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Nontrivial Stable Control Economic Model of Sports Facilities

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Abstract

Given that multiple regions have their own development speeds of supply and demand for public sports facilities, a dynamic supply and demand control model of multiple regional public sports facilities is built in this paper. With the range of residents' income increases and the public sports facilities supply investment as the control variables, we established a unified construction controlling means by using the Linear Matrix Inequality approach derived from Lyapunov stability theory. As such, this means guarantees that the systems of public sports facilities in multiple regions with various developments are asymptotically stable; meanwhile, it can lower the difficulty of the control.

Keywords*: Lyapunov stability theory, linear matrix inequality, demand control model*

1. Introduction

A new conception of ten-minute sports and fitness circle has been put forward by the Chinese government since 2011. Thereafter, the concept of public sport service is winning more and more support in China, which highlights the importance of public sports facilities construction. Local governments are also actively promoting regional public sports facilities construction. However, with the growth of population and residents' income in China, what can we do to realize the balance of supply and demand for regional public sports facilities in multiple regions? And what can we do to coordinate such stable balance with the residents' income and the public sports facilities investment? These issues are worthy of both government and scholars study.

The existing studies have made a lot of achievements on the relations between public sports facilities and the economy.

Siegfried J, et al [1] think the boom in stadium and arena construction, along with the rise in public support for such facilities, generates many economic questions, such as why public subsidies arise, why the subsidies take the form of new facilities instead of payment transfer, and whether the public investments financially are attractive. Ahlfeldt G M, et al [2] develop a hedonic price model explaining standard land values in Berlin. The model assesses the impact of three multifunctional sports arenas situated in Berlin-Prenzlauer Berg. Results show that sports arenas have significant positive impacts within a radius of about 3,000m. Coates D, et al [3] investigate the relationship between professional sports franchises and venues and real per capita personal income in 37 standard metropolitan statistical regions in the United States over the period 1969-1994.Johnson A T, et al [4] think one cannot evaluate sports facilities by focusing on economic data only. They assess the value of sports facilities from the importance of noneconomic factors based on a tennis facility building with public funds in 1990 in New Haven, Connecticut, the United States. The article explores the economic impact of the facility, discusses the shortcomings of economic impact studies, and defines appropriate criteria for evaluating the community value of sports facilities.

To the investment issues and the explore countermeasures for the mass sports facilities in Xinjiang region, China, Zhao W, et al [5] analyze them from the viewpoints of policy making and industrial internal factors analysis. Geng X L, et al [6] analyze the social benefits of

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investment of National Fitness Project in small town of southern Jiangsu, China. HE Z J [7] thinks the key factors resulting in scarce sports resources of resident community are cost-sharing fuzzy, national burden overweight and individual sharing insufficient, so he suggests expanding multiple financing channels, establishing scientific investment management system and strengthening the supervision and management. From the perspective of architectural planning and implementation, Zheng H H [8] explores the construction strategy of sports facilities in China's urban communities. Xu H L, et al [9] think that the main reason affecting residents to participate in physical exercise is the contradiction between the rising population and the lack of sports facilities. However, the government's policy support, investment return and multi-channel funding preparation will lead to a steady rise in investment in sports facilities.

Generally, the existing studies have the features of static analysis on the relation between public sports facilities and the economy, yet the dynamic economic analysis and dynamic economic control model research have been still comparatively rare. The dynamic economic analysis, especially the dynamic economic control method, however, for the long-term effective application of public sports facilities, plays an important role as it does in others fields [10-12]. For this reason, taking the range of residents' income increases and the public sports facilities supply investment as the control variables, we construct a dynamic economic control model of multiple regions public sports facilities in this paper. The prominent advantage of this approach is that it can achieve stabilizing control of the public sports facilities supply and demand in multiple regions; meanwhile, it can lower the difficulty of the control.

2. Model

With respect to demand for public sports facilities in m different development level regions, the state equation of the demand function $D(t)$ of the region i at the time t can be expressed as:

$$\dot{D}(t) = \alpha_i D(t) + \beta_i R(t), i = 1, \cdots, m \quad (1)$$

Where $R(t)$ is the range of residents' income increases, α_i, β_i are the parameters of the model and $\dot{D}(t)$ is the change rate of the demand of the local residents for public sports facilities. $\dot{D}(t)$ is influenced by the current value of $D(t)$, and it can be regulated by the range of residents' income increases $R(t)$. The different values of α_i, β_i have different effects on $\dot{D}(t)$, and they can make $D(t)$ show a form of life curve model [13].

Firstly, for the rapidly urbanizing region with high-speed economic and social development, the demand for residents' public sports facilities is growing increasingly and strong, therefore the value of α_i is relatively large.

For the economically and socially developed region that has completed urbanization, the demand for residents' public sports facilities has been largely satisfied, thus the value of α_i is relatively small.

For the low urbanization rate region with relatively slow economic and social development, the demand for residents' public sports facilities is also growing relatively slow, therefore the value of α_i is smaller than the above situations.

Secondly, under the condition that the ranges of residents' income increases in different regions are the same, in general, the residents of the economically and socially developed regions generally are more willing to spend in meeting their demands for sports services than the residents of the less developed regions. This will cause the public sports facilities demand grows faster in the regions and the value of β_i can be given a larger value. So the range of residents' income increases $R(t)$ can be a control variable, with different values of $R(t)$, it can regulate the demand for public sports facilities $D(t)$.

In the aspect of public sports facilities supply, the production function of public sports facilities of the region i at the time t can be expressed as:

$$S(t) = \theta_i k(t), i = 1, \cdots, m \tag{2}$$

Where $k(t)$ is the capital stock of the public sports facilities supply organization, θ_i is the output capital ratio.

In the light of the economic theory [14], capital is an important factor of the public sports facilities production. Generally speaking, the output capital ratio of the developed regions is more than that of the underdeveloped regions.

In the light of the investment theory [15], the value of the public sports facilities supply investment, the depreciation of the capital stock and the change of the capital stock can be described as the following dynamic state equation.

$$\dot{k}(t) = I(t) - \delta_i k(t), i = 1, \cdots, m \tag{3}$$

Where $I(t)$ is the public sports facilities supply investment, δ_i is the depreciation rate of the capital stock.

Calculating the derivative of formula (2) on the both sides, and substituting into formula (3), there is:

$$\dot{S}(t) = \theta_i \dot{k}(t) = \theta_i \left(I(t) - \delta_i k(t) \right) = \theta_i I(t) - \delta_i S(t), i = 1, \cdots, m \tag{4}$$

Formula (4) is the state equation of the production function of public sports facilities of the region i.

Associating formula (1) and formula (4), and representing them as a matrix form, the supply and demand model of public sports facilities of the region i can be described as:

$$\begin{pmatrix} \dot{D}(t) \\ \dot{S}(t) \end{pmatrix} = \begin{pmatrix} \alpha_i & 0 \\ 0 & -\delta_i \end{pmatrix} \begin{pmatrix} D(t) \\ S(t) \end{pmatrix} + \begin{pmatrix} \beta_i & 0 \\ 0 & \theta_i \end{pmatrix} \begin{pmatrix} R(t) \\ I(t) \end{pmatrix}, i = 1, \cdots, m \tag{5}$$

The control variables of the model are the range of residents' income increases $R(t)$ and the public sports facilities supply investment $I(t)$. Through their dynamic values, the trajectory of the change of the system state can be controlled, the goal of the public sports facilities supply and demand balance can be achieved, and the system finally can reach a stable state.

The following work is to seek for a unified controlling means for $R(t)$ and $I(t)$. By implementing this controlling means, the supply and demand system of the public sports facilities of m different development level regions can realize asymptotically stable.

The advantage of the controlling means is that a unified controlling means can be applied to a number of regions, thus the cost and the complexity of the control process will be greatly reduced.

3. Controlling Means

The above-mentioned problem can be solved by using the linear matrix inequalities approach based on Lyapunov stability theory [16].

Formula (5) can be described as:

$$\dot{x} = A_i x + B_i u, i = 1, \cdots, m \tag{6}$$

Where $x = \left(D(t) \quad S(t)\right)^T$ is the two-dimensional state vector of the system, $u = \left(R(t) \quad I(t)\right)^T$ is the two-dimensional control input, and $A_i = \begin{pmatrix} \alpha_i & 0 \\ 0 & -\delta_i \end{pmatrix}, B_i = \begin{pmatrix} \beta_i & 0 \\ 0 & \theta_i \end{pmatrix}, i = 1, \cdots, m$ are the two-dimensional known constant matrices.

The state feedback controller is designed as:

$$u = Kx \tag{7}$$

Where K is the 2×2 dimensional state feedback gain matrix.

The derived closed-loop system is:

$$\dot{x} = \left(A_i + B_i K\right) x, i = 1, \cdots, m \tag{8}$$

In the light of the stability theorem of the linear time invariant system [17], the sufficient and necessary condition of the asymptotic stability of the closed-loop system (8) is the existence of a symmetric positive definite matrix P, which makes the following formula be true:

$$\left(A_i + B_i K\right)^T P + P\left(A_i + B_i K\right) < 0, i = 1, \cdots, m \tag{9}$$

Therefore, the stabilization controller design problem comes down to the following problem: to find a matrix K and a symmetric positive definite matrix P, which can make the matrix inequality (9) be true. That is, to solve the problem of matrix inequality group (9) with the matrices K and P as variables.

To solve this problem, first the matrix inequality group (9) can be described as:

$$PA_i + A_i^T P + K^T B_i^T P + PB_i K < 0, i = 1, \cdots, m \tag{10}$$

As the matrix P^{-1} is symmetrical, formula (10) multiplied by matrix P^{-1} on both sides of the left and right respectively, there is:

$$A_i P^{-1} + P^{-1} A_i^T + \left(P^{-1} K^T\right) B_i^T + B_i \left(KP^{-1}\right) < 0, i = 1, \cdots, m \tag{11}$$

Let,

$$X = P^{-1}, Y = KP^{-1} \tag{12}$$

Substituting formula (12) into formula (11), there is:

$$A_i X + XA_i^T + Y^T B_i^T + B_i Y < 0, i = 1, \cdots, m \tag{13}$$

For the matrix variables X and Y, the inequality group (13) is a linear matrix inequality group. As the positive definiteness of the matrix P is equivalent to that of the matrix X, and therefore if the following linear matrix inequality system.

$$\begin{cases} A_i X + XA_i^T + Y^T B_i^T + B_i Y < 0, i = 1, \cdots, m \\ X > 0 \end{cases} \tag{14}$$

It has a feasible solution, the system (6) has the stabilizing controller (7).

Furthermore, if X and Y are the feasible solution of linear matrix inequality system (14), then from formula (12),

$$K = YX^{-1}, P = X^{-1} \tag{15}$$

Where K is the stabilizing state feedback gain matrix of the system (6), P is the Lyapunov matrix of the corresponding closed-loop system.

4. Example

The following example will illustrate the application process of the above controlling means.

There are three different regions A, B and C. Their economic and social development levels are respectively high, medium and low. All of them need to construct public sports facilities to meet the local residents' demand. The public sports facilities supply and demand system of the three regions can be described by the above dynamic model.

As both urban and rural residents income elasticity of household equipment and services have all been greater than 1 since 1994 in China [18], and the output capital ratio and economic and social development levels have positive relationship [19], we can set the system parameters as follows:

A: $\alpha = 0.05, \beta = 0.09, \theta = 0.8, \delta = 0.05$.

B: $\alpha = 0.08, \beta = 0.05, \theta = 0.7, \delta = 0.04$.

C: $\alpha = 0.02, \beta = 0.01, \theta = 0.6, \delta = 0.03$.

We seek to a unified controlling means for the range of residents' income increases $R(t)$ and the public sports facilities supply investment $I(t)$. With the unified controlling means, all of the three regions with different development levels can achieve the public sports facilities supply and demand system asymptotically stable. Meanwhile, the difficulty of the control process can be lower.

Next we use the above controlling means to solve the problem.

Firstly, we should solve a feasible solution of the linear matrix inequalities (14). Here is:

$$m = 3,$$
$$A_1 = \begin{pmatrix} 0.05 & 0 \\ 0 & -0.05 \end{pmatrix}, B_1 = \begin{pmatrix} 0.09 & 0 \\ 0 & 0.8 \end{pmatrix},$$
$$A_2 = \begin{pmatrix} 0.08 & 0 \\ 0 & -0.04 \end{pmatrix}, B_2 = \begin{pmatrix} 0.05 & 0 \\ 0 & 0.7 \end{pmatrix},$$
$$A_3 = \begin{pmatrix} 0.02 & 0 \\ 0 & -0.03 \end{pmatrix}, B_3 = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.6 \end{pmatrix}.$$

Secondly, substituting the above parameters into formula (14), using the Matlab software to calculate, we obtain a feasible solution of formula (14) as follows:

$$X = \begin{pmatrix} 1.1409 & 0 \\ 0 & 1.2163 \end{pmatrix}, Y = \begin{pmatrix} -9.4747 & 0 \\ 0 & -0.7891 \end{pmatrix}$$

Thirdly, substituting the value of X, Y into formula (15), we get:

$$K = \begin{pmatrix} -8.3050 & 0 \\ 0 & -0.6488 \end{pmatrix}, P = \begin{pmatrix} 0.8765 & 0 \\ 0 & 0.8222 \end{pmatrix}$$

Where K is the stabilizing state feedback gain matrix of the problem, P is the Lyapunov matrix of the corresponding closed-loop system.

Finally, substituting the value of K into formula (7), we get:

$$\begin{pmatrix} R(t) \\ I(t) \end{pmatrix} = \begin{pmatrix} -8.3050 & 0 \\ 0 & -0.6488 \end{pmatrix} \begin{pmatrix} D(t) \\ S(t) \end{pmatrix}$$

So far, we have got a unified controlling means for the range of residents' income increases $R(t)$ and the public sports facilities supply investment $I(t)$. It is a negative feedback controlling means. When the state variable value is lower than the value of the steady state, the control variables will increase its value. And when the state variable value is higher than the value of the steady state, the control variables will decrease its value.

Eventually this controlling means can achieve system control target, it can make all of the three regions A, B and C with different development levels achieve the public sports facilities supply and demand system asymptotically stable. In addition, because the same controlling means is used in all of the regions, it makes the difficulty of control to be also decreased. At present, the dynamic economic control method applying to public sports facilities has been still comparatively rare.

5. Conclusion

For multiple regions with different speed and extent of economic and social development, the changing rate of the public sports facilities demand and supply is also different accordingly. However, the public sports facilities supply and demand system for all the regions is required to be asymptotically stable. For this purpose, we take the range of residents' income increases and the public sports facilities supply investment as the control variables, use a unified controlling means to solve the problem. The controlling means can make the system asymptotically stable. Meanwhile, the difficulty of control can also be decreased by this means.

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