Networked Control Over Noisy Channel With Time Delays

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Abstract

This paper investigates networked control problems for linear control systems, where the sensors and the controller are connected by a bandwidth-limited, noisy digital communication channel with time delays. For this case, we examine the role communication has on quantized feedback stabilization problems, present quantization, coding, and control schemes, and derive a sufficient condition on the data rate of stabilization of the unstable plant. It is shown in our results that, communication constraints have important effects on stabilization of networked control systems. An illustrative example is given to demonstrate the effectiveness of the proposed method.

Keywords: *networked control systems, time delays, communication constraints, quantization, feedback stabilization*

1. Introduction

Networked control systems have attracted great interests in recent years, which arise in advanced manufacturing, industrial automation, national defence, and intelligent transportation [1-3]. Networked control systems have many advantages, such as easy installation, low cost, reduced weight and power requirements, and high reliability. However, communication constraints also bring up many challenges.

There is a vast amount of research on networked control problems for linear control systems under communication constraints. The intuitively appealing result was proved [4-7], indicating that it quantifies a fundamental relationship between unstable physical systems and the rate at which information must be processed in order to stably control them. This result was generalized to different notions of stabilization and system models, and was also extended to multi-dimensional systems [8-10]. The research on Gaussian linear systems was addressed in [11-13].

Liu and Yang investigated quantized control problems for linear time-invariant systems over a noiseless communication network [14]. You and Xie investigated the minimum data rate for mean square stabilization of linear systems over a lossy digital channel, where the packet dropout process is modeled as a Markov chain [15]. Furthermore, Liu addressed coordinated motion control of autonomous and semiautonomous mobile agents in [16], and derived the condition on stabilization of unmanned air vehicles over wireless communication channels in [17]. The survey papers [18] gave a historical and technical account of the various formulations.

In this paper, we examine a class of networked control problems which arise in the coordinated motion control of autonomous mobile agents, such as unmanned air vehicles (UAVs), unmanned ground vehicles (UGVs), and unmanned underwater vehicles (UUVs). The case with packet dropout, time delay, and bandwidth limitations is considered at the same time. The quantization, coding, and control schemes are designed to stabilize the system in the mean square

sense. Our work here differs in that we present the tradeoffs among the data rate, time delays, the dropout probability, and stabilization.

The remainder of this paper is organized as follows: Section 2 introduces problem formulation. Section 3 deals with networked control schemes. Section 4 presents a sufficient condition on data rates for stabilization of networked control systems. The results of numerical simulation are presented in Section 5. Conclusions are stated in Section 6.

2. Problem Formulation

Consider the following stochastic linear time-invariant control system

$$X(k+1) = AX(k) + BU(k) + FW(k) \tag{1}$$

where $X(k) \in \mathbb{R}^n$ denotes the plant state, $U(k) \in \mathbb{R}^m$ denotes the control input, and $W(k) \in \mathbb{R}^d$ denotes the disturbance satisfying $E\|W(k)\|^2 < \phi_W < \infty$, respectively. The system is depicted in Fig.1. Here, A, B, and F are known constant matrices with appropriate dimensions. The initial state $X(0)$ is assumed to be a random variable, satisfying $E\|X(0)\|^2 < \phi_0 < \infty$. Without loss of generality, we assume that (A, B) is a controllable pair, and the system is also fully observable.
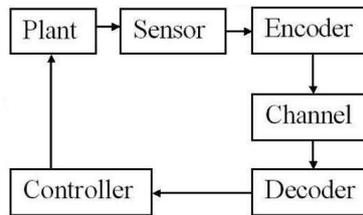

Figure 1. Networked control systems.

In this paper, we consider the case where the sensors and the controllers are geographically separated and connected by bandwidth-limited, noisy digital communication channel with time delays. The channel is also assumed to be memoryless. Let d denote the channel propagation delay. As in [15], the packet dropout process of the channel is modeled as a time-homogeneous Markov process. Here, we set $\gamma_k = 1$ when the packet has been successfully delivered to the decoder, and set $\gamma_k = 0$ when the packet is lost. The Markov chain has a transition probability matrix defined by

$$(P(\gamma_{k+1} = j|\gamma_k = i))_{i,j \in s} = \begin{bmatrix} 1-q & q \\ p & 1-p \end{bmatrix} \tag{2}$$

where $s := \{0, 1\}$ is the state space of the Markov chain (see [15]).

Clearly, the information of the plant states needs to be quantization, coded, and sent to the decoder. Then, for the case with the limited data rate, time delays, and packet dropout, our main task here is to design the quantization, coding, and control schemes to stabilize the system (1) in the mean square sense

$$\limsup_{k \to \infty} E\|X(k)\|^2 < \infty \tag{3}$$

employing the finite data rate R (bits/sample) provided by the packet-dropout channel with the time delay d. In addition, a key problem is to give a sufficient condition on the data rate for stabilization of the unstable plant.

3. Networked Control Under Communication Constraints

In networked control systems, communication constraints, such as packet dropout, time delays, and data-rate limitations, often degrade the control performances and destabilize the control system. Thus, we present the quantization, coding, and control schemes to guarantee stabilization of the system (1) in this section.

As in [15], let $\{T_k\}_{k\geq 0}$ denote the stochastic time sequence given by

$$T_n := \inf\{k : k \geq T_{n-1}, \gamma_k = 1\} + 1$$

where $T_0 = 1$ and $\gamma_0 = 1$. Then, the time duration $\bar{T}_n$ is defined by

$$\bar{T}_n := T_n - T_{n-1}$$

where $\bar{T}_n$ denotes the time duration between two successive packet received times (see [15]).

Let $\hat{X}(k)$ and $\grave{X}(k)$ denote the estimate value and the predictive value of the plant state $X(k)$, respectively. Here, we employ a predictive control law of the form

$$U(k) = K\breve{X}(k) \tag{4}$$

where

$$\breve{X}(k) := \begin{cases} A^d\hat{X}(k-d) + \sum_{l=0}^{d-1} A^{d-l-1}BU(k-d+l), & \text{if } \gamma_k = 1 \\ A^d\grave{X}(k-d) + \sum_{l=0}^{d-1} A^{d-l-1}BU(k-d+l), & \text{if } \gamma_k = 0 \end{cases}$$

where we define

$$\grave{X}(k-d) = (A+BK)\breve{X}(k-d-1).$$

Let λ_i denote the ith eigenvalue of A $(i = 1, \cdots, n)$. Then, there exists the unitary matrix H that diagonalizes $A = H'\Lambda H$ where $\Lambda =$ diag$[\lambda_1, \cdots, \lambda_n]$, if system matrix A has only real eigenvalues each with geometric multiplicity one. On the contrary, we have $\Lambda =$ diag$[J_1, \cdots, J_m]$ where each $J_i(i = 1, \cdots, m)$ is a Jordan block. It is derived in [11] that there are the same results for the two cases above. Here, we consider the first case in order to avoid extraneous complexity. Then, we define

$$\bar{X}(k) = HX(k),$$
$$\tilde{X}(k) = H\hat{X}(k),$$
$$\vec{X}(k) = H\grave{X}(k).$$

We define the predictive error of the plant states by

$$Z(k) := \bar{X}(k) - \vec{X}(k).$$

Here, we may quantize, encode $Z(k)$, and sent the information of $Z(k)$ to the decoder in the channel output. Let $\hat{Z}(k)$ and $V(k)$ denote the quantization value and the quantization error of $Z(k)$, respectively. Similar to that in [19], we present the quantization scheme. Given a positive integer M_i and a nonnegative real number $\Delta_i(k)$ $(i = 1, \cdots, n)$, define the quantizer $q : \mathbb{R} \to \mathbb{Z}$ with sensitivity $\Delta_i(k)$ and saturation value M_i by the formula

$$q(\bar{z}_i(k)) = \begin{cases} M^+, & \text{if } \bar{z}_i(k) > (M_i + 1/2)\Delta_i(k), \\ M^-, & \text{if } \bar{z}_i(k) \leq -(M_i + 1/2)\Delta_i(k), \\ \lfloor \frac{\bar{z}_i(k)}{\Delta_i(k)} + \frac{1}{2} \rfloor, & \text{if } -(M_i + 1/2)\Delta_i(k) < \bar{z}_i(k) \leq (M_i + 1/2)\Delta_i(k) \end{cases} \tag{5}$$

where define $\lfloor \bar{z} \rfloor := \max\{k \in \mathbb{Z} := k < \bar{z}, \bar{z} \in \mathbb{R}\}$. The indexes M^+ and M^- will be employed if the quantizer saturates. The scheme to be used here is based on the hypothesis that it is possible to change the sensitivity (but not the saturation value) of the quantizer on the basis of available quantized measurements. The quantizer may counteract disturbances by switching repeatedly between "zooming out" and "zooming in" (see [19]).

4. A Lower Bound of the Data Rate for Stabilization

In this section, we examine the networked control problem under communication constraints, and derive a sufficient condition on the data rate for stabilization of the unstable plant.

First, we give the following lemma following from [20]

Lemma 1: Consider an errorless digital channel. Let $z \in \mathbb{R}$ denote a Gaussian source and $\hat{z}$ denote an estimate of z. Define $R(D)$ as the information rate distortion function between $\hat{z}$ and z. The expected distortion constraint is defined as $D \in \mathbb{R}^+$. Let h denote the sampling period. Given $D \geq E(z - \hat{z})^2$, there must exist a quantization and coding scheme if the data rate R of the channel satisfies

$$R > \tfrac{1}{h} R(D) \geq \tfrac{1}{2} \log_2 \tfrac{\sigma^2(z)}{D} \, (\text{bits/sample})$$

where we define $\sigma^2(z) := E(z - Ez)^2$.

Proof: The proof is given in [20]. □

The data rate of the channel has important effects on the control performances. Here, we have the following result.

Theorem 1: Consider the system (1) with the control law (4). Assume that all eigenvalues of $A + BK$ lie inside the unit circle. The communication channel which connects the sensors and the controller is assumed to be a bandwidth-limited, noisy digital channel with the time delay d. The packet dropout process of the channel is modeled as a time-homogeneous Markov process (2). Then, the system (1) is stabilizable in the mean square sense (3) if the date rate of the channel satisfies the following inequality:

$$R > (2d + \tfrac{1}{q} + 2)\Sigma_{i \in \Theta} \log_2 |\lambda_i| \; (\text{bits/sample})$$

where $\Theta := \{i : |\lambda_i| > 1, i = 1, 2, \cdots, n\}$.

Proof: Consider the system (1)

$$\begin{aligned} X(k+1) &= AX(k) + BK\breve{X}(k) + FW(k) \\ &= A(X(k) - \breve{X}(k)) + (A + BK)\breve{X}(k) + FW(k). \end{aligned} \tag{6}$$

Notice that

$$\grave{X}(k+1) = (A + BK)\breve{X}(k),$$

and

$$\breve{X}(k) := \begin{cases} A^d \hat{X}(k-d) + \sum_{l=0}^{d-1} A^{d-l-1} BU(k-d+l), & \text{if } \gamma_k = 1 \\ A^d \grave{X}(k-d) + \sum_{l=0}^{d-1} A^{d-l-1} BU(k-d+l), & \text{if } \gamma_k = 0 \end{cases}$$

Substitute the two equalities above into (6), and obtain

$$H' Z(k+1) := \begin{cases} A^{d+1}[X(k-d) - \hat{X}(k-d)] + \sum_{l=0}^{d} A^{d-i} FW(k-d+l), & \text{when } \gamma_k = 1 \\ A^{d+1}[X(k-d) - \grave{X}(k-d)] + \sum_{l=0}^{d} A^{d-i} FW(k-d+l), & \text{when } \gamma_k = 0 \end{cases}$$

Notice that

$$\begin{aligned} V(k-d) &= H[X(k-d) - \hat{X}(k-d)], \\ Z(k-d) &= H[X(k-d) - \grave{X}(k-d)]. \end{aligned}$$

Thus, it follows that

$$Z(k+1) := \begin{cases} \Lambda^{d+1} V(k-d) + \sum_{l=0}^{d} H A^{d-i} FW(k-d+l), & \text{when } \gamma_k = 1 \\ \Lambda^{d+1} Z(k-d) + \sum_{l=0}^{d} H A^{d-i} FW(k-d+l), & \text{when } \gamma_k = 0 \end{cases}$$

This implies that

$$\begin{aligned} Z(T_n) &= \Lambda^{d+1} V(T_n - d - 1) + \sum_{l=0}^{d} H A^{d-l} FW(T_n - d - 1 + l), \\ Z(T_n - d - 1) &= \Lambda^{d + \bar{T}_n} Z(T_{n-1}) + \sum_{j=0}^{\bar{T}_n - 2} \sum_{l=0}^{d} H A^{\bar{T}_n - d - 1 + l} FW(T_n - d - 1 + l). \end{aligned}$$

Then, we get

$$\begin{aligned} tr[\textstyle\sum_{Z(T_n)}] &= tr[\Lambda^{2(d+1)} \textstyle\sum_{V(T_n - d - 1)}] + \sum_{l=0}^{d} tr[F'(A^{d-l})' A^{d-l} F \textstyle\sum_W], \\ tr[\textstyle\sum_{Z(T_n - d - 1)}] &= tr[\Lambda^{2(d + \bar{T}_n)} \textstyle\sum_{Z(T_{n-1})}] + \sum_{j=0}^{\bar{T}_n - 2} \sum_{l=0}^{d} tr[F'(A^{\bar{T}_n - d - 1 + l})' A^{\bar{T}_n - d - 1 + l} F \textstyle\sum_W]. \end{aligned}$$

It means that

$$\sigma^2(z_i(T_n)) = \lambda_i^{2(d+1)}\sigma^2(v_i(T_n - d - 1)) + \phi_i,$$
$$\sigma^2(z_i(T_n - d - 1)) = \lambda_i^{2(d+\bar{T}_n)}\sigma^2(Z(T_{n-1})) + \varphi_i$$

where we define

$$\phi_i = [\sum_{l=0}^{d} F'(A^{d-l})'A^{d-l}F\sum_W]_{ii},$$
$$\varphi_i = [\sum_{j=0}^{\bar{T}_n-2}\sum_{l=0}^{d} F'(A^{\bar{T}_n-d-1+l})'A^{\bar{T}_n-d-1+l}F\sum_W]_{ii}.$$

Here, let R_i denote the data rate corresponding to $z_i(k)$. Then, it follows from Lemma 1 that

$$R_i = \tfrac{1}{2}\log_2 \frac{\sigma^2(z_i(T_n - d - 1))}{\sigma^2(v_i(T_n - d - 1))}.$$

Namely,

$$\sigma^2(z_i(T_n - d - 1)) = \sigma^2(v_i(T_n - d - 1)).$$

Thus, we have

$$\sigma^2(z_i(T_n)) = \frac{\lambda_i^{2(\bar{T}_n+2d+1)}}{2^{2R_i}}\sigma^2(z_i(T_{n-1})) + \frac{\lambda_i^{2(d+1)}}{2^{2R_i}}\varphi_i + \phi_i.$$

Furthermore, notice that

$$E\bar{T}_n = 1 + \tfrac{1}{q}.$$

Thus, we have

$$\limsup_{k\to\infty}\sigma^2(z_i(k)) < \infty,$$

if the data rate R_i satisfies the following inequality

$$R_i > (2d + \tfrac{1}{q} + 2)\log_2|\lambda_i| \text{ (bits/sample)}$$

with $|\lambda_i| > 1$. Then, we get

$$\limsup_{k\to\infty}E\|X(k)\|^2 < \infty,$$

if the data rate R of the channel satisfies the following inequality

$$R > (2d + \tfrac{1}{q} + 2)\Sigma_{i\in\Theta}\log_2|\lambda_i| \text{ (bits/sample)}$$

where we define $\Theta := \{i : |\lambda_i| > 1, i = 1, 2, \cdots, n\}$.

Namely, the system (1) is stabilizable in the mean square sense (3) if the data rate R is larger than the lower bound above. □

5. Numerical Example

We present an example to illustrate the proposed quantization, coding, and control method. Consider a discrete-time linear control system

$$X(k+1) = \begin{bmatrix} 5.46 & 0.71 & -0.52 \\ 0.41 & -2.63 & 0.61 \\ 0.82 & 0.91 & 4.71 \end{bmatrix} X(k) + \begin{bmatrix} 0.51 \\ 5.42 \\ 3.52 \end{bmatrix} U(k) + \begin{bmatrix} 0.62 \\ 0.73 \\ 2.81 \end{bmatrix} W(k).$$

For the channel, we set the time delay $d = 4$, and set the transition probability matrix

$$P(\gamma_{k+1}|\gamma_k) = \begin{bmatrix} 0.12 & 0.88 \\ 0.89 & 0.11 \end{bmatrix}.$$

Because of full state feedback, we compute the feedback gain, and obtain $K = [2.61 \ -0.41 \ 2.64]$. Let the initial value $X(0) = [80 \ 50 \ -120]'$.

First, we send the information of the plant state to the decoder over the channel with the data rate $R = 240$ bits/s. Such data rate is less than the lower bound given in the Theorem 1. The corresponding simulation is given in Fig.2. Clearly, the system is unstable.

In order to guarantee stabilization of the system, we set the data rate $R = 420$ bits/s which is larger than the lower bound given in the Theorem 1. The corresponding simulation is given in Fig.3. Clearly, the system is stabilizable. It states that the control law (4) can ensure stabilization of the system if the data rate is large enough.
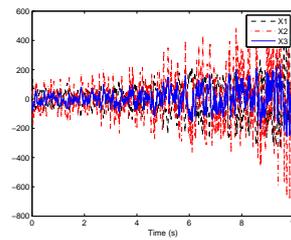
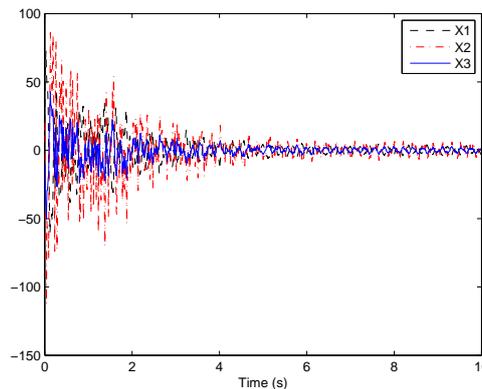Figure 2. The plant state responses when $R = 240$ bits/s.

Figure 3. The plant state responses when $R = 420$ bits/s.

6. Conclusions

In this paper, we addressed a class of networked control problem, which arise in the coordinated motion control of autonomous mobile agents, such as unmanned air vehicles (UAVs), unmanned ground vehicles (UGVs), and unmanned underwater vehicles (UUVs). In particular, we considered the case with a bandwidth-limited, noisy digital communication channel with time delays, gave the quantization, coding, and control schemes, and derived the sufficient condition on the data rate for stabilization of the unstable plant. The simulation results have illustrated the effectiveness of the quantization, coding and control schemes.

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