three single objective problems:

$$\min f_i(t), i = 1,2,3$$
$$\text{s.t. } 1mm \le t_1, t_2, t_3, t_4, t_5 \le 3mm$$

Set the optimal solution as t^j, and note $f_i(t^j) = f_i^j$, $j = 1,2,3; i = 1,2,3$ calculate normalized $g_i(t^i) = g_i^i$, use formula (3) and solve linear weighted sum problem (4)..

The optimal solution is $(1, 3, 3, 1, 1)$, the objective function values are respectively:

$$Mass = 1675.4896kg,\ A_{in} = 1.6428m/s,\ Intrusion = 0.2639m.$$

Table 1 gives the calculation results of SO method, interior point method and active-set method based the above improved α -method. As the weighting coefficients are all greater than 0, the effective solution of the vehicle crashworthiness problem is got.

Table 1. The Comparison of Calculated Results

Algorithm	Initial Points	Optimal Solution(mm)	Mass(kg)	Ain(m*s^-1)	Intrusion(m)	Number of Iteration
SO method of [9]	[1,3,1,1,1]	[1.0,3.0,1.0,1.0,1.0]	1666.4	6.9448	0.0925	
interior-point	[1.1;2.9;1.1;1.1;1.1]	[3.0,3.0,1.0,2.1,3.0]	1688.4	9.8012	0.0445	30
	[2;2;2;2;2]	[1.0,1.0,1.0,1.1,3.0]	1671.5	8.6095	0.0537	29
active-set	[1.1;2.9;1.1;1.1;1.1]	[3.0,3.0,1.0,2.1,3.0]	1688.4	9.8012	0.0445	16
	[2;2;2;2;2]	[1.0,1.0,1.0,1.1,3.0]	1671.5	8.6093	0.0537	9

Figure 2-5 illustrates respectively the convergence process of the interior-point method and active-set methods for two different initial points= ($x1 = [1.1; 2.9; 1.1; 1.1; 1.1]$, $x2 = [2; 2; 2; 2; 2]$):

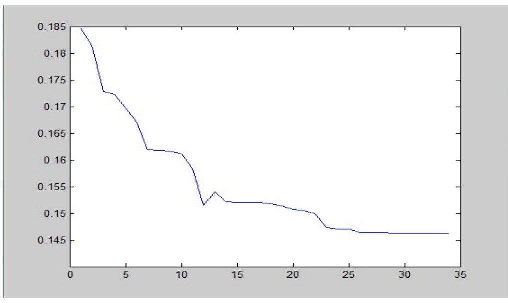

Figure 2. Interior-point Method on $x1$

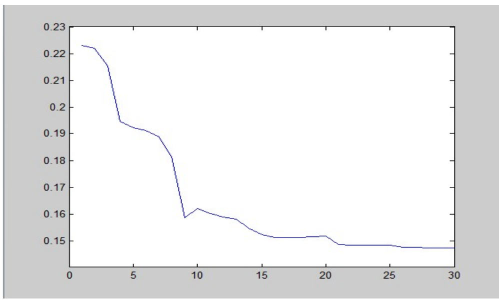

Figure 3. Interior-point Method on $x2$

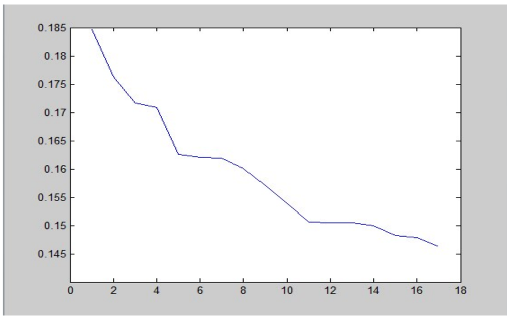

Figure 4. active-set method on $x1$

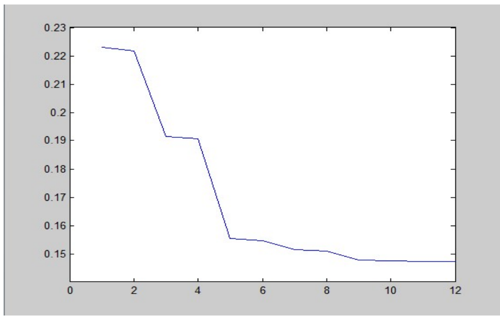

Figure 5. active-set method on $x2$

5. Conclusion

This paper establishes the multi-objective nonlinear programming model for vehicle crashworthiness, uses the thickness of five reinforcement of the established vehicle model as design variables, vehicle mass, integral value of B-pillar acceleration of 0.04-0.08 seconds of 100% head-on collision and pedals intrusion of 40% head-on offset collision are objective function. Improved α -method is used to determine the weighting coefficients of the three target weighting, thus converts the multi-objective problem into a single objective problem.

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