

# Measures of Impact Under Cascading Failure of Power Grid

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## Abstract

*A centrality measure has been proposed considering the directionality of power flow of future smart grid. Applicability of the proposed method has been evaluated in several standard IEEE test systems. Various comparisons are shown between impacts of removal of critical nodes found from two different models: nondirectional and bidirectional. Larger impact of removing critical nodes found from bidirectional flow model shows the utility of the proposed method. Measures of impacts considered are changes in path length, loss of connectivity and load lost during cascade.*

**Keywords:** Centrality Measure, bidirectional power flow, path length, connectivity loss, load loss.

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## 1. Introduction

Existing power transmission grids around the world are being made much more smarter by integrating smart and new technologies by utilities [1]. The scope of smart grid includes various generation options, primarily in the distribution side – near consumers. Engagement of customers with the energy management systems is the most lucrative part of smart grid from the point of view of regulating energy usage. Excess of generation after local use can be transmitted long distance to meet the energy shortage of the destination area.

This introduces a new concept of power flowing from customer end towards the grid. The bidirectional power flow changes the whole power flow pattern of the existing grid [2]. Analytical methods, technical strategies, control system and protecting devices need to be changed along with, to mention a few. Metering and protecting equipments will experience flows coming from the reverse side. Proper operation of the equipments used earlier can be ensured either by changing the instruments themselves or by incorporating new measurement techniques [3].

Recent years have seen several very large scale blackouts initiating from small disturbances [4]. In August 1996, a cascading outage occurred in the Western power grids of North America in USA and Mexico [5]. More than 4 million people suffered the consequences. Most affected areas were out of electricity for about 4 days. Another large scale blackout which affected around 55 million people happened in August 2003 [6]. Several northeast and midwestern states of USA and some provinces of Canada were affected.

From the frequent events of large scale-blackouts it is clear that the existing dynamics security assessment and monitoring system has not been working well [7]. The motivation of complex network framework based analysis approach comes from the necessity of new, alternative and improved methodologies to assess the risk involved with cascading events in power system. Degree centrality, betweenness centrality and closeness centrality measures are commonly used in social network research to find a person with most influence. [8].

Power grid topology has been analyzed by various researchers recently to explore its strength and weakness using complex network framework. The strength of the grid is found to be, from a pure topological analysis of USA power grid, small-world property [9]. This implies that various nodes within the system can be reached easily, which will make the communication that comes along with the smart grid easy and effective. The scale freeness of the topology of the grid

is shown to be a weakness of the grid since it makes the system very much vulnerable to targeted attack [10]. This targeted attack can trigger cascading failure which will lead to blackout.

The research on power grid from a system point of view has been triggered after the publications of the preliminary topology based analytical results. Since results from pure topological approach is quite misleading [11], several researchers have a mix of both topological and electrical characteristics based complex network analysis of power system to find reasonably improved results [12, 13].

Motivated by the topology based analytical results, that found the power grid robust against random failure but vulnerable to targeted attacks [10], critical node and link analysis of power grid have been carried out to explore the criticality of the power grid. If critical components can be spotted out which can initiate cascading effect, special preventive actions could be exercised to prevent large scale blackouts from happening.

Network efficiency, a topological measure of performance change after the inclusion or removal of nodes or lines from a grid, is analyzed in [14]. A weighted line betweenness based approach is utilized to find out critical lines responsible for spreading of large scale blackouts from small initial shock [15]. Vulnerable regions of power system are identified employing complex network theory based qualitative simulation in [16]. Transmission line reactance is incorporated to compute a new vulnerability index to identify critical lines [17].

A link is explored between power system reliability and small world effect [18]. Maximum flow based centrality approach is used to find out critical lines which remove the shortcoming of the assumption of power flowing through the shortest paths between source and load nodes [19]. The flow based method has slow convergence but can be useful when used in conjunction of planning issues. A DC power flow model is used and hidden failure of protective equipment is considered to model the structural vulnerability of power grid [20]. Electrical parameters are incorporated extensively to improve the centrality indices for power system [21].

An extended topological approach proposed in [22] takes into consideration traditional topological metrics as well as operational behavior of power grids like real power flow allocation and line flow limits. Power Transfer Distribution Factor (PTDF) is used to simulate cascading events in an attempt to identify correlated lines [23].

All these analysis are carried out mainly on nondirectional models where the direction of power flow has not been considered. But since with the inclusion of distributed generations the power flow pattern is going to change, new methodologies have to be proposed which take into account bidirectional power flow. Since communication is an important factor in smart grid, identifying those nodes in the system would be very much useful. These issues have not yet been addressed in literature as per authors best knowledge. Bidirectional power flow based centrality measure has been proposed in [24]. A closeness based centrality measure considering bidirectional power flow have been analyzed in [25] while [24] focuses on betweenness based variant.

In this paper, a comparison of the bidirectional flow based method has been made with nondirectional power flow based method. This method is a modification of closeness centrality which takes into account power flow distribution among various transmission lines during steady state. This is a reasonable extension of previous work carried out by researchers since it captures the power flow in smart grid environment. The impact of removing critical components are identified using well known impact metrics like path length, connectivity loss and load loss.

The organization of the rest of the paper is as follows. Section 2. provides a model for the analysis of smart power grid under complex network framework. A new model based on bidirectional power flow is considered and a method is discussed to find critical nodes in the power grid. The effect of removal of critical nodes on various topological and electrical measures are addressed in Section 3.. Conclusion is drawn and future research direction is provided in Section 4..

## 2. System Model and Methodology

The first step of analyzing power grid under complex network framework is to model the system as a directed graph [7]. Vertices in the graph represent generating stations, substations, loads etc. Edges of links represent transmission lines that connect various generating stations, substations and load points. In this model, only transmission system is considered. The overall distribution system is regarded as a lumped load at the distribution substation terminal.

A power system network is represented by a graph  $G = (V, E, W)$  comprising of a set  $V$ , whose elements are called vertices or nodes, a set  $E$  of ordered pairs of vertices, called edges or lines. An element  $e = (x, y)$  of the edge set  $E$ , is considered to be directed from  $x$  to  $y$ , where  $y$  is called the head and  $x$  is called the tail of the edge. The elements of  $W$  are weights of edge set elements. There exists a one-to-one correspondence between set  $E$  and set  $W$ . In this model, transmission line impedances in pu is considered as weights of the edges between nodes. Absolute value of impedance of the transmission line is taken as the weight of the edge. This weight is required to find shortest electrical path between various pairs of vertices. Johnson's algorithm is used to find shortest path set in the network [26].

Power flow analysis is conducted for the given test system during nominal condition. Newton-Raphson method is used to solve the simultaneous nonlinear algebraic power flow equations [27]. The direction of real power flowing through the lines is taken as the direction of edges in the modeled graph as shown in Fig. 1. From this point this graph will be known as nominal unidirectional flow graph. In order to consider the bidirectional flow in smart grid, a backward unidirectional flow graph is also modeled. The direction of edges in the reverse unidirectional flow graph is exactly opposite to the nominal unidirectional flow graph as shown in Fig. 2. Superposition of the two models give bidirectional model. Here we show the difference of two modeling approaches: (a) nondirectional model and (b) bidirectional model.

Assume that,  $k$  represents the intermediate bus within the shortest path originating from bus  $s$  and ends at bus  $t$ . Let  $P_{st}$  represents the maximum power flowing in the shortest electrical path between buses  $s$  and  $t$ , and  $P_{st}(k)$  is the maximum of inflow and outflow at bus  $k$  within the shortest electrical path between buses  $s$  and  $t$ . Then, let their fraction is represented by  $r_{st}(k)$  as in:

$$r_{st}(k) = \frac{P_{st}(k)}{P_{st}} \quad (1)$$

where the ratio  $r_{st}(k)$  is an index of the degree to which buses  $s$  and  $t$  need bus  $k$  to transmit power between them along the shortest electrical path. If a double sum is taken of (1) over all intermediate buses  $k$  and all destination buses  $t$  for the source buses  $s$ ,

$$C_C^E(s) = \sum_{k=1}^n \sum_{t=1}^n \frac{P_{st}(k)}{P_{st}}, s \neq t \neq k \in V \quad (2)$$

a centrality measure for bus  $s$  within the grid is obtained. This measure (2) adds up the real power of the lines originating at bus  $s$  and terminating at all other buses. This quantity takes high values if the difference between numerator and denominator term is low. This fact represents that very few amount of power is lost in the shortest path. Such buses might have more direct influence on other buses since very few amount of power is lost. Table 1 lists top ten critical nodes in IEEE 30 bus test system [27] found from nominal and backward unidirectional as well as bidirectional model.

In summary the method to identify can be summarized as:

1. Model a power system as a directed graph.
2. Calculate power flowing through various lines.
3. Construct a reverse directional graph.
4. Find the shortest path set of the graph from source nodes to load nodes.

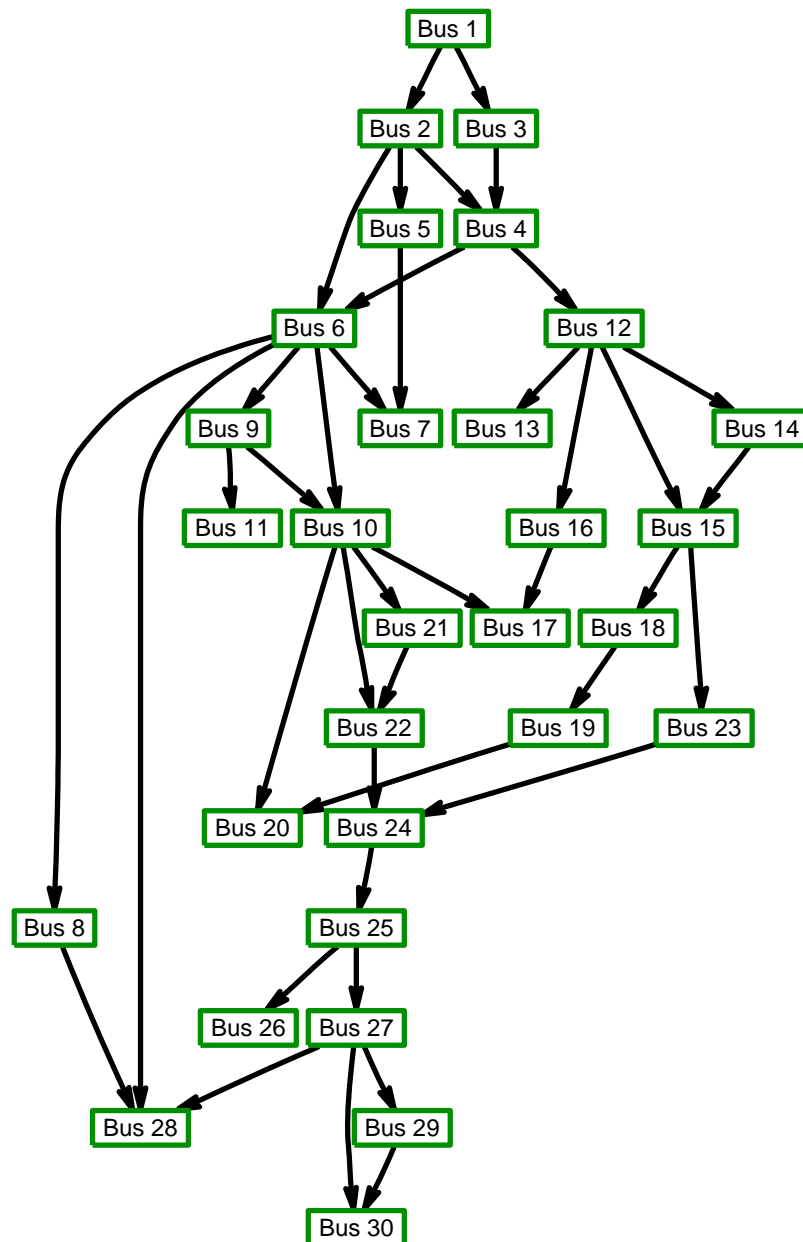


Figure 1. Nominal unidirectional flow in IEEE 30 bus test system.

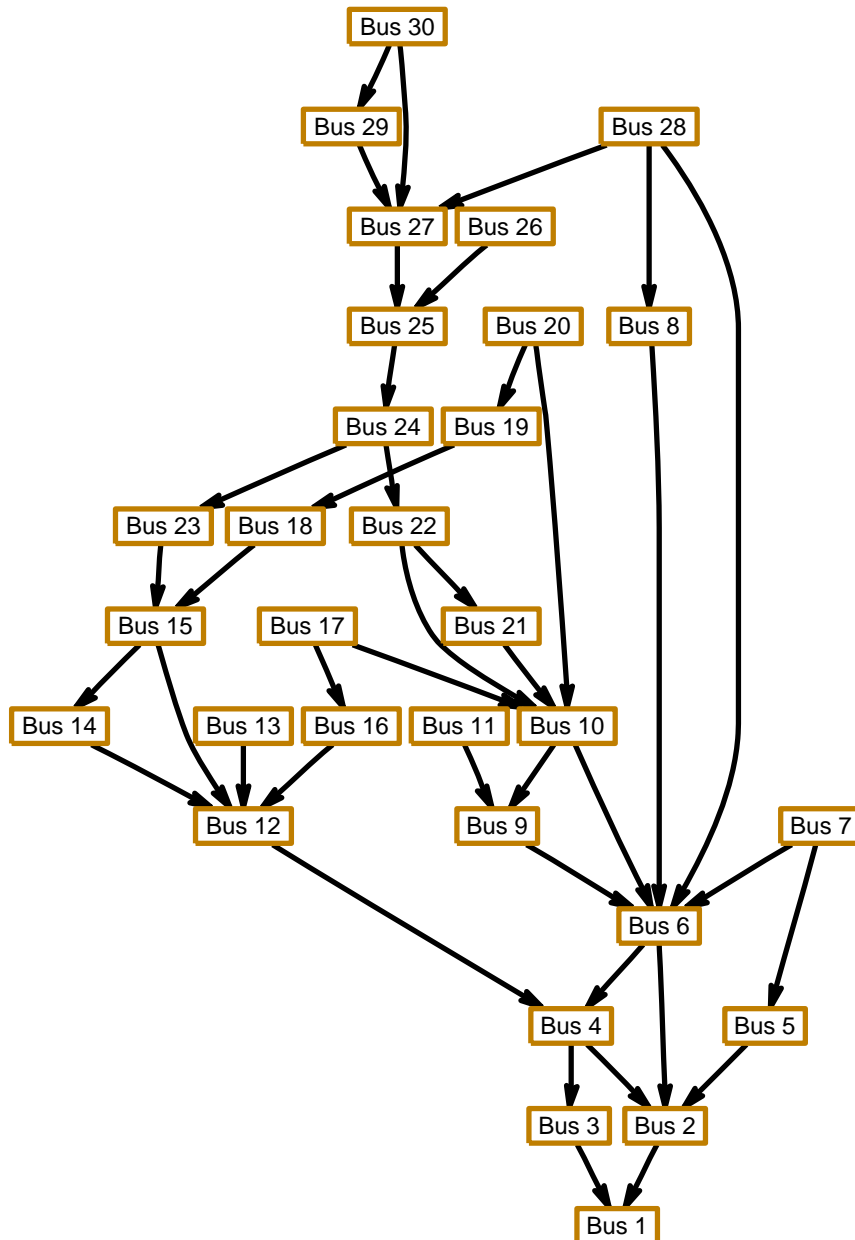


Figure 2. Reverse unidirectional flow in IEEE 30 bus test system.

Table 1. Top Ten Nodes in Nondirectional &amp; Bidirectional Power Flow Models

Nondirectional	Bidirectional
1	5
3	8
2	7
4	6
6	4
13	24
12	19
9	13
14	12
28	14

5. Find  $r_{st}(k)$  and calculate  $C_C^E(k)$ .
6. Sort and rank in the descending value of  $C_C^E(k)$ .

### 3. Measure of Impact

At first, the nominal network is solved and nodes are removed from the system one by one in the descending order of centrality measure as shown in Table 1. In order to measure the impact of removing critical nodes from the system various measures are being used. The criteria are to find out whether there is a cascading failure initiated with removal of nodes. If there is a cascading failure upon the removal of nodes, they are considered critical, otherwise not. And, the measure of impacts quantify the criticality. In this paper, four measures are considered. The first two of them, path length and connectivity loss are purely topological. The last measure is percentage of load lost due to the removal of critical nodes.

#### 3.1. Path Length

The path length is used by researchers as a measure of network connectedness. It is the average length of the shortest paths between any two nodes in the network [28]. It is found that if a node is removed from a system, it generally increases the distance between other nodes. So, the increase in network characteristic path length is considered as a measure of impact analysis of removing critical nodes from the system.

Distance between two vertices can be computed as:

$$d(u, v) = \min|P| \quad (3)$$

where  $P$  is a path from  $u$  to  $v$ . Path length can be defined as:

$$\bar{d} = \frac{1}{k} \sum_{u \neq v \in V} d(u, v) \quad (4)$$

where  $0 \leq d(u, v) \leq \infty$ .  $k$  is the number of connected pairs.

This is topological path length. Another electrical path length is also measured where the distance is computed in terms of impedance of transmission lines. A simple IEEE 57 bus test system is used to simulate the consequence of node removal on path length and the result is depicted in Fig. 3. It is clear that the impact of removing critical nodes based on bidirectional flow rather than bidirectional flow model is comparatively higher. Initially the impact is higher in nondirectional measure but after four nodes removal the bidirectional model shows impact in large scale. Electrical path length based measure shows similar characteristic. In the later measure the impact is always higher in bidirectional flow model.

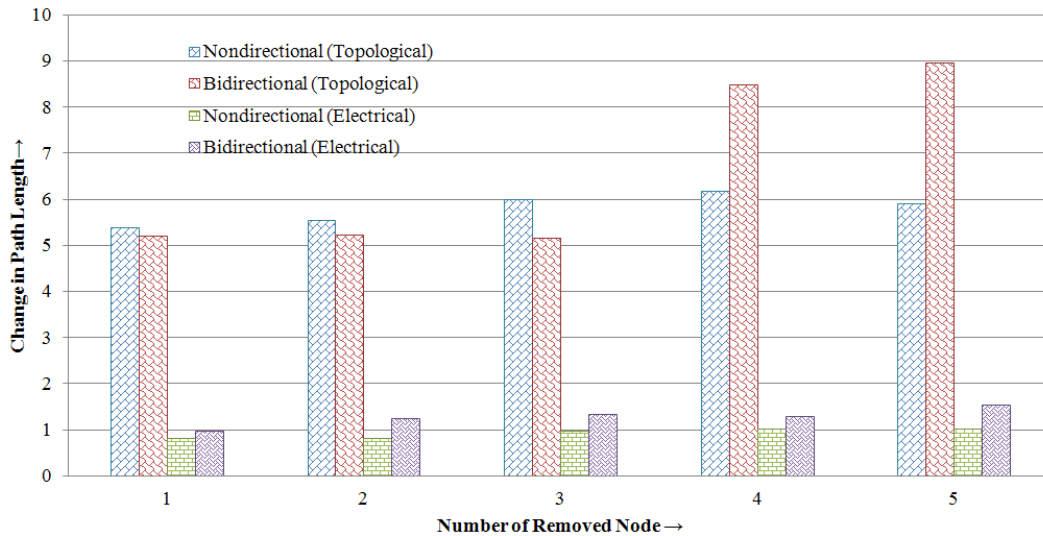


Figure 3. Change in path length in IEEE 57 bus test system for removal of critical nodes based on two different measures.

### 3.2. Connectivity Loss

This is a purely topological measure of impact a power grid encounters when some nodes are removed from the system. In this measure we calculate how much connectivity is lost in terms of how many generators a transmission or distribution node can access due to effect of removing a node from the system. The less is the number of generators a node is connected with, the less is the redundancy and the more is the vulnerability of the node. It is given as (5) originally proposed in [29].

$$C = 1 - \left\langle \frac{N_g^i}{N_g} \right\rangle_i \quad (5)$$

where the averaging is done over each intermediate nodes, i.e., substations.  $N_g$  is the total number of generators and  $N_g^i$  is the number of generators that a node  $i$  can reach. Impact on connectivity loss for two different models are presented in Fig. 4 for IEEE 157 bus test system.

It is found that connectivity is lost to a great extent in both cases, although the effect is higher in case of bidirectional flow model.

In case of bidirectional flow model almost 50% connectivity is lost after six nodes removal while if we remove nodes according to nondirectional model even after 10 nodes removal the connectivity is very high. It takes 17 nodes removal according to nondirectional model to decrease the connectivity loss to 50%.

### 3.3. Load Loss

Last measure of impact is found from a simple model of cascading failure that is presented here. Since it is not possible to exactly model the blackout, various approximate measures have been taken by several researchers to mimic the situation [30, 31, 32, 11].

Power system is a very much complex interconnected system whose exact modeling would require consideration of dynamics of rotating machines and devices within the system, discrete dynamics of switchgear elements, non-linear algebraic equations that govern line flows and social dynamics of governing and operating bodies.

In this paper, a fairly simple model of cascading failure of the power grid is proposed by incorporating important electrical features ignoring those which are too complicated but have little effects. The detail of the model is described here.

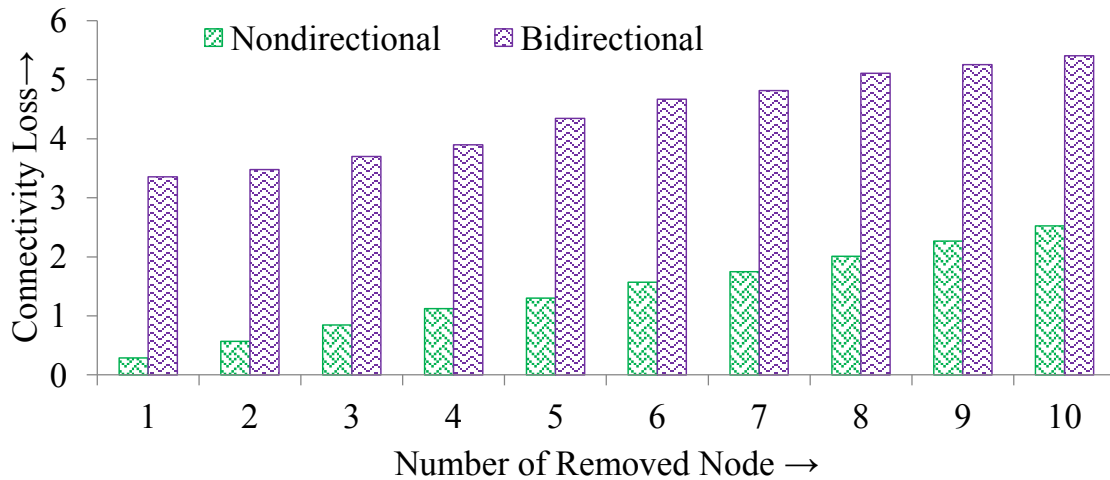


Figure 4. Connectivity loss of IEEE 118 bus test system as a function of removal of critical nodes from two different point of views.

At first AC power flow is used to calculate the steady state condition of the network. Real and reactive power of transmission lines are found from numerical solution of line flow equations given in (6) and (7)

$$P_i = \sum_{j=1}^n |V_i||V_j||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \quad (6)$$

$$Q_i = - \sum_{j=1}^n |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j) \quad (7)$$

where the symbols have their usual meanings as found in power system literature.

During the analysis, generator and load dynamics are not included. Although the limitation of not using dynamics of generators and loads are well understood but it is at least useful for modeling one mechanism of cascading failure that is cascading overload. Also, Generation Shift Factors (GSF) and Line Outage Distribution Factors (LODF) [33] are used to recalculate flows in lines after disturbance. This helps achieving fast results without using actual load flow after each disturbance. The speed and accuracy of the result and comparison with actual load flow is out of the scope of this paper and will be addressed in another research article in future.

The transmission lines are removed if overloaded. Also, time delayed over current relays are used in every line so if there is a lot of overload it trips fast and if there is a little bit of overload it trips slowly. Another thing that is added to the model is ramping up of generators. As the system separates into sub grids, generators are allowed to ramp up or ramp down to rebalance a little bit.

So, if a component failure disturbs the supply-demand balance, through generator set-point adjustment this balance is achieved. But if there is not enough ramping ability, then the ultimate choice is to trip lowest possible system load. The total amount of load lost during the successive removal of nodes is used as a measure of impact.

Fig. 6 shows load loss as a percentage of total system load for IEEE 300 bus test [34] system as shown in Fig. 5. Up to six node removal the load loss is nearly equal and does not increase much for both unidirectional models. After five node removal, more than 50% load of the system need to be shedded to ensure secure and reliable operation of the remaining system.

The overall steps are summarized below while the flowchart in Fig. 7 represents the same.

**Step 1** AC power flow is solved to find out the steady-state condition of the network.

**Step 2** Find out if the power flow is converged or not.



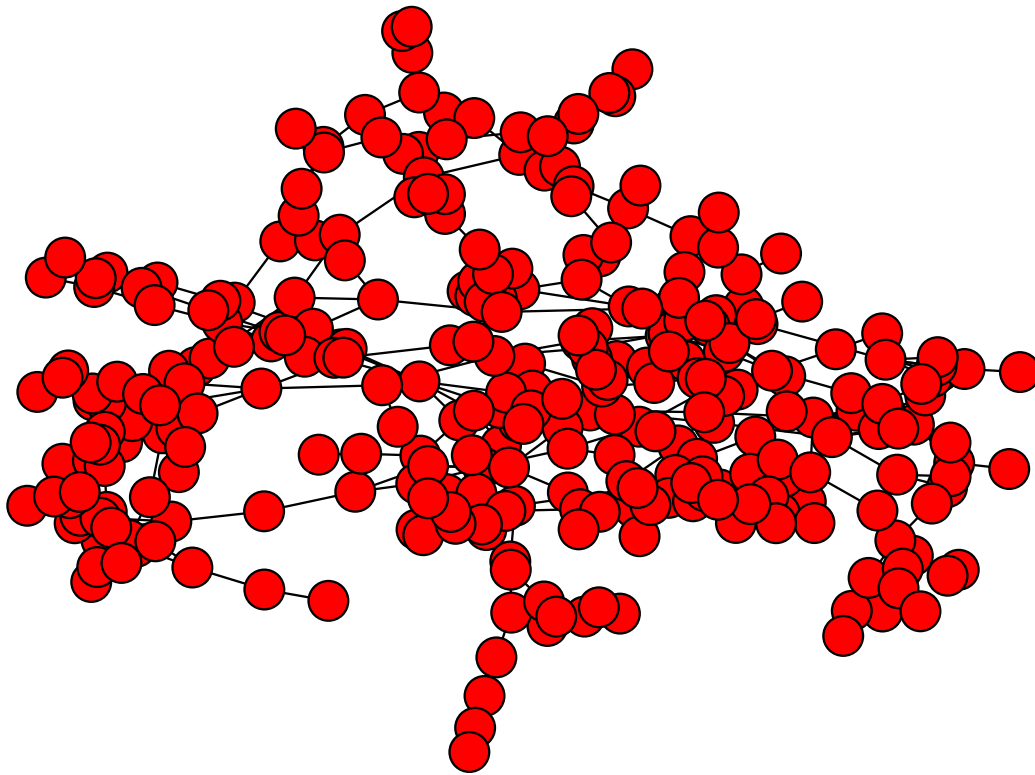


Figure 5. Topology of IEEE 300 bus test system.

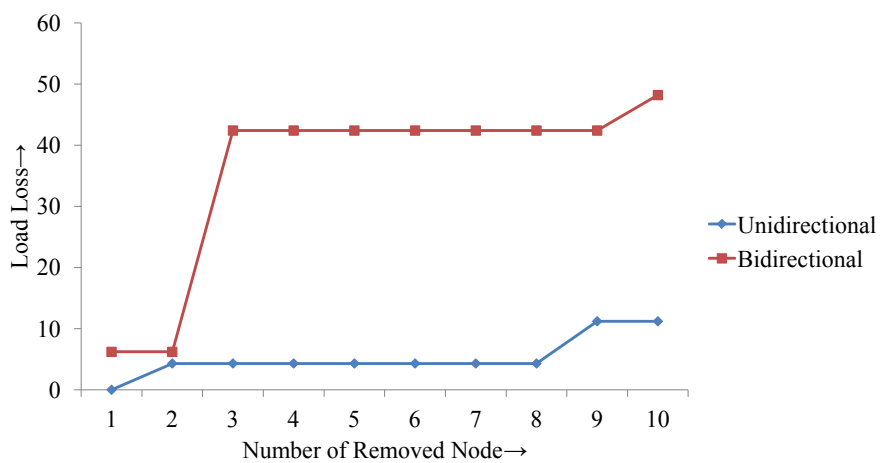


Figure 6. Two different effects on load loss due to loss of functionality of important nodes in IEEE 300 bus test system.

- Step 3** If power flow is not converged, reduce some load from the system and go to Step 1.
- Step 4** If power flow is converged, initialize counter  $i$ .
- Step 5** Increment counter  $i$  by 1.
- Step 6** Record total load shedded during the process.
- Step 7** Remove  $i$ -th critical node from the critical list.
- Step 8** Line Outage Distribution Factor (LODF) [33] is used to calculate redistribution of power flow without actually solving the load flow problem again.
- Step 9** Check for system load-generation balance. If the balance is not achieved, ramp-up or ramp-down generators accordingly to adjust generator set-point in order to achieve the balance. Generation Shift Factor (GSF) is used to calculate redistribution of power flowing among lines.
- Step 10** Check if there is any overloaded line.
- Step 11** If any overloaded line is found trip the line and go to step 8. Time-delayed overcurrent relays are used in every line so if there is a lot of overload they trip faster and if there is a little bit of overload they trip slowly.
- Step 12** If there is no overloaded line, go to Step 5.

#### 4. Conclusion

The changing power flow pattern demands for the improved analytical techniques to solve the global problem of cascading failure. This issue has been addressed under complex network framework. An improved model of closeness centrality measure has been proposed. Bidirectional power flow pattern of future smart power system has been taken into consideration while modeling the system. Three different measures of impact have been evaluated to observe the effect of removal of critical nodes found from centrality measures.

Closeness centrality based critical node analysis has been carried out and results from nondirectional and bidirectional flow based methods have been compared. Large changes in path length, connectivity and load loss implies the efficacy of the proposed method. IEEE 30 bus, 57 bus, 118 bus and 300 bus test systems has been used to demonstrate the applicability of proposed modified centrality measures in critical node analysis of power systems. The analysis of a real Polish transmission system based on the proposed methodology is our future work.

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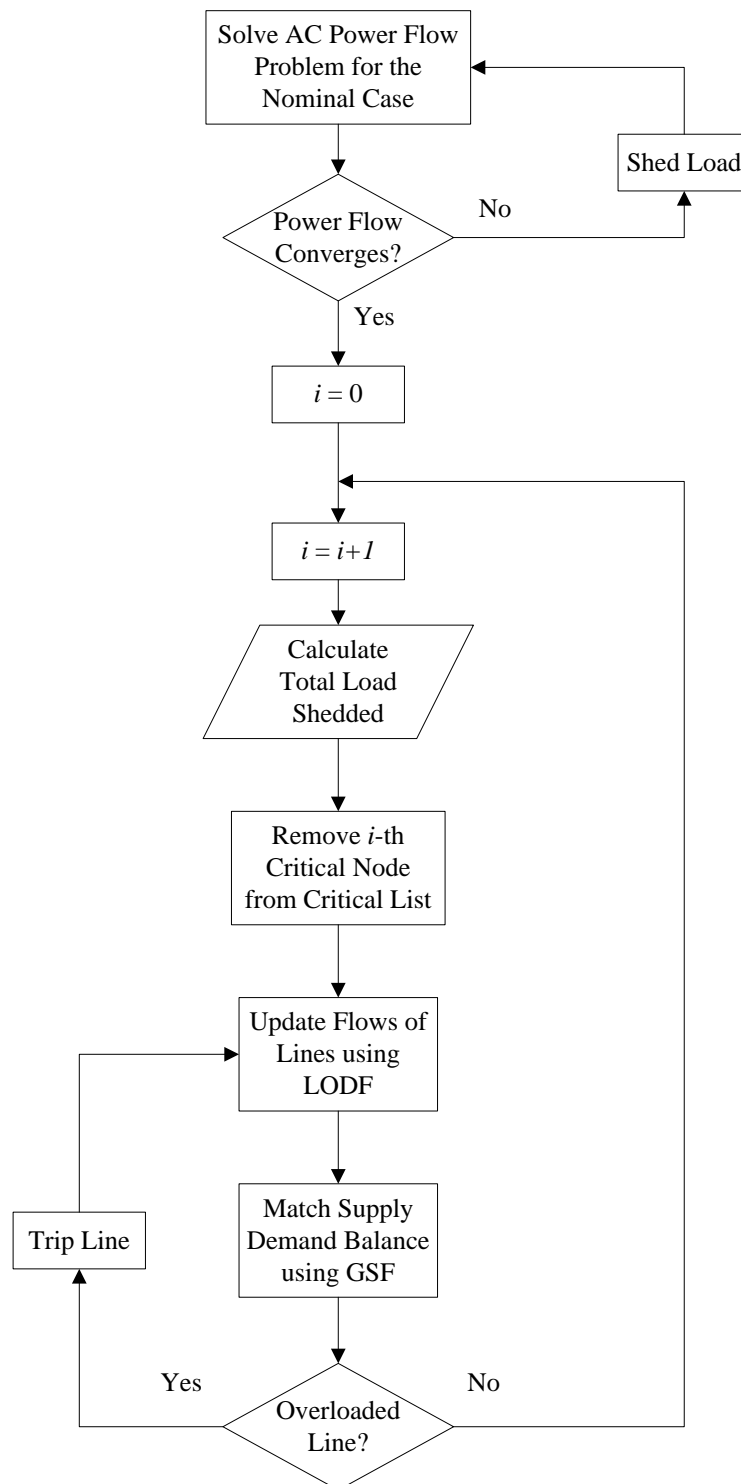


Figure 7. Simple cascading failure model.

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