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Comparison Methods for Converting a Spindle Plant to Discrete System

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Abstract

This study presents comparison methods in the conversion of a spindle plant in order to obtain an accurate discrete system when compared to a continuous system of spindle. The accurate conversion results of the continuous system into discrete form are required for implementing the control system of spindle. Comparison methods that will be conducted to convert the continuous system of spindle plant into discrete system through zero-order hold (ZOH), first-order hold (FOH), impulse invariant dicretisation, tustin (bilinear), and pole-zero matching methods. The performances of each method in the conversion process have been presented. Conversion performances of continous spindle plant into discrete form using FOH method, showed more accurate compared to other methods. Performances of the conversion accuracy of FOH method have been evaluated in terms of transient responses analysis that closed similar results with a continuous system of spindle plant. At the closed similar transient responses for the discrete system using FOH method show the final value, time to peak, percertages overshoot and setlling time are 0.863 v, 0.910 s, 0 % and 0.550 s respectively.

Keywords*: Comparison, continous, discrete, conversion, method*

1. Introduction

The digital signal processing was began with explosive growth in the decade of the 1960s, when researchers discovered how to use recursive digital filters for simulating analog filters [1]. The improvement of conversion techniques from continuous system into discrete form is still required to support the implementation especially when using computational algorithms. In the current issue, almost all control systems using digital control system based computing algorithm. In fact most of the system is generally shaped in continous plant. It will require any techniques for converting into a discrete form. Thus it would be in harmony with the use of digital control systems.

Most industrial processes especially engineering systems are constructed using discrete models. Chemical processes and electrical systems are typical examples. The recursive features of these discrete models and the recent availability of high performance low cost microcontrollers have enabled a new consideration of control and analysis of these systems. Shieh and Wang [2] presented methods for model conversions of continuous-time state-space equations and discrete-time state-space equations, many well-developed theorems and methods in either continuous or discrete domains can be effectively applied to a suitable model in either domain.

Melwin and Frey [3] described continuous-time to discrete-time conversion with a novel parametrised s-to-z-plane mapping. A parametrised s-to-z-plane map was introduced, where the conventional bilinear map and backward and forward Euler rules appear as special cases. With a simple technique for applying this map to adaptively reduce truncation error in the continuous-time to discrete-time conversion problem. In order to convert continous signal to discrete, Keller et al [4] explored a method for the discrete-time simulation of continuous-time sigma-delta modulators. Via the application of lifting, correction values for each state variable of a modulator are calculated, which subsequently are used to calibrate online these state variables during a discrete-time simulation of the continuous-time system.

Korlinchak and Comanescu [5] explained discrete time integration of observers with continuous feedback based on Tustin's method with variable prewarping, improved improvement to trapezoidal integration the state equations of observers are integrated using a discrete-time filter that is prewarped as a function of the drive's operating frequency.

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Charles [1] also explained that Hurewicz theory showed how a z-transform could describe a sampled data transfer function. On the other hand Cagatay et al [6] proposed a definition of the discrete fractional Fourier transform that generalises the discrete Fourier transform in the same sense that the continuous fractional Fourier transform generalises the continuous ordinary Fourier transform. Xiao et al [7] designed the filtering module with filter using high precision and wide dynamic response range, can meet the requirements of speed and precision of laser gyro demodulation aerospace fields.

Other wise, several control methods are formulated using continuous models for which several theories and practical methods have been developed. A large practical control methods consist of both continuous-time and discrete-time sub-systems. For effective analysis and synthesis of these composite systems it is often necessary to convert a discrete sub-system to an equivalent continuous model. Gahinet and Shampine [8] presented the framework for modeling linear time-invariant (LTI) systems with delays, the delays in feedback loops are general enough for most control applications, and lends itself well to computer-aided analysis. Antonie and George [9] developed the first framework of system approximation that applies to both discrete and continuous systems by developing notions of approximate language inclusion, approximate simulation, and approximate bi-simulation relations. Other way, Indah et al [10] presented the comparison between the conventional particle filter and particle filter with Gaussian weighting methods, the weight was calculated in each particle, the remain particle's weight was calculated using the Gaussian weighting.

However, most of the published work on building filters to convert contionous signal to discrete signal with limited possibility for comparing several methods. Moreover, not much works on comparison of continous conversion into discrete form using without method, ZOH, FOH, Impulse-Invariant, Tustin Approximation and Zero-Pole Matching Methods. Generaly it can be seen when ZOH method are used by most researchers for converting a continous system into a discrete system. Including matlab by default also uses the ZOH method. But, it is necessary to clarify that whether ZOH method is the most accurate method to convert a continuous system into a discrete system. This study will examine the comparison of various methods in order to obtain the most accurate method for conversion of a continuous system into a discrete system. This is a challenging task for finding the closest method compared to continuous system is affected by several factors. This paper presents comparison methods for converting the spindle continous plant into discrete system using several methods to find the most accurate method. It is found that the several methods has not been explored to use in the same plant to find out the closest the spindle continous plant with discrete system. Using without method, the ZOH, FOH, Impulse-Invariant, Tustin Approximation and Zero-Pole Matching Methods to obtain the closest discrete system compared into continous system. For performance assesment, the spindle continous plant will compared into discrete system in terms of transient responses analysis. The analysis results show that better system performance and closest similar with continous system are achieved with FOH method.

2. Research Method

In this work used several methods for converting the continous of spindle plant into discrete system. The methods that will be used in this work consist of ZOH, FOH, and Impulse-Invariant, Tustin Approximation and Zero-Pole Matching Methods. This work also presented the conversion result without any method.

In the early part will be presented comparisons between the methods of ZOH, FOH and Impulse-Invariant. The ZOH, FOH, and impulse-invariant methods produce exact discretizations in the time domain for: (1) systems without time delays, (2) systems with time delays on the inputs and outputs only (no internal delays). Because of the exact match, it can use these discretisation methods for time-domain simulations. In this context, exact discretisation means that the time responses of the continuous and discretised models match exactly for the following classes of input signals: (1) Staircase inputs for ZOH, (2) Piecewise linear inputs for FOH, (3) Impulse trains for impulse IMP. For systems with internal delays (delay in feedback loops), the ZOH and FOH methods results in approximate discretisations. An internal delay is illustrated in the following Figure 1.

For such systems, the continous to discrete (c2d) performs the following actions to compute an approximate ZOH or FOH discretization [11]: (1) Decompose the delay τ as

$\tau = kT_s + \rho$ with $0 \le \rho \le T_s$. (2) Absorbs the fractional delay ρ into *H(s)*. (3) Discretises *H(s)* to *H(z)*. (4) Represents the integer portion of the delay kT_s as an internal discrete-time delay z^{-k} . The final discretised model appears in the following Figure 2. The impulse-invariant method does not support systems with internal delays.

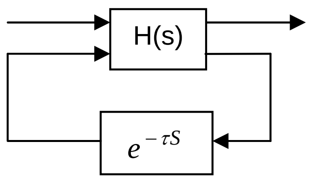

Figure 1. An internal delay

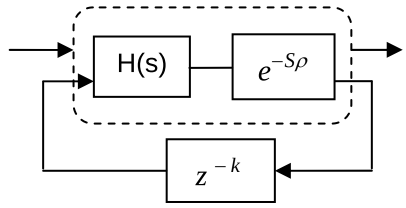

Figure 2. The final discretized model

Tustin Approximation and Zero-Pole Matching Methods. When discretising a system with time delays, the Tustin and matched methods: (1) Round any time delay to the nearest multiple of the sampling time, (2) Approximate the fractional time delay. When discretising tf and zpk models using the Tustin or matched methods, c2d first aggregates all input, output, and input-output (i/o) delays into a single input-output delay τ_{TOT} for each channel. The c2d then approximates τ_{TOT} as a Thiran filter and a chain of unit delays in the same way as described for each of the time delays in statespace models [11].

2.1. The Spindle of Lathe Machine

The spindle of lathe machine in this work was presented in Figure 3. The rig consists of three main parts: a spindle, sensors and a processor. The spindle is rotated by the main motor, holds the cutting tool, which cuts the work piece, then the cutting forces are generated which effects the spindle accuracy directly. The system identification for the spindle of lathe machine was implemented within the Matlab and Simulink environment on Intel Pentium 1.80 GHz and 2.00 GB RAM as detailed in M. Khairudin [12]. The data obtained in the form of collection of variations with the tacho output voltage. The transfer function form is obtained by Matlab program. Through the identification data obtained transfer function. Experimental works were conducted using the experimental rig for system identification results.

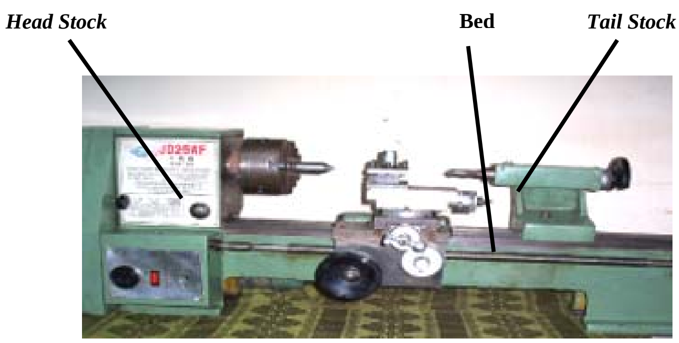

Figure 3. The Experimental Setup of a Spindle

For the spindle without cutting process, the transfer function can be writen as:

$$G_1(s) = \frac{9.925\,s^2 + 1794\,s + 1.424e005}{s^3 + 137.5\,s^2 + 2.487e004\,s + 1.648e005} \quad (1)$$

3. Results and Analysis

The continous system of spindle plant can be convert into discrete system without any method. It means the converting process without using any method. The converting process in this work wil use the sampling time of 0.01 s. The conversion result of discrete system without any method can be seen in equation (2) below:

$$G_1(z) = \frac{0.09911z^2 - 0.04855z + 0.01246}{z^3 - 1.107z^2 + 0.4324z - 0.2528} \quad (2)$$

When the input step is given to the continuous system and discrete system in equation (1) and (2) respectively, the responses can be seen at Figures 4 and 5.

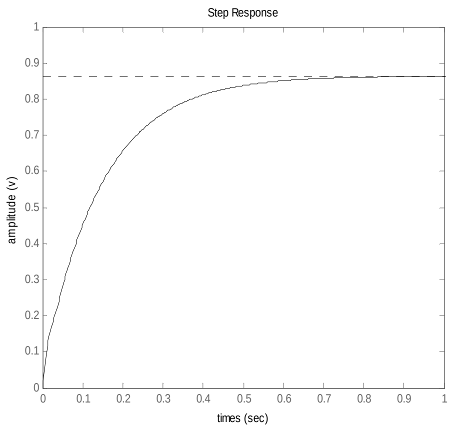

Figure 4. Step response of continous spindle system

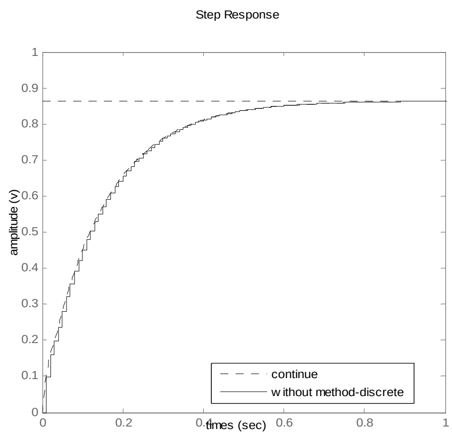

Figure 5. Step response of discrete system without method

Figure 5 presents the conversion process into discrete signal using without any method; the discrete signal is lagging a few moments compared to continous system. It means that in the case of discrete signal have delay times. The steady state error between continuous and discrete signal appears without any differences. To analyse the transient responses of the discrete system without method can be seen at Table 1.

To convert the contionous spindle plant into discrete system, there are several methods can be used. The methods will be used in this work consist of ZOH, FOH, and Impulse-Invariant, Tustin Approximation and Zero-Pole Matching methods. Generaly it can be seen when ZOH method are used by most researchers for converting a continuous system into a discrete system. Including matlab by default also uses the ZOH method. But, it is necessary to clarify that whether ZOH method is the most accurate method to convert a continuous system to a discrete system. This study explored the comparison of various methods in order to obtain the most accurate method for conversion of a continuous system into a discrete system.

The continuous system in equation (1) will be converted by ZOH method with sampling time of 0.01 s. The discrete system will be obtained as equation (3) below

$$G_1(z) = \frac{0.09911z^2 - 0.04855z + 0.01246}{z^3 - 1.107z^2 + 0.4324z - 0.2528} \quad (3)$$

It is obtained the same discrete equations between using ZOH method conversion in equation (3) and the conversion using without any method in equation (2). It has been proven that the matlab by default with automatically process for converting into a discrete form using the ZOH method. Although when conducting the conversion using without any method, it will give the impact that the result in the conversion process is the same as when using the ZOH method. It also will provide the same responses when the discrete system with ZOH method given the step input. It is noted the responses of ZOH method is same as the responses of discrete systems using without any method.

Subsequently presented comparison using FOH and impulse-invariant methods. The continuous system in equation (1) will be converted by FOH method with sampling time of 0.01 s. The discrete system is obtained in equation (4) below.

$$G_1(z) = \frac{0.05107z^3 + 0.02557z^2 - 0.02254z + 0.008922}{z^3 - 1.107z^2 + 0.4324z - 0.2528} \quad (4)$$

Otherway the continuous system in equation (1) when the conversion into the discrete form is conducted by impulse-invariant method with sampling time is 0.01 s, it will yield the following system in equation (5).

$$G_1(z) = 0.00949 * \frac{9.925z^3 - 2.609z^2 - 0.6768z - 1.503e-016}{z^3 - 1.107z^2 + 0.4324z - 0.2528} \quad (5)$$

Based on the equations (4) and (5) show the different equations compared to the previous equations. It means will give impact for the system responses. Figures 6 and 7 presented the step responses of the discrete system with FOH and impulse-invariant methods respectively.

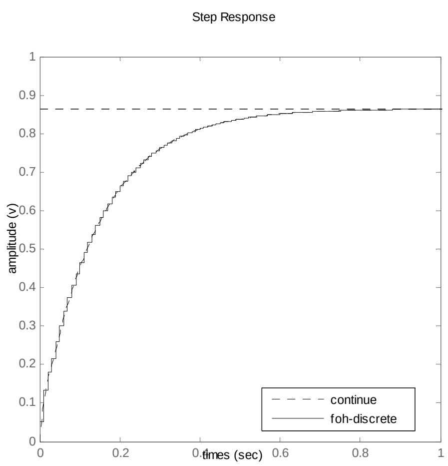

Figure 6. Step response of discrete system with FOH method

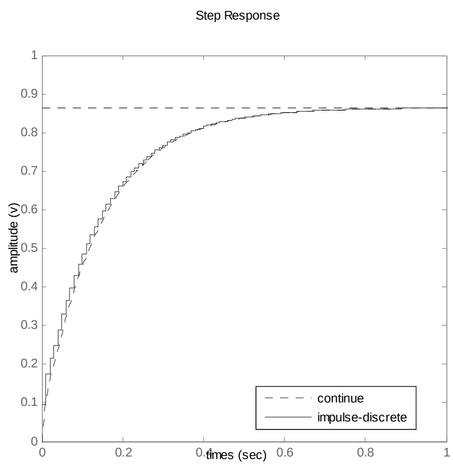

Figure 7. Step response of discrete system with impulse-invariant method

Figure 6 shows the conversion results of continuous spindle plant using FOH methods. The discrete signal using FOH method provides the signal located at the same position or chimed in one line with a continuous signal. It means that the discrete signals using FOH method almost simultaneously at one time with continuous spindle plant. In other words, the discrete signal using FOH methods obtained a signal with a little delay. This condition provides more ideal conditions for a time of discrete signal appearing almost simultaneously with the continous original signal. Certainly it will impact that the responses analysis which consist of the final value (v), time to peak (s), percentage of overshoot and settling time (s) on a discrete system with FOH method will be almost the same as the continuous spindle plant. For the details responses analysis of discrete system using FOH can be seen at Table 1.

Figure 7 shows the results of the conversion of continous spindle plant using impulse-invariant that the discrete signal is leading compared to continous system. This means the discrete system with the impulse-invariant method appears more faster than continous spindle plant. Obviously this condition resulted in the differences between the signal discrete and continuous signal for responses analysis.

The next challenges methods that will be used in this work are the Tustin approximation and Zero-Pole Matching methods. The continuous system in equation (1) can be converted into the discrete form by Tustin approximation method with sampling time is 0.01 s. The discrete system will be obtained as equation (6) below.

$$G_1(z) = \frac{0.04819z^3 + 0.02087z^2 - 0.01763z + 0.009689}{z^3 - 1.289z^2 + 0.7522z - 0.3921} \qquad (6)$$

Using the same technique, the continuous system in equation (1) can be converted into the discrete form by Zero-Pole Matching method with sampling time is 0.01 s. The discrete system will be obtained as equation (7) below.

$$G_1(z) = \frac{0.1065z^2 - 0.06097z + 0.01748}{z^3 - 1.107z^2 + 0.4324z - 0.2528} \qquad (7)$$

Based on the equations (6) and (7) reveal the different equations compared to the previous equations. It means will give impact for the system responses. Figures 8 and 9 show the step responses of the discrete system with the Tustin approximation and Zero-Pole Matching methods respectively.

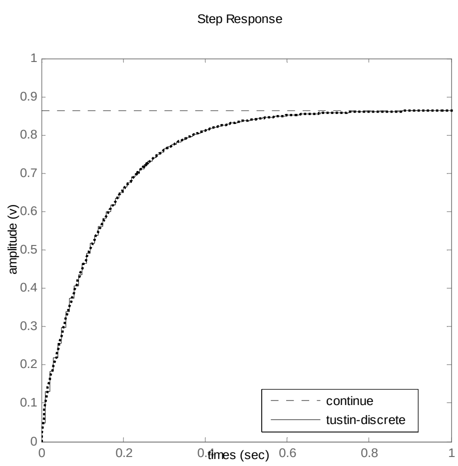

Figure 8. Step response of discrete system with Tustin method

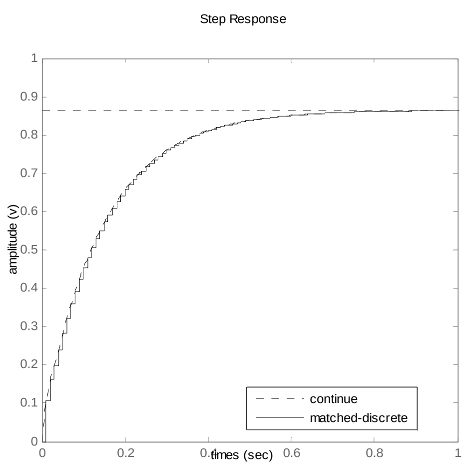

Figure 9. Step response of discrete system with matched method

Figure 8 shows that the conversion results of continuous spindle plant using Tustin method results. The discrete signal using Tustin method provides the discrete signal like the FOH method. But the discrete signal using Tustin methods obtained a signal with a delay. The time to peak of 0.920 is more slower than the continous system. For the details responses analysis of discrete system using Tustin method can be seen at Table 1.

Figure 9 shows the results of the conversion of continous spindle plant using Zero-Pole Matching method that the discrete signal is lagging a few moments compared to continous system. It means that in the case of discrete signal have delay times. The steady state error between continuous and discrete signal appears without any differences.

To provide the exhaustive comparison between several methods that presented previously, Figure 10 presents the comparison for the responses of Tustin, zoh, foh, matched and impulse methods.

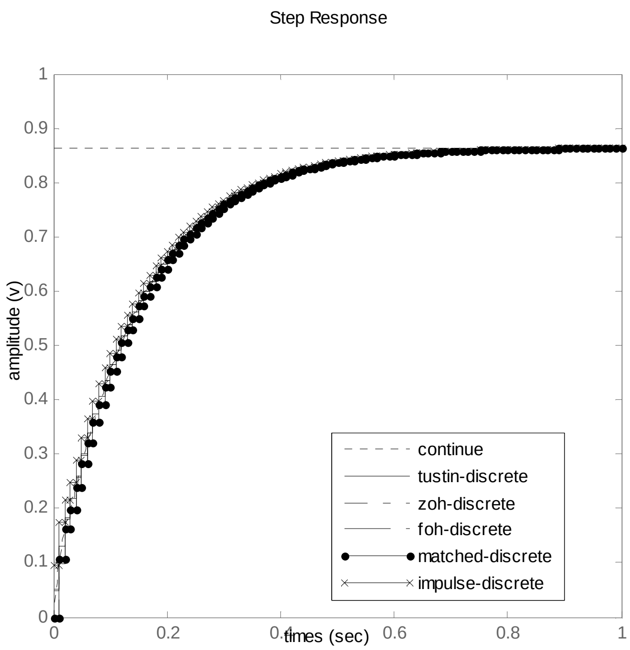

Figure 8. Comparison step response of discrete system with several methods

To analyse the responses performance of a discrete system, it is noted the best performance is the most similar with the original system or the continuous system. It is necessary to find the most accurate of conversion method compared to other. Therefore the discrete system responses need to be analysed by comparing the performances of results for each method. Table 1 presents the transient responses analysis for all methods mentioned.

Tabel 1. Comparison Transient Responses Analysis of Continue and Descrete System

No	Plant with method	Final Value (v)	Time to Peak (s)	% Overshoot	Settling time (s)
1	Continue	0.863	0.911	0	0.552
2	Tustin	0.863	0.920	0	0.550
3	ZOH	0.863	0.910	0	0.560
4	FOH	0.863	0.910	0	0.550
5	Matched	0.863	0.920	0	0.560
6	Impulse	0.863	0.930	0	0.560
7	Without method	0.863	0.910	0	0.560

Table 1 describes the transient responses analysis, it can be stated that the FOH method with the final value approaching a continuous signal. In other hand a time to peak is almost the same as the continous system. Also a setlling time is almost equal to the continous system. It is noted that the FOH method is the most accurately method for converting the continous system into the discrete system compared to other methods.

4. Conclusion

The development of converting a continous spindle plant into discrete system with various methods has been presented. A set of linear model of spindle plant has been developed. The closest discrete system is required to develop the control system. Comparison methods have been conducted to convert the continuous system of spindle plant into discrete system through the method of zero-order hold (ZOH), first-order hold (FOH), impulse invariant dicretisation, Tustin (bilinear), and pole-zero matching methods and also converting the continous system without any method. Conversion performances of continuous spindle plant into discrete system using FOH method, showed more accurate compared to other methods. Performances of the conversion accuracy of FOH method have been evaluated in terms of transient responses analysis that closed similar results with a continuous system of spindle plant.The responses analysis namely final value, time to peak, percertages overshoot and setlling time.

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