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Underdetermined direction of arrival estimation for multiple input and multiple outputs sparse channel based on Bayesian learning framework

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ABSTRACT

Direction of arrival (DOA) estimation for a sparse channel has attracted serious attention recently. Better signal analysis and denoising achieve accuracy in DOA determination. This paper proposes an underdetermined DOA estimation for multiple input and multiple outputs (MIMO) sparse channels. A novel multi-kernel-based non-negative sparse Bayesian learning (MK NNSBL) framework is implemented using the multiplied form of basis vector within the manifold matrix for a defined grid. Meanwhile, virtual antenna locations are reconfigured by exploiting the conventional cuckoo search algorithm (CCSA) for the fine reception of incoming signals on a non-uniform linear array (NULA). The simulated results reveal that the novel approach outperforms in its optimal root mean square error (RMSE) for various signal-to-noise ratio (SNR) limits and the compilation time. The convergence comparative graph indicates the improved performance in the proposed framework over existing algorithms.

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1. INTRODUCTION

Wireless mobile communication plays a vital role in today's life. To satisfy the stringent demands of dense users' requirements such as higher data rates, signal-to-noise ratio (SNR), and high accuracy with fewer errors different generations of cellular networks have evolved right from early first-generation (1G) to current fourth generation (4G). Now, the fifth generation (5G) network is the new version of the mobile communication system which is commercialized to deploy for usage. Nowadays, the need for direction of arrival (DOA) estimation is increasing rapidly day by day in 5G wireless mobile communication systems, radar, sonar, electronic surveillance, seismology, re-configurable intelligent surfaces (RIS), and medical diagnosis. DOA estimation is performed virtually using computers rather than the manual method to avoid the need for physical adjustment of an antenna and extra phase shifter for beam steering. DOA estimation means finding the exact direction of transmitted electromagnetic signals that impinges on the receiving antenna elements in an array over a noisy channel.

In general, DOA estimation techniques have three main classifications such as spectral estimation, parametric subspace-based estimation (PSBE), and deterministic parametric estimation (DPE) [1]. The popular parametric subspace-based methods multiple signal classification (MUSIC) and estimation of signal parameters via rotational invariance technique (ESPRIT) [2] are implied to a greater extent among others, but sparse Bayesian learning (SBL) is a popular method that can be used for sparse signal recovery (SSR) in compressive sensing (CS) [3]. The MUSIC and ESPRIT algorithms comparison based on implementation time is discussed [4] where, the MUSIC technique uses noise eigenvectors, and ESPRIT uses signal eigenvectors. A new improved MUSIC method DOA approximation is performed on circular and non-circular sparse wave forms to improve the degrees of freedom (DOF) [5]. A cross dipole array is exploited with augmented quaternion (AQ) ESPRIT type for DOA estimation for low SNR limits [6]. For increasing the small step size between the elements and over low SNR values, an enhanced standard ESPRIT (ES ESPRIT) based DOA finding method is developed [7]. The source localization method is a tough task in a sparse channel so, in this regard, the SSR procedure provides good accuracy without knowing the number of sources, without taking many snapshots, and with the correlation of signals [8]. The extremely effective SBL technique was first introduced by Tipping for finding sparse solutions by making use of basis functions [9].

Further, in this regard to address the problems in a joint direction-of-departure (DOD) and DOA estimation over bi-static MIMO radar SBL is adopted [10], and the Kalman filter method [11]. To overcome the complexity only real-value-based SBL is presented on arbitrary linear arrays [12], and on sparse arrays for off-grid targets using underdetermined criteria [13]. A novel SBL with phase errors (SBLPE) approach is proposed with acceptable complexity [14]. An analysis is described of the recent SBL based algorithms with their challenges [15]. A joint SBL method is implemented to find two targets at bistatic radar working passively [16]. Non-negative based SBL for an underdetermined DOA finding is adopted with the cumulative and hybrid form of basis vector using the optimized antenna reconfiguration method [17]. Basis pursuit denoising (BPDN) based multi-kernel approach is used for a non-uniform linear array (NULA) [18]. By using a symmetric MIMO array joint DOA estimation along with the range and reflectivity of back scattered waves in near-field applications are derived [19]. Time-varying SBL-based DOA estimation is sequentially performed for an unknown variance over the channel [20], and for block sparse signals [21]. The important objective of this paper is to find solutions to all the above discussed issues in literature such as off-grid models showing high computational complexity, simulations on low SNR values [22], low accuracy algorithms, need for more compilation time with memory, high RMSE, lack of DOF flexibility, and lower optimized results for NULA. The main contributions in this paper are divided into two sections. Firstly, multiplicative basis vector formation within the manifold matrix for beam rotation as per need along with expectation maximization (EM) method. Secondly, the variation in the virtual antenna locations for fine impinging of incoming signals on the array employing the simple conventional cuckoo search algorithm (CCSA) technique. On simulations, the results of the novel method show that it achieves good results on different SNR limits than the non-negative sparse Bayesian learning (NNSBL) approach. The remainder of the paper is assembled as follows. A step-by-step analysis of the proposed multi-kernel based SBL algorithm is introduced in section 2. The virtual optimal antenna reconfiguration method is described in the flowchart in section 3. Evident results and discussions are plotted in section 4 followed by a conclusion of the new proposed model in section 5.

2. THE PROPOSED MULTI-KERNEL-BASED SPARSE BAYESIAN LEARNING FRAMEWORK

The multi-kernel approach in the NNSBL framework is applied for an underdetermined DOA estimation scenario by considering the multiplicative basis vector. In MIMO applications if the number of incoming sources is greater than or equal to the number of elements in an array it is termed an underdetermined condition. The posterior values are maximized by exploiting the prior values via NNSBL [22]. Adaptive beam forming accuracy is enhanced with multiplicative basis vector and optimal antenna reconfiguration model. The DOA estimation proposed method includes two approaches: i) multiplicative basis vector formation within the manifold matrix for beam rotation as per need and ii) the variation in the virtual antenna locations for fine impinging of incoming signals on an array.

2.1. Multiplicative basis vector-based multi-kernel NNSBL method

The proposed implementation includes the multi-kernel basis vector in the form of multiplication within the steering matrix. To reduce most of the sparsity, BPDN-based SSR methods are employed for calculating the DOA. The improvements in the conventional DOA approximation approach for signal recovery are briefed. This type of basis vector consideration is a novel development especially made for MIMO applications.

2.2. Signal model

Consider 'K' narrowband far-field incoming signals from different directions in a MIMO sparse channel falling on an omnidirectional antenna. An analysis of NULA of antenna elements that are separated by integer multiples of half-wavelength is carried out. Similar to Rao subspace method, the transmitter count is double or huger than the number of receiving sensors is considered [23]. Minimum redundancy positioning of antenna elements in an array enhances the DOF [24]. The independent variable is the distance that is considered for antenna reconfiguration as per the application. So, let us assume the distance between the antenna elements is ‘d’ and the used total element number is ‘E’ then the difference in distance between the elements is given by:

$$\Omega = d_{e1} - d_{e2}, e1 = 0,1, \ldots, E-1 \; and \; e2 = 0,1, \ldots, E-1 \tag{1}$$

assume narrowband uncorrelated source signals are given by:

$$sb(t), b = 1, 2, \ldots, K \tag{2}$$

introducing noise into the channel concerning E antenna elements is assumed as:

$$ne(t), e = 0,1, \ldots, E-1 \tag{3}$$

the array signal model is designed to be:

$$\mathrm{u(t) = As(t) + n(t), t = 1,2, \ldots, T} \tag{4}$$

where $u(t) = [u1(t), \ldots, uE(t)]$ is the received signals at an array end, the predefined DOA grid is $\Theta = [\theta 1, \ldots, \theta K], s(t) = [s1(t), \ldots, sK(t)]$ and $n(t) = [n1(t), \ldots, nE(t)]$ are the transmitted signals with unknown channel noise vectors at time ‘t’ respectively. Array manifold matrix or steering matrix is used to steer the antenna elements which contains a group of steering vectors as:

$$A = [a(\theta 1), a(\theta 2), .., a(\theta K)] \tag{5}$$

$a(\theta K)$ is known as the steering vector of the K^{th} source signal. Each steering vector is denoted as $a(\theta k) = [1, v\ (d_{e1}, \theta k) \ldots v\ (dE-1, \theta K)]\ T$, which includes multiplied basis functions related to phase given by $v\ (de, \theta k) = exp\ [-j2\pi(de/\ \lambda) sin\ \theta k]$, $\{\}^T$ indicates transpose notation.

Some general assumptions are also made concerning the source and the noise signals having zero-mean with independent variances ‘σ_n^2’, acting upon all the signals. The aim is to find arriving angles ‘θ’ of various sources ‘K’ so that $u(t)$ must fall within the on-grid angles. The basis vector used in the manifold matrix is responsible to predict the fine DOA angles. From the assumptions made earlier, the covariance matrix ‘R_u’ is:

$$R_u = E\{u(t)u(t)^H\} = A(\theta)GA(\theta)^H + H \tag{6}$$

where (6) includes E(.) and $(.)^H$ which indicates the expectation and hermitian operation. G and H are the sources and channel noise covariance matrix respectively.

DOA estimation is performed on the assumption that the incident source signals are symmetric circularly gaussian distributed, and the residual error is of covariance matrix estimation [25]. On vectorizing ‘R_u’ virtual form of the manifold is formed using a covariance matrix. The residual error is the difference between estimated and actual received DOA values given by:

$$\hat{y} - y = vec(\widehat{R}_u) - vec(R_u) \sim \mathbb{CN}(0, R_u^T \otimes R_u/T) \tag{7}$$

where, ‘$\hat{y}$’ is the estimated received signal and ‘vec $(\widehat{R}_u)$’ is the estimate of the covariance matrix vectorized. The actual received signal is ‘y’ and vectorized covariance matrix is ‘vec (R_u)’.

Further, the vectorized R_u is equated as y= vec $(R_u) = (A^* \otimes A)g + \sigma_n^2 1_m$ where ⊗ denotes kronecker product. g= $[\sigma_1^2, \sigma_2^2, \sigma_3^2, \ldots \sigma_K^2]^T$ is the source variance vector with $\{\}^T$ as transpose notation. The array manifold matrix in virtual is considered as $\overline{A}$= $(A^* \otimes A)$. So, by considering $\widetilde{R}_u = R_u^T \otimes R_u/T$ with T as the period and all the above-mentioned assumptions, (7) is transformed as $\hat{y} \sim \mathbb{CN}\ (\overline{A}g + \sigma_n^2 1_m, \widetilde{R}_u)$.

BPDN is widely used nowadays to estimate DOA in sparse channels. Later by considering ′Φ’ as an overcomplete basis matrix for $\overline{A}$ parameterized by all the directions on the grid Θ and on multiplying the basis vectors gives the best DOA estimates described by (8).

$$\hat{y} \sim \mathbb{CN}(\Phi w + \sigma_n^2 1_m, \widetilde{R}_u) \tag{8}$$

From (8) describes that only positive real values are present in the sparse matrix ‘w’. This matrix includes the positions at actual DOA as ones and other positions as zeros. The vector 1_k= $[e_1^T, e_2^T, e_3^T \ldots e_K^T]^T$ with e_K vector is zeros leaving the K^{th} entry as one where $\{\}^T$ indicates transpose notation. The building block of the manifold matrix is the product of multiple basis vectors leading to the name called multi-kernel based SBL approach [26]. By exploiting prior values, expected posterior values maximizing is the primitive aim of the research.

The method adopted for doing this is called EM. The main key parameter RMSE is calculated using the mean of residual errors and by taking the square root on it. This RMSE is formulated in (9). For a better understanding, the table of notations used for the analysis in the novel method is defined in Table 1.

$$\mathrm{RMSE} = \sqrt{1/\mathrm{K}\sum_{\mathrm{i}=1}^{\mathrm{K}}(\hat{y}_{\mathrm{i}} - y_{\mathrm{i}})^2} \tag{9}$$

Table 1. Table of notations with definitions

Notation	Definition
A	The manifold matrix
K	The narrowband far field incoming signals fall on receiving array from different directions
E	Maximum elements in an array
s (t)	The transmitted signals matrix
a	Steering vector in steering matrix
v	The basis function corresponding to a particular steering vector
d	Distance between antenna elements
n (t)	AWGN noise matrix at the element
u (t)	The received signals matrix at the array end
R_u	The covariance matrix of 'u'
G	Source covariance matrix
H	Channel noise covariance matrix
$\hat{y}_i$	The estimated DOA of the received signal
y_i	The true DOA of the received signal

3. VIRTUAL OPTIMAL ANTENNA RECONFIGURATION METHOD

Antenna locations are varied virtually to reduce the objective parameter RMSE to an optimal value by exploiting the CCSA. It is an efficient, easy, and iterative optimization approach followed in recent days. The best antenna location search is initialized with the guess position. The significant steps are proceeded with the random walks.

3.1. Conventional cuckoo search algorithm (CCSA)

The stochastic CCSA is one of the simple meta-heuristic algorithms employed to solve most optimization problems. It is a bio-inspired algorithm. The general nature of the cuckoo bird is that it does not build its own nest and manipulates the host bird by laying its egg in host nests. This caliber of cuckoo makes its younger generations succeed further. In comparison, cuckoo and host birds are considered as signals whereas nests are taken to be antenna element locations. Levy distribution is the sum of the small steps of the cuckoo bird. The goal is to find the best possible locations on replacing the old ones until the optimal RMSE value is reached. During the implementation, CCSA works on an outer loop and NNSBL is simulated as an inner loop to find the exact DOA. The differential distance is calculated over iterations until the least RMSE is reached. The flowchart includes the local refinements and global best finding as the significant steps in CCSA as shown in Figure 1.

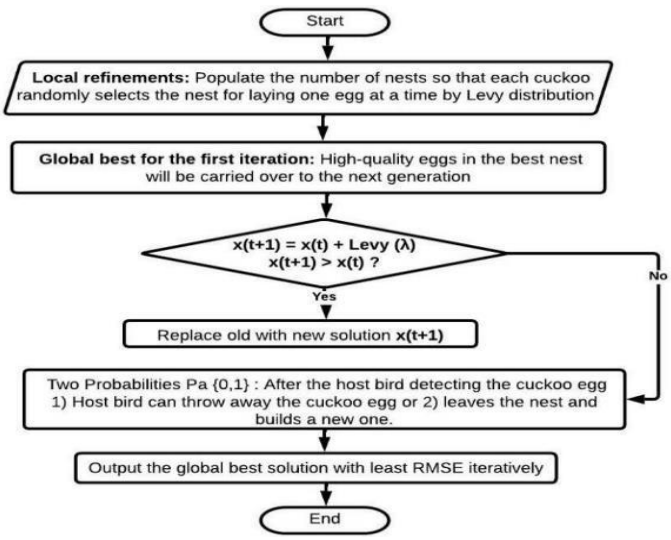

Figure 1. Flowchart of the CCSA method

4. RESULTS AND DISCUSSION

Exploiting the unique CCSA model with the proposed NNSBL framework resolves most of the sparse DOA estimation problems in a virtual manner. CCSA works on an outer loop and NNSBL is simulated as an inner loop to estimate the sharp DOA. The additive white gaussian noise (AWGN) is the noisy channel analyzed with a 1-degree grid difference for *'T'* snapshots. The simulations are performed using Matlab software. Thirteen NULA element positions are deployed at [1]–[10], [12]–[14] and are spaced by integer multiples of half the wavelength. The different key parameters considered for the simulation in terms of frequency of the carrier signal, number of antennas in the MIMO model, and predefined grid angles are shown in Table 2.

Table 2. Key simulation parameter of the proposed MIMO model

Key parameters	Configuration
Array pattern type	Non-uniform linear array
Number of receive antennas	13
Number of transmit signals	15
Range of antenna coverage	-70^0 to 70^0
Measuring bearing limit	-60^0 to 60^0
Frequency of carrier signal	200 MHz
Velocity of propagation	380 m/s
Transmit signals angles	-68.4^0 -55.1^0 -42.2^0 -30.2^0 -22.6^0 -10.4^0 -6.2^0 2.4^0 7.3^0 15.6^0 23.2^0 31.4^0 42.2^0 55.2^0 68.2^0

4.1. Various stochastic signals generations and its diversity

This section illustrates the generation of stochastic uncorrelated fifteen narrowband source signals falling on the receive sensor array. Uncorrelated signals limit the interference because their transmission time is very less than that of the correlation time. Real-time-varying signals are influenced when transmitted over the AWGN channel. Thus, the entire fifteen noise-influenced transmitted signals falling on the array further undergo beam rotating operation as per the steering matrix. Different signals show unique amplitude variations concerning time because of their random generation as shown in Figure 2.

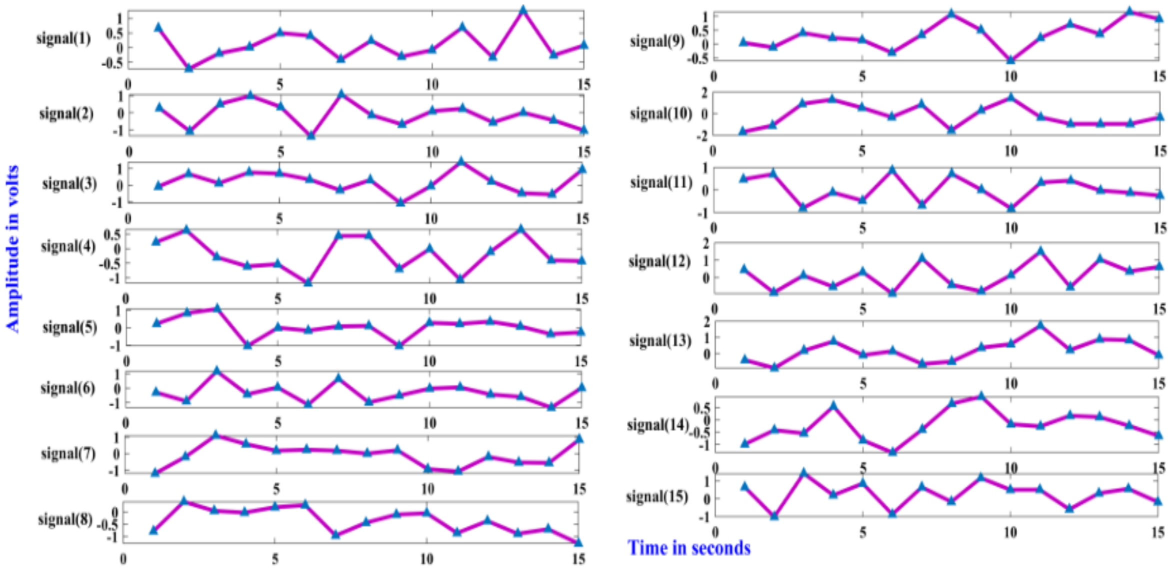

Figure 2. Various narrowband source signals generations at multiple sources

Time-varying AWGN signals affect the transmitted signals when sent over a MIMO sparse channel. There are about thirteen such kinds of noisy signals at receiving array. The range of a such noisy signal is taken from -20 dB to 30 dB. A well-known BPDN method in wireless communications is further employed to reduce the sparsity of the impinging signals on an array during the implementation.

The uniqueness of the employed MIMO sparse array is that it follows Khatri Rao criteria. Initially, in this new method, all the prior's values are assumed to be unknown. But, while implementing the MK NNSBL method entire prior values, differential distance, and the predefined grid is initialized to start the learning procedure in the machine. Later, by using the EM method these priors are increased to maximum posterior values.

The manifold matrix is also known as the steering matrix responsible for exact beam rotation operations. The generated unique manifold matrix is a row matrix containing the steering vectors of each element. Such thirteen steering vectors are formed by multiplying the basis vectors that depend on the differential distance, and wavelength. Stochastic steering vectors require the BPDN method for fine DOA peak estimation with low errors. Various stochastic signal generations and their diversity is implemented over 400 snapshots and 100 iterations for each SNR value. Thus, the framed manifold matrix with the multiplicative form of basis vector is called as multi-kernel approach and is represented in Figure 3.

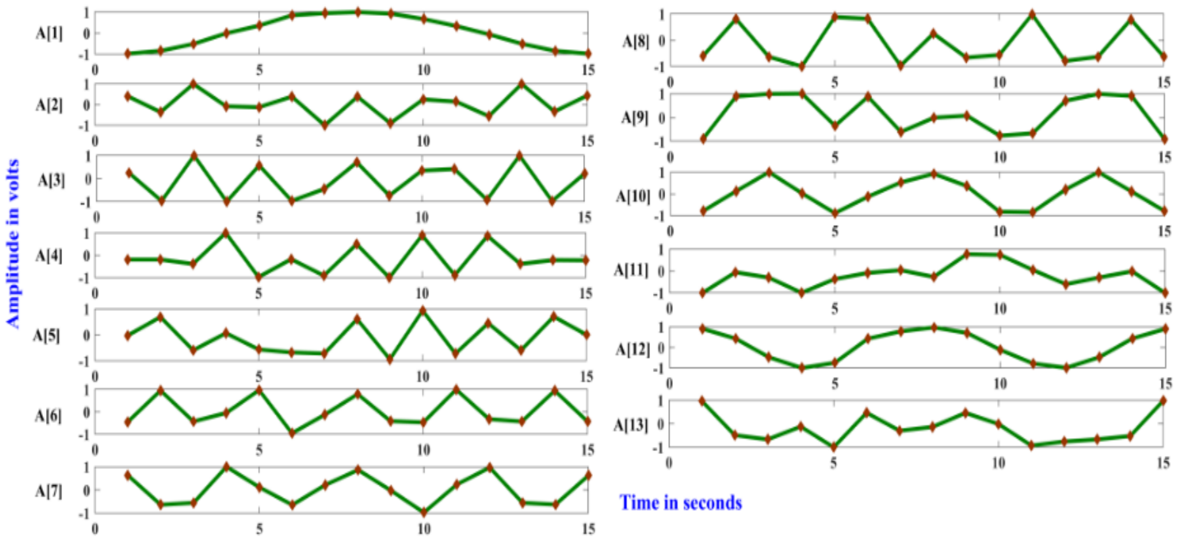

Figure 3. Manifold matrix "A" on combining all the multiplied basis vectors

4.2. Comparison of novel MK NNSBL DOA peaks over existing NNSBL DOA peaks estimation

This performance comparison experiment between the novel MK NNSBL DOA peaks estimation over existing NNSBL DOA peaks approximation is executed by assuming the uncorrelated signals. The basic NNSBL method working in a stochastic nature shows less sharpened DOA peaks in the normalized spatial spectrum. The reason is it is employed with only one basis vector so-called single kernel implementation. Since only positive or real prior values are used to find posterior values, it is termed a non-negative approach.

A 15×13 MIMO sparse wireless communication system model based on conventional NNSBL is compiled and the respective simulation plots for various SNR values are shown in Figure 4. By varying SNR in terms of -20, -10, 0, 10, 20, and 30 dB, the DOA estimation with fifteen incident signals on a thirteen elements antenna array is performed. And it is represented in Figures 4(a)-(f) respectively.

The proposed MK NNSBL algorithm works in a stochastic nature that varies multiplied basis vectors continuously for producing the overall manifold matrix. This method shows fine-sharpened DOA peaks in the normalized spatial spectrum because of multi-kernel implementation. The estimated output is the product of the manifold matrix vectors and the incident signals which are then added with AWGN signals.

A 15×13 MIMO sparse wireless communication system model based on MK NNSBL is compiled and the respective simulation plots for various SNR values are plotted in Figure 5. In the same way as NNSBL, by varying SNR in terms of -20, -10, 0, 10, 20, and 30 dB the DOA estimation for MK NNSBL is also performed. And it is given in Figures 5(a)-(f) respectively. From the simulation plots, it can be observed that the estimated DOA matches very well with the true DOA and shows very sharp peaks using MK NNSBL than traditional NNSBL.

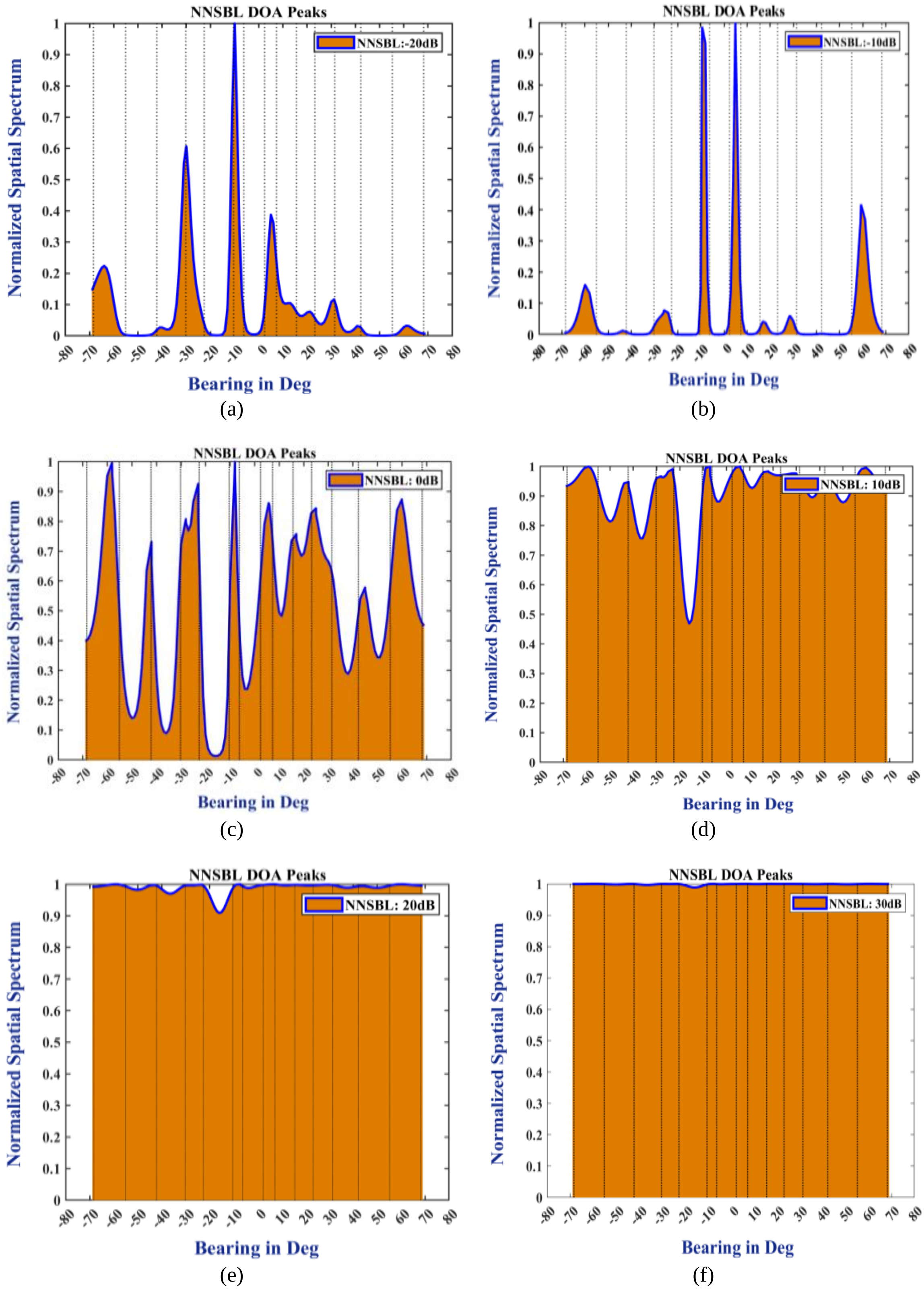

Figure 4. DOA estimation using conventional NNSBL for various SNR values: (a) -20 dB, (b) -10 dB, (c) 0 dB, (d) 10 dB, (e) 20 dB, and (f) 30 dB, where dotted vertical lines indicate actual DOAs, and brown plots are estimated ones

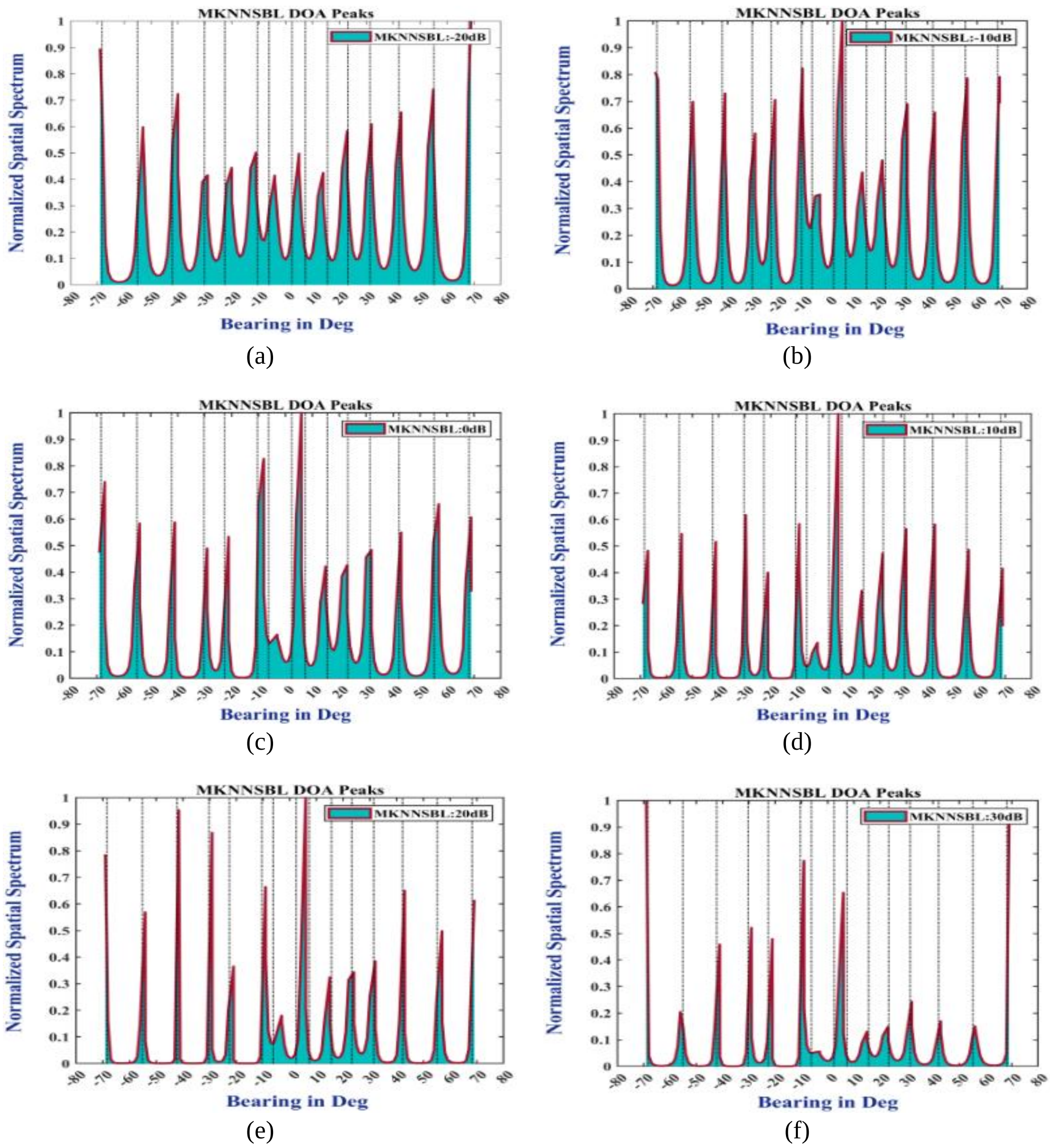

Figure 5. DOA estimation using multi-kernel NNSBL for various SNR values: (a) -20 dB, (b) -10 dB, (c) 0 dB, (d) 10 dB, (e) 20 dB, and (f) 30 dB, where vertical lines show true DOAs, and sharp blue peaks are estimated DOA ones

4.3. Convergence comparative graph

Performance evaluation of an underdetermined DOA estimation is validated in terms of the RMSE vs SNR convergence graph. CCSA method is exploited to find the optimized differential co-array distance for the antenna reconfiguration purpose in NULA because of its simplicity. Further, the SNR is varied from -20 to 30 dB and the RMSE is estimated for 15 iterations over different decibels in the defined range. RMSE vs SNR convergence plot for the conventional NNSBL and MK NNSBL DOA estimation is given in Figure 6. Overall, RMSE for the basic NNSBL algorithm is reduced from 2.9 to 2.31. Whereas RMSE for the proposed MK NNSBL algorithm is dominating with its values around 1.55 to 1.67.

Compilation time is the significant parameter in the MIMO model that represents the speed of the model in seconds. It depends on the computational complexity of the method adopted as well. The Table 3. discusses the compilation time needed for each algorithm’s execution for different SNR values from -20, -10, 0, 10, 20, to 30 dB, and that indicates the complexity in implementation. The simulation results show that the MK NNSBL algorithm with CCSA-based antenna reconfiguration takes less time and is having superior performance in finding fine DOAs.

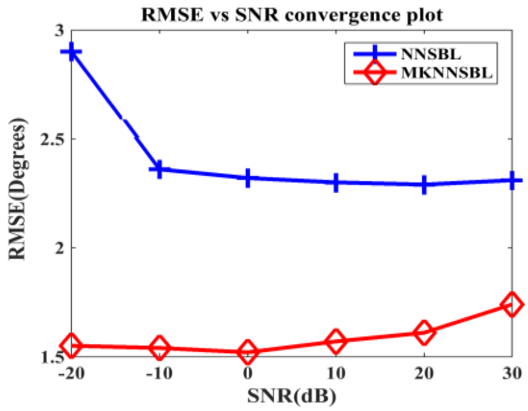

Figure 6. RMSE vs SNR convergence plot for conventional NNSBL and MK NNSBL

Table 3. Comparison of NNSBL vs MK NNSBL algorithm's compilation time

SNR (in decibels)	Compilation time of NNSBL (in seconds)	Compilation time of MK NNSBL (in seconds)
-20	2.9830	3.0800
-10	3.0514	3.5497
0	3.3812	3.3670
10	3.4306	3.3135
20	3.4055	3.2970
30	3.4180	3.2370

5. CONCLUSION

Based on the literature survey the existing DOA estimation methods have more computational complexity, low accuracy, more compilation time with memory, high RMSE for low SNR, and lower optimized results for NULA. To overcome all these issues, sparse signal recovery-based methods are employed. Two significant methods are proposed in this paper. Firstly, multiplicative basis vector formation within the manifold matrix for beam rotation as per need along with EM method. Secondly, the variation in the virtual antenna locations for fine impinging of incoming signals on an array by utilizing the simple CCSA. The improvement in DOA estimation accuracy is observed through MATLAB simulation plots. Based on these simulation plots it is found that the estimated DOA almost matches with actual values with sharp peaks. The convergence comparative graph of root mean square error versus various signal-to-noise ratio values proves that the novel algorithm dominates with its reduced error. Further, the complexity increases whereas compilation time decrease in the multi-kernel approach comparatively. Finally, the overall results show that there is a satisfactory improved performance in the proposed method when compared with the conventional one. And therefore, this novel method can be used for 5G massive MIMO array real-time applications.

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