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Fractal and Chaos Characteristics in Rock Milled Process

Cui Xin-Xia, Luo Chen-Xu*
School of Mechanical and Electrical Engineering, China University of Mining and Technology, Xuzhou 221116, China, tel: 86−15150028506, fax: 0086 8388 5830
*corresponding author, e-mail: xzhqingting@163.com

Abstract

In order to research the mechanism and to reveal the natural characteristics in rock milled process, the milling load was deemed to be a component of the deterministic nonlinear dissipation system. The characteristic factor model of milling load was built on the basis of time delay method, and the phase-space of milling load was rebuilt. the dimensional phase-type of broken attractor was described. The results indicated that the broken attractor was a fractal set, which acquired through the scale conversion of phase-space developed by each dimension. The relevant dimension of broken attractor can be as the identification to reflect the change of rock broken mechanism. And the Lyapunov exponential spectrum and the maximum Lyapunov exponent were acquired by confirming the system reconstruction dimension. The chaos phenomenon was existed in the rock milled process, which provides the basis for building the deterministic model of rock milled process.

Keywords*: Rock, Milling, Fractal, Chaos, Lyapunov exponent*

1. Introduction

There were remarkable dispersion and irregular characteristics in rock milled process, which caused by a lot of different scale flaws existed in rock. The structure of rock was neither perfect continuation, nor fully discrete. It made the works described by mathematics lose meaning for guiding production. The rock milled process was shown as Figure 1, the compound motion of rotary and linear was done when the milling mechanism working, the angle of cutting tools installed on it were different, and the stress state was very complex.

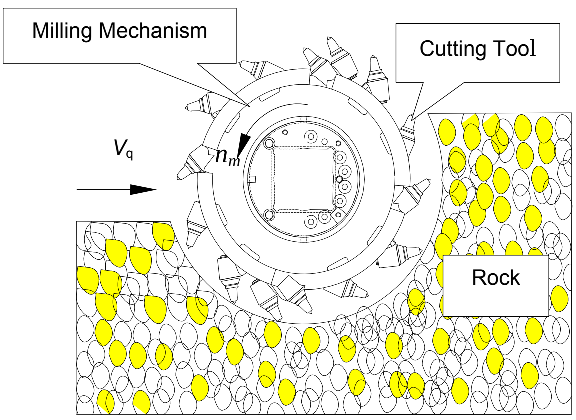

Figure 1. Rock milled process

The milling load was the actual record for the change when rock broken. For a long time, the scholars followed the research methods of statistic theory, and taking the milling load as a random signal.But, the reasons of the nonhomogeneity and irregularity existed in signal were not clarified by the classical signal analysis method. In recent years, the nonlinear dynamic

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system theory was developed rapidly, especially the fractal and chaos theory,which were used widely in disposing the seemingly no rules phenomenon [1~7]. Dan B. Marghitu [8] researched on the dynamics characteristics of the metal milled processing, indicated that the cutting force signal had chaos characteristics. Shu Karube [9] built the mechanical model of the vibration cutting system, indicated that the cutting load had nonlinear dynamic action, and had chaos characteristics. Primoz Potocnik [10] built the nonlinear dynamic model of the metal cut processing, indicated that the cutting load included high time nonlinear factors, which resulted the mechanics characteristics of cutting load is extremely complex. DUAN Xiong [11] researched on the dynamic characteristics of the water jet assisted cutting rock, indicated that there was chaos characteristics in the cutting load. XIA Yi-min [12~13] researched on the dynamic characteristics of the rock broken by spiral cutting method, indicated that the broken load has fractal characteristics. Basing on the above researches, the cutting load is taken as a deterministic signal in this paper, and the fractal and chaos methods are utilized to explain the broken mechanism of the rock.

2. Fractal Characteristic Factor Model Building

There are a lot of methods for copulating the fractal dimension, each has its advantages and disadvantages. The root mean square method provided in the literature [14] is used in this paper, which can representing the complexity of curve better and its physical meaning is more clear. The fractal dimension is related with the time series, the method as follow:

The time series $Z(\tau)$ has fractal characteristics and satisfies the Equation (1).

$$Z(\tau)-Z(\tau_0)=\zeta\left|\tau-\tau_0\right|^{2-D_f} \tag{1}$$

if $\tau_0=0$ and $Z(0)=0$, then the variance or the covariance of the time series is:

$$Var(\tau)=E\left[Z(\tau)-\overline{Z(\tau)}\right]^2 \sim \tau^{4-2D_f} \tag{2}$$

or

$$\sigma(\tau)=Var(\tau)^{1/2}=C\tau^{2-D_f} \tag{3}$$

where C -scale coefficient; τ -time-domain scale; D_f -fractal dimension.

The Equation (3) indicates that the relationship between the covariance and time-domain scale (τ) is power exponent, and the power exponent is correlate with the fractal dimension. According to the Equation (3), the digitized load curve can be treated as a time series, and the covariance $\sigma(\tau)$ is calculated with *n* time domain τ_i ($i=1,2,3,\cdots,n$). And the regression line of $\lg\sigma-\lg\tau$ can be calculated according to the Equation (3). The relationship between the slope (α) of $\lg\sigma-\lg\tau$ and the fractal dimension is:

$$D_f=2-\alpha \tag{4}$$

But, the Equation (4) is not established only the sampling length less than the relevance length of the curve, shown as Figure 2.

The fractal dimension D_f is the reflection of the complexity and irregularity of the curve. It is the measuring parameter of similarity; the scale coefficient *C* is the absolute measuring parameter of the curve. They couldn't reflect the characteristic of the curve alone. Therefore, in order to get the characteristic factor for representing the curve, the Equation (3) is transformed as follow:

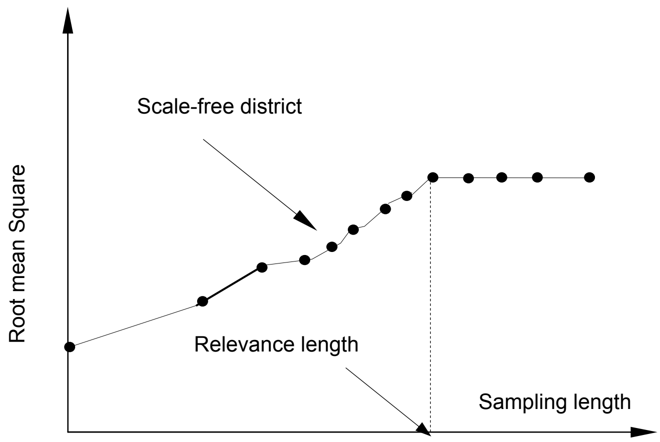

Figure 2. Relationship between scale-free district and relevance length

$$\lg\sigma(\tau)=\lg C+(2-D_f)\lg\tau \tag{5}$$

If $\sigma(\zeta)=1$, then :

$$\lg C+(2-D_f)\lg\zeta=0 \tag{6}$$

So, the characteristic factor (ζ) of milling load is:

$$\zeta=C^{-\frac{1}{2-D_f}} \tag{7}$$

3. Inversion of Chaos Motion

The extremely complicated chaos motion could be inversed by a very simple nonlinear dissipation dynamical system. For one *n* dimension dynamical system:

$$\frac{dx_i}{dt}=f_i(x_1,x_2,\cdots,x_n),i=1,2,\cdots,n \tag{8}$$

It is equivalent with one *n* order dynamical system:

$$x^{(n)}=f\left(x,\dot{x},\ddot{x},\cdots,x^{(n-1)}\right),x\in R \tag{9}$$

That is to say, if one time series is got, and the dimension of the phase space is known, the system dynamics can be reappeared by computing the each order derivatives of the time series. According to the time delay method, *m* time series could be constructed from one time series, and the dynamical system model could be rebuild, too. The process as follows [15~16]:

The original time series is discrete by equal time interval:

$$x_0=(w_0,w_1,w_2,w_3,\cdots,w_n,\cdots) \tag{10}$$

And *m* elements are delayed by the time interval $\tau = k\Delta t$ (*k* is integer). The new *m* dimension phase space is constructed by moving a shift-able window along the one dimension time series according to the time interval $\tau_c = (m-1)\tau$. The shift-able window can show *m* data. The τ insuring each vector of X_m is interdependence, but not depend on the m.

The delay window time τ_c is depended on the m, and changing with the τ. The constructed m dimension phase space is shown as follow:

$$X^m = \begin{cases} w_0, w_1, w_2, w_3, \cdots, w_{m-1} \\ w_\tau, w_{1+\tau}, w_{2+\tau}, w_{3+\tau}, \cdots, w_{m-1+\tau} \\ \vdots \\ w_{(n-1)\tau}, w_{1+(n-1)\tau}, w_{2+(n-1)\tau}, w_{3+(n-1)\tau}, \cdots, w_{m-1+(n-1)\tau} \\ \vdots \end{cases} \tag{11}$$

According to above, the confirmation of the delay time interval τ and the phase reconstruction dimension m are the main problem for the reconstruction process. if the value of τ is bigger, a lot of details information will be lost, and the data demanded for the original signal will be increased; if the value of τ is smaller, the value of X_i and $X_{i+\tau}$ will be not distinguish. X_i is a time series of the X^m, $i = 1, 2, 3, \cdots, M$, N is the length of the original series. The selecting principle of the τ is to reduce the relevance between the series in the Equation (11), and the information included in the dynamical system couldn't lost. The autocorrelation function method is better to solve the delay time τ, which basing on the linear relevance between the series. The autocorrelation function $C(\tau)$ of the discrete variable X_i is:

$$C(\tau) = \lim_{M\to\infty} \frac{1}{M} \sum_{i=1}^{M-1} X_i X_{i+\tau} \tag{12}$$

And $C(\tau)$ is the level of similarity between the two time series (i and $i+\tau$). When the value of X_i is fixed, a higher value of $C(\tau)$ indicates the level of similarity between X_i and $X_{i+\tau}$ is bigger. While the smaller value of τ indicates the level of similarity between X_i and $X_{i+\tau}$ is bigger, and the value of $C(\tau)$ is bigger, too. Otherwise, a higher value of τ indicates the difference between X_i and $X_{i+\tau}$ is bigger, and the X_i and $X_{i+\tau}$ will be uncorrelated completely in last; At the same time, the value of $C(\tau)$ will decrease until zero. From this, the value of τ when the $C(\tau)$ tends to zero is required. And the reconstitution dimension is confirmed according to the effect of it on the relevance dimension of the series in X^m. When the value of τ is fixed, the relevance dimension of the series will not change with the reconstitution dimension in X^m, and the value of reconstitution dimension is the required by this time.

The reconstitution dimension (*m*) can be gained from the relevance integral of the time series, the process as follows:

$$C(m,N,r,t) = \frac{2}{M(M-1)} \sum_{1\le i\le j\le M} \theta\left(r - d_{ij}\right), r > 0 \tag{13}$$

where $t=\tau$, $d_{ij}=\left\|X_i-X_j\right\|$; if $x<0$, then $\theta(x)=0$; if $x\geq 0$, $\theta(x)=1$; r is the relevance scale.

Definition of the relevance dimension is:

$$D(m,t)=\lim_{r\to 0}\frac{\log C(m,r,t)}{\log r} \tag{14}$$

and

$$C(m,r,t)=\lim_{N\to\infty}C(m,N,r,t) \tag{15}$$

But, the length of the time series is limited and the relevance scale (*r*) can't to zero. Based on this, the relevance dimension is replaced by the slope of a linear range:

$$D_C=D(m,t)=\frac{\log C(m,N,r,t)}{\log r} \tag{16}$$

when the m change and the D_C not to change, the gained m is the required reconstitution dimension. The reason is that when the m less than the required phase space dimension of the dynamic system, the information included in the time series can't reflect completely, which lead to the change of the relevance dimension (D_C) obviously with the change of the reconstitution dimension; When the m greater than the required phase space dimension of the dynamic system, the information included in the time series can be reflected completely by the reconstituted phase space; and the relevance dimension can't change with the m at this time, which reveals the boundedness of the phase space.

4. Characteristic of Milling Load

It is a irreversible process that the energy of the milling system acting on the rock transforms to the rock broken surface energy. So, the load signal of the rock acting on the milling system is dissipative. In order to research the characteristic of the milling load, the experiment is done. The experiment condition as follows: the diameter of milling mechanism is 530mm, the helical angle of vane is 20°, the impact angle of cutting tool is 45°, the cutting line distance is 30mm, the cutting width is 210mm, the rotary speed is 80r/min, the haulage speed is 1.5m/min. The milling load is shown in Figure 3. To research whether there is the fractal characteristic in milling load, the milling torque is analyzed according to the calculating method of the load characteristic factor. And the front five points are taken as the scale-free interval, shown as Figure 4.

It can be seen that, there is scale-free interval in milling load, and the relationship between the mean-square deviation logarithm and the time domain logarithm is linear in the given interval. That is to say there is obvious self-similar hierarchy structure, which indicates there is hierarchy structure in milling broken dynamical system.

In order to research the characteristic of the milling load, the experiment data will be dispersed by same interval and the phase space will be reconstituted according to the Equation (11). But, it can't be expressed when the phase space more than three-dimension. So, the three-dimension phase graph of the broken attractor and the two-dimension phase graph are given in Figure 5 - Figure 8.

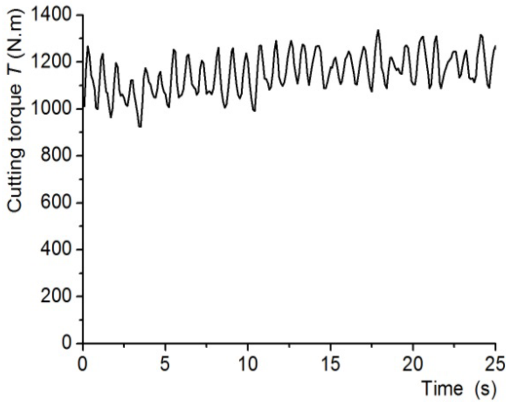

Figure 3. Experiment load

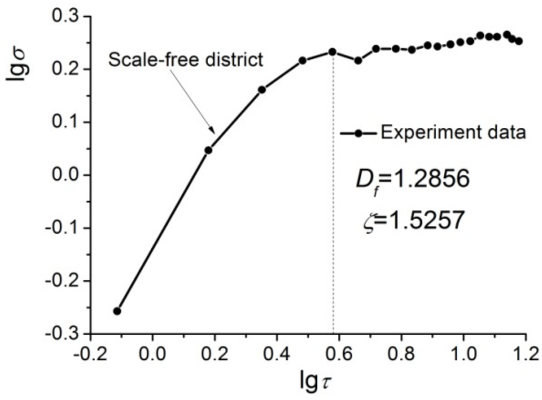

Figure 4. Scale-free district curve

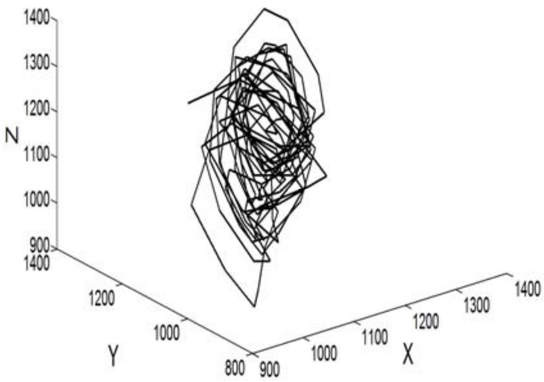

Figure 5. Three-dimension phase graph

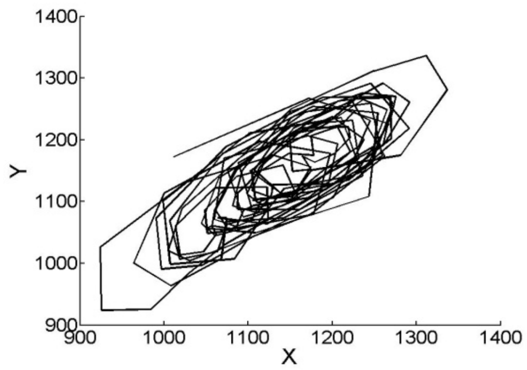

Figure 6. Two-dimension phase graph of X-Y plane

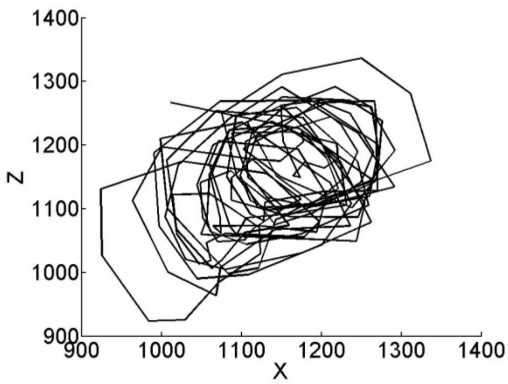

Figure 7. Two-dimension phase graph of X-Z plane

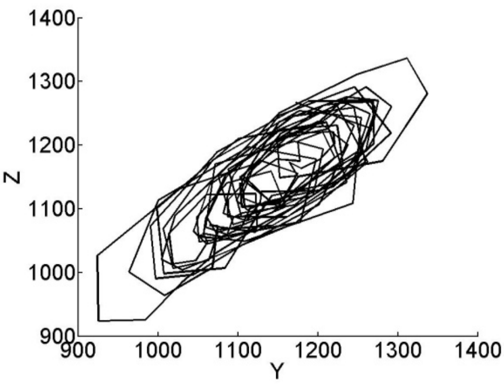

Figure 8. Two-dimension phase graph of Y-Z plane

It can be seen from the figures that the phase track of the broken attractor don't stop on a point or a limit cycle, which indicates the system isn't a simple damp motion or a periodic motion. Although the phase track seems to be a limit cycle, but the period of it is infinite and the track is never close. The phase track is always in a certain area and never repeats, and it repeats folding and crosses each other. So, it can be seen that there is the characteristic of strange attractor from the presentation appeared by the system attractor. But, this is only a superficial phenomenon. if we want to make clear the characteristics of the milling system, the Lyapunov exponential spectrum and the max Lyapunov exponent should be researched.

In order to acquire the Lyapunov exponential spectrum and the max Lyapunov exponent, the reconstitution dimension *m* is needed to be confirmed. Therefore, the relevance dimension graph under different reconstitution dimension is acquired, which basing on the experiment data in Figure 3 and according to the computing principle of the Lyapunov exponent in the literature [19-20]. The relation curve of $\log\theta \sim \log r$ is given when the $m=3$ and the *Dc*=1.4299, shown as in Figure 9. It can be seen from Figure 9 that the relation between the logarithm of relevance integral and the logarithm of relevance scale is about linear, which indicates the system has self-similar structure. And the relationship between the relevance dimension *Dc* and the reconstitution dimension *m* is shown in Figure 10. When the reconstitution dimension *m*=6, the changing of the relevance dimension *Dc* is gently. According to the analysis about the relationship between the *Dc* and the *m*, the signal included in the system can be reflected completely when the *m*=6.

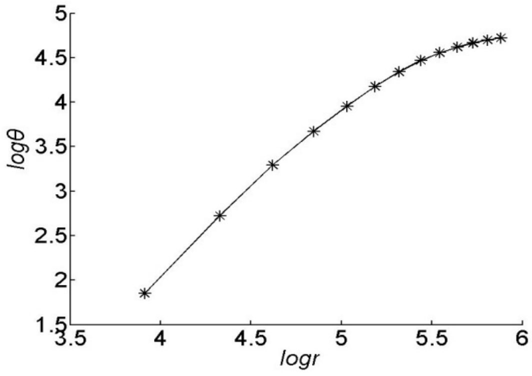

Figure 9. Relationship between logθ and log$r$ (m=3)

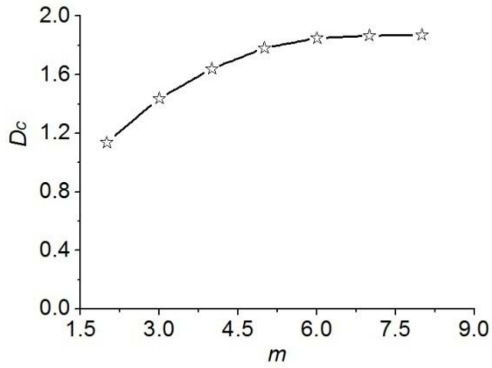

Figure 10. Relationship between D_C and m

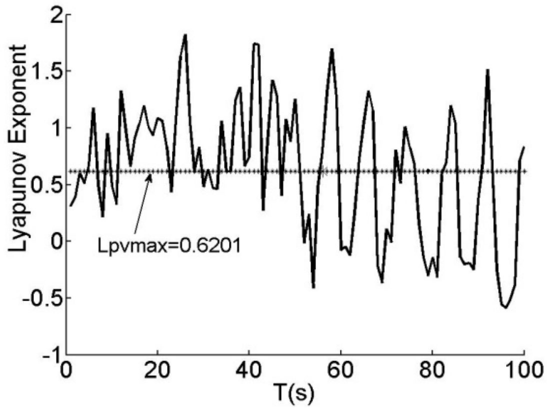

Figure 11. Lyapunov exponential spectrum

Basing on the determining of the system reconstitution dimension (*m*), the Lyapunov exponential spectrum and the max Lyapunov exponent are gained, shown as Figure 11, and the Lpvmax instead of the max Lyapunov exponent in Figure 11. The value (0.6201) of the max Lyapunov exponent is bigger than zero, which indicates there is chaos character in the milling torque. According to the above analysis, there is fractal and chaos characteristics in the rock milled process.

5. Conclusion

(1) Basing on the fractal theory and utilizing the root mean square method, the characteristic factor of milling load is built:

$$\zeta = C^{-\frac{1}{2-D}}$$

According to this equation, the rock with different characteristics can be distinguished.

(2) Basing on the experiment data, the phase space is reconstituted, and the strange attractor phase graph of milling load is acquired.

(3) Simultaneously, the Lyapunov exponent of milling load is got basing on the system reconstruction dimension, and the Lyapunov exponential spectrum and the max Lyapunov exponent are acquired.

(4) The researching results indicate that the milling system has fractal and chaos characteristics.

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