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Analysis of Brownian Particles for Finding the Shortest Path in Networks

Bin Hu[1,2], Jia-li Xu[2], Huan-yan Qian*[1]
[1]School of Computer Science and Technology, Nanjing University of Science and Technology, Xiaolingwei 200, Nanjing, China, 210000
[2]College of Information Science and Technology, Nanjing Agricultural University ,Nanjing, Weigang 1, Nanjing, China, 210095
*Corresponding author, e-mail:hyqian@njust.edu.cn

Abstract

In this paper, we propose a method to analyze the shortest path finding between two nodes in complex networks. In this method, we first find that single Brownian particle follows the shortest path between source node i and destination node j in the probability of $\sum_{a=1}^{n}[B(j)^{d_{ij}-1}-B(j)^{d_{ij}}]_{ia}$ where d_{ij} denotes the shortest path steps between two nodes. To be compared with single particle utilization, then we specially analyze the multiple particles. We compute the probability of m particles' taking the shortest path between i and j when s particles starts simultaneously from the source and head to the destination as $P\{S_{ij}(s)=t\}=C_s^m(\sum_{a=1}^{n}[B(j)^{d_{ij}-1}-B(j)^{d_{ij}}]_{ia})^m(\sum_{a=1}^{n}(B(j)^{d_{ij}})_{ia})^{s-m}$. It's very clear that there must be particles taking the shortest path to arrive at the destination in the multiple particles environment. And with the number of m increasing, the arriving probability first arise and then drop down rapidly until to zero. In the end, we make the experiments and confirm our theoretical analysis. Our results would provide valuable usage for some applications such as finding the optimal routing in wireless sensor networks.

***Keywords**: shortest path; Brownian particle; networks*

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1. Introduction

Network searching plays an important role in data exchange, resource utilization, information share and so on which makes the method of finding the shortest path become a key research area in complex networks [1, 2].

At present the shortest path finding strategies in complex networks are usually based on passing packets. The packets are initiated from the source node and passed its' neighbors. Such process are repeated until these packets arrive at the destination node. So the shortest path has been found at the same time. Inspired by this idea, there are three main strategies of finding the shortest path between nodes in networks. Theses are the Break First Search (BFS) [3], the Random Walk (RW) [4] and some local search methods that utilize high degree nodes. Among them, BFS has high accuracy, but is has high flux of inquiring data packets on the network [5], which has great impact on the utilization of network resources. Although the local search methods can reduce the flux of inquiring packets, the accuracy of them mainly depend on the network topology. RW is an important process in complex networks. In recent years RW attracts extensive attention. First passage time is the characteristic of RW and often used to solve the shortest path finding problem [6, 7, 8, 9, 10, 11, 12]. But most of them are just constructed on the basis of single Brownian particle.

In this passage, besides the single Brownian particle utilization, we will also use the multiple Brownian particles to analyze the problem of shortest path finding. By analyzing single particle's random walking, we find that this particle follows the shortest path between source

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node i and destination node j in the probability of $\sum_{a=1}^{n}[B(j)^{d_{ij}-1}-B(j)^{d_{ij}}]_{ia}$ where d_{ij} denotes the shortest path steps between two nodes. Under this basis ,we finally deduce that with s particles' random walking on the network, there are m particles first arriving at the target node in the probability of $P\{S_{ij}(s)=d_{ij}\}=C_s^m(\sum_{a=1}^{n}[B(j)^{d_{ij}-1}-B(j)^{d_{ij}}]_{ia})^m(\sum_{a=1}^{n}(B(j)^{d_{ij}})_{ia})^{s-m}$. From the elementary mathematical knowledge, we can infer that with $s->+\infty$ and $m=0$, $P\{S_{ij}(s)=t\}->0$ .So we can easily conclude that $P\{S_{ij}(s)=d_{ij}\}_{m>0}=1-P\{S_{ij}(s)=d_{ij}\}_{m=0}=1$. So we may conclude that there must be particles taking the shortest path to arrives at the destination.[13,14,15]

2. Research Method

2.1. Shortest Path Finding of Single Brownian Particle

We first pay attention to the shortest path finding of a single Brownian particle on a generic network. The network could be basically considered as a connected and undirected graph G(m,n) where m denotes the number of edges and n denotes the number of nodes. If node i and j are connected with at least one edge, we let $n_{ij}=1$ and $n_{ji}=1$, otherwise $n_{ij}=0$ and $n_{ji}=0$. We also assume all $n_{ii}=0$ .So the adjacent matrix of such network is expressed by N with all entries $n_{ij}, i, j=1,2,...n$ .The variable d_i is used to stand for the degree of node i and so $d_i=\sum_{j=1}^{N}n_{ij}$ .When we select the node i as the start position for one singe Brownian Particle random wandering on the network, it is clear that the possibility of the particle's transferring to any neighboring node is $\frac{1}{d_i}$.Thus it's easy to see the Markov transition matrix P of such network could be calculated as follow:

$$P=ND \tag{1}$$

where $N=\begin{pmatrix} n_{11} & \cdots & n_{1n} \\ \vdots & \ddots & \vdots \\ n_{n1} & \cdots & n_{nn} \end{pmatrix}$ is the adjacent matrix and $D=\begin{pmatrix} \frac{1}{d_1} & 0 & 0 \\ & \ddots & \\ 0 & 0 & \frac{1}{d_n} \end{pmatrix}$ is the diagonal matrix.

Now let's randomly select on the network node i as the source and node j as the destination. The particle starts from the source node and take randomized walk on the network to the destination. The random variable S_{ij} stands for the number of steps the particle uses to get to the destination node j from the source node i for the first time. So expression $P\{S_{ij}=t\}$ denotes the probability that the particle first arrives at the destination at the exact t'th step. According to the famous C-K equation, we can infer that

$$P\{S_{ij}=t\}=\sum_{\sigma_1\sigma_2...\sigma_{t-1}}p_{i\sigma_1}p_{\sigma_1\sigma_2}...p_{\sigma_{t-1}j},\sigma_1,\sigma_2.....,\sigma_{t-1}\neq j \tag{2}$$

where $p_{i\sigma_1}, p_{\sigma_1\sigma_2}....., p_{\sigma_{t-1}j}$ are entry items of matrix P.To calculate the $P\{S_{ij}=t\}$, we first introduces the other matrix $B(h)$. $B(h)$ is acquired by simply making the $h'th$ column of matrix P to zero. So

$$B(h)=(p_1, p_2,..., p_{h-1}, 0, p_{h+1},...., p_n) \quad (3)$$

And we can calculate $P\{S_{ij}>t\}$ as

$$P\{S_{ij}>t\}=\sum_{a=1}^{n}(B(j)^t)_{ia} \quad (4)$$

In equation(4), $P\{S_{ij}>t\}$ denotes the probability of the single particle first reaching node j after setting out from node i beyond t steps. Then we can easily infer that

$$P\{S_{ij}=t\}=P\{S_{ij}>t-1)-P(S_{ij}>t) \quad (5)$$

According to the basic knowledge of probability theory, the mean first reaching steps X_{ij} can be calculated by

$$\begin{aligned} X_{ij} &= \sum_{t=0}^{\infty} tP(S_{ij}=t)=\sum_{t=0}^{\infty} t(P(S_{ij}>t-1)-P(S_{ij}>t)) \\ &= P(S_{ij}>1)+P(S_{ij}>2)+....=\sum_{t=0}^{\infty} P(S_{ij}>t) \end{aligned} \quad (6)$$

And we can finally get

$$X_{ij}=\sum_{h=1}^{n}(\frac{1}{I-B(j)})_{ih} \quad (7)$$

where I is the identity matrix and $\frac{1}{I-B(j)}$ denotes the inverse operation.

2.2. Shortest Path Finding of Multiple Brownian Particles

Supposing d_{ij} denotes the shortest path between node i and j, we can clearly see that single particle follows this link in the probability of $\sum_{a=1}^{n}[B(j)^{d_{ij}-1}-B(j)^{d_{ij}}]_{ia}$ according above discussion. Based on this, we can further investigate the shortest path problem of multiple Brownian particle. Like former description, we randomly select on the network node i as the source and node j as the destination. All particles simultaneously start from the source node and go to the destination node. Variable s stands for the number of particles walking on the network and variable m denotes the number of particles which take the shortest to first arrive at the destination node. Random variable $S_{ij}(s)$ expresses the steps that m arriving particle take from source node i to destination node j.Let X_{ij}^{b} be the number of steps the b'th Brownian particle takes to get to the destination for the first time. It's obvious that all the particles' walking are independent. So we can get

$$P\{S_{ij}(s)=t\}=\sum_{m=1}^{s} P\{PX_{ij}^{1}=t\}...P\{X_{ij}^{m}=t\}P\{PX_{ij}^{m+1}>t\}....P\{PX_{ij}^{s}>t\} \quad (8)$$

According to Eq.(4) and (5),

$$P\{S_{ij}(s)=t\}=C_s^m(\sum_{a=1}^{n}(B(j)^t)_{ia})^{s-m}(\sum_{a=1}^{n}[B(j)^{t-1}-B(j)^t]_{ia})^m, m=0,1,2.... \quad (9)$$

It's very clear that when $m=0$, $P\{S_{ij}(s)=t\}=C_s^0(\sum_{a=1}^{n}(B(j)^t)_{ia})^s$ .With $s->+\infty$, $P\{S_{ij}(s)=t\}->0$ .So we can easily conclude that $P\{S_{ij}(s)=t\}_{m>0}=1-P\{S_{ij}(s)=t\}_{m=0}=1$. That is to say there must be particles arrive at the destination in t steps when $s->+\infty$.

We also can also compute the mean first reaching steps of multiple Brownian particles as:

$$X_{ij}(s)=\sum_{t=0}^{\infty}P(S_{ij}(s)>t)=\sum_{t=0}^{\infty}(\sum_{a=1}^{n}(B(j)^t)_{ij})^s \quad (10)$$

3. Results and Analysis

As a check of our analysis above, we performed two experiments. In first experiment, we use a small random tree-like model to test the shortest path finding both in the case of single Brownian particle and in the case ofmultiple Brownian particles. And in second experiment, we will use a mesh-like model.

3.1. Tree-Like Network Analysis

From model as Figure 1, we can see that the shortest path steps between source node 1 and destination node 12 are 4. So it's very easy to calculate the probability of single particle's taking such shortest path is 0.0333.

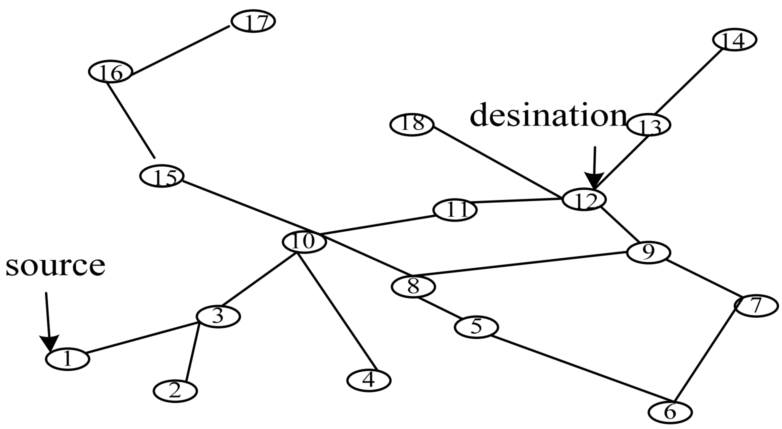

Figure 1. Tree-like network model

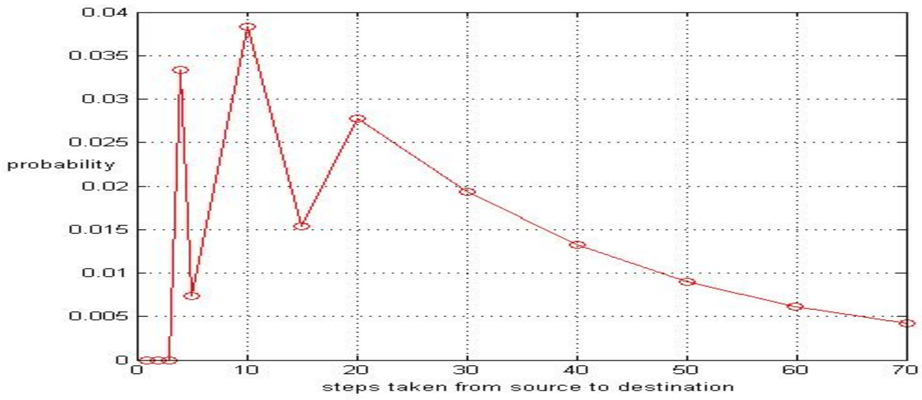

Figure 2. Single particle’s first arriving at the destination in t’th steps

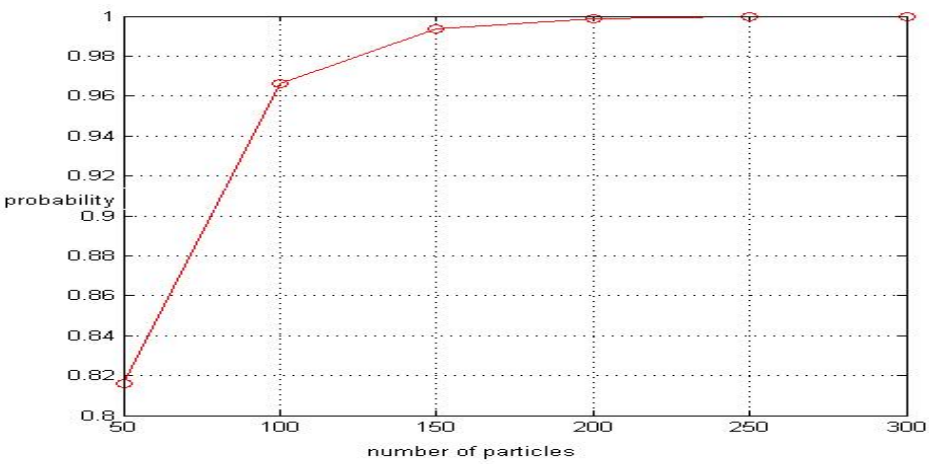

Figure 3. Arriving probability in multiple particles environment with the number of s

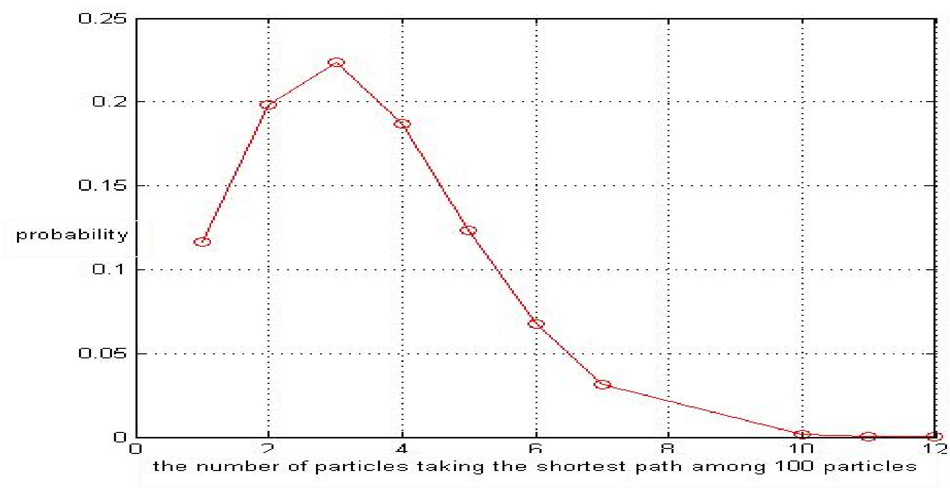

Figure 4. The probability of taking the shortest path with the increase of m

3.2. Mesh-Like Model

The Mesh-like model has been widely used nowadays, especially in wireless sensor networks. Figure 5 gives a simple example.

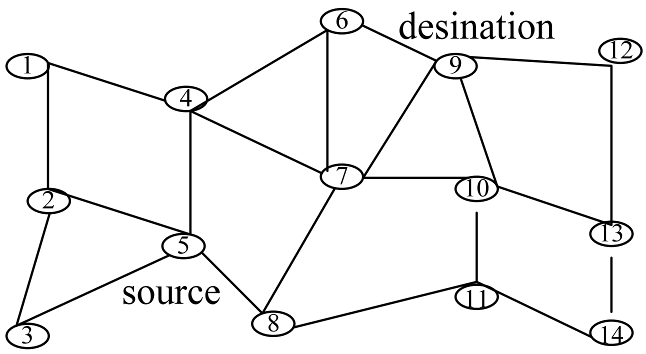

Figure 5. Mesh-like network model

From above model, we can see that the shortest path steps between source node 5 and destination node 9 are 3.So it's very easy to calculate the probability of single particle's taking such shortest path is 0.05

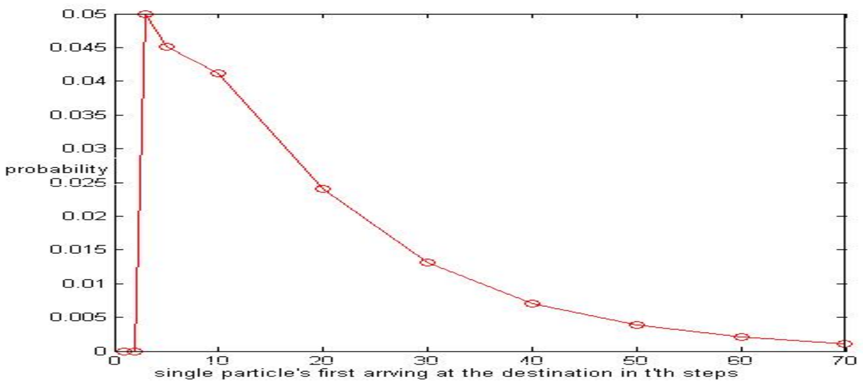

Figure 6. Single particle's first arriving at the destination in t'th steps

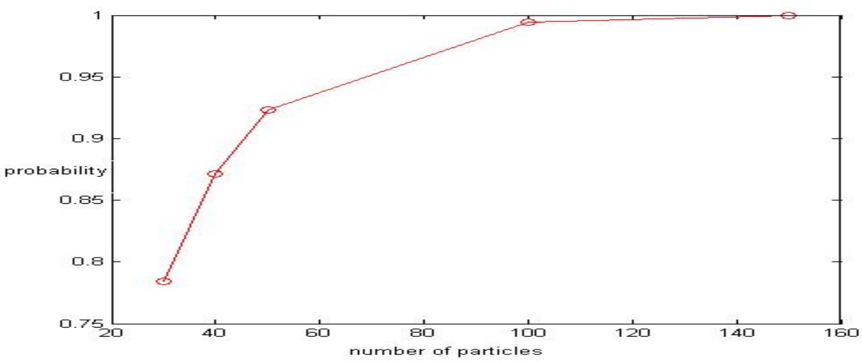

Figure 7. Arriving probability in multiple particles environment with the number of s

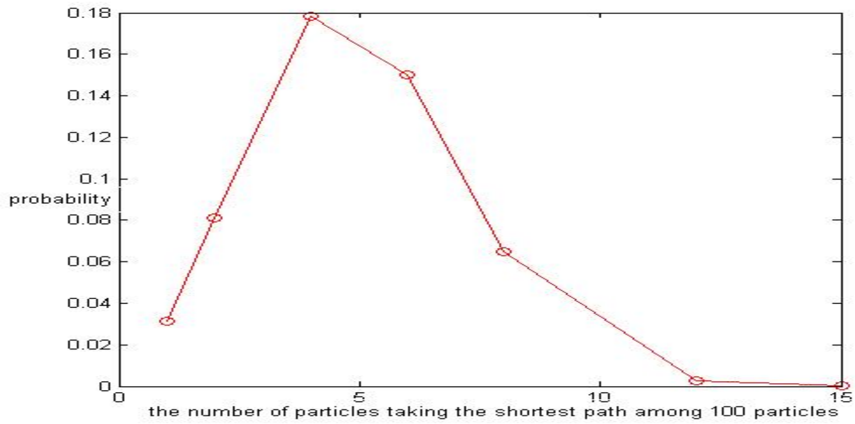

Figure 8. The probability of taking the shortest path with the increase of myy

4. Conclusion

In this paper, we analyze the shortest path finding with the use of single and multiple Brownian particle in detail. From above analysis, we can easily get a conclusion that there must be particles to take the shortest path in multiple particles' usage and it is more efficient than the single particle's application. In our example, it has at least increased its' finding probability as much as four times. In spite of this, the number of particles which take the shortest path in multiple environment are limited. In our experiment, when 100 particles simultaneously start from the source node, no more than 15 particles take the shortest path.

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