TELKOMNIKA Indonesian Journal of Electrical Engineering
Vol.12, No.1, January 2014, pp. 487 ~ 495
DOI: http://dx.doi.org/10.11591/telkomnika.v12i1.3577

Takagi-Sugeno Fuzzy Model Identification for Small Scale Unmanned Helicopter

Arbab Nighat Khizer*[1], Dai Yaping[2], Syed Amjad Ali[3], Xu Xiang Yang[4]
[1, 2, 4] School of Automation, Beijing Institute of Technology, Beijing 100081, P. R. China
[3]School of Mechatronics, Beijing Institute of Technology, Beijing 100081, China
[1, 3]Mehran University of Engineering and Technology, Jamshoro, Sindh, Pakistan
*Corresponding author, e-mail: arbabnighat@gmail.com

Abstract

This paper presents a system identification method for small unmanned helicopter based on Takagi Sugeno fuzzy models under hover. Firstly, the nonlinear discrete model of small unmanned helicopter with unknown parameters is determined by Takagi Sugeno system. Secondly, derivative based gradient method is used to identify the unknown parameters of TS fuzzy model because in gradient adaptation, fuzzy system is not supposed to be linear in the parameters, so all fuzzy sets for input and output could be adjusted. The proposed method showed its effectiveness in terms of data matching obtained by the X-Plane© flight simulator. Obtained simulation results show high accuracy of the modeling method and better justification for real time applicability of the approach.

Keywords: *Small unmanned helicopter (SUH), Modeling, Identification, Takagi-Sugeno fuzzy system, Gradient method.*

1. Introduction

Controlling of a small scale unmanned helicopter is more challenging than its full scale counterpart because of its complex system which exhibits a high degree of nonlinearity and instability [1]. Taking off and landing vertically is the ability of SUH makes it more successful in civilian and military applications, such as surveillance, monitoring and data acquisition. Its unique features such as lower flight speed and hovering proves more suitable for challenging environment [2]. The SUH is typical a nonlinear system having 6DoF and four control inputs. Due to cross-coupling and time-varying ability, it is considered as unstable system [3]. Therefore building of an accurate and stable model for SUH is great challenge to the researchers [4]. During recent years, major research work has been done for establishing a simple model, which has ability to cover the complexity of helicopters. Two methods are generally used for system modeling and identification, the first principle method and system identification method. In the first method many unknown parameters are required to estimate. Therefore this method is suitable for full scale helicopters but as far as small scale helicopters are concerned, this method is highly unsuitable [5-6]. Second method is system identification method which further divides into time domain and frequency domain techniques. Mettler used frequency system identification techniques (Comprehensive Identification from FrEquency Responses (CIFER)) for establishing a linear state space model at hover for small helicopter system [7]. For identification of dynamic model process the time-domain system identification is often suitable. Under this assumption, a helicopter can be treated as a single input and single output system with three independent SISOs. A 3DoF helicopter was modeled by a new autoregressive (AR) model using the Least Squares Estimation (LSE) [8]. Artificial Intelligence (AI) like neural network was also used for the identification and control of nonlinear systems [9-11]. The modeling method by neural network is usually complex method and could not be used to examine the characteristics of the dynamics.

In this paper, a nonlinear discrete model for small unmanned helicopter is derived. The discrete velocity dynamics are obtained using nonlinear equation of motion of helicopter. A Takagi-Sugeno (TS) model is then developed based upon the translational velocity dynamics. To estimate the unknown parameters of TS fuzzy system, a gradient method is used due to its simplicity, accurate learning and quick convergence of all fuzzy rule base parameters including

Received June 19, 2013; Revised August 22, 2013; Accepted September 15, 2013

membership function. The frequency sweep input excitation signal is used as input because of its adequate acceptance in flight vehicle identification process. The X-plane simulator is used to obtain experimental data and to verify the modeling results. This commercially available simulator gives a better indication of the approach applicability in real-time flight applications. The verification result shows the successful potential of proposed approach in terms of simple and accurate identification methodology.

The paper is organized as follows: Section 2 described system dynamics including coordinates and frames, rigid body dynamics along with main rotor and tail rotor force generation. Lastly the section covers the complete nonlinear discrete small unmanned helicopter model. Section 3 described Takagi-Sugeno Fuzzy Models in detail. Section 4 discussed the proposed Takagi-Sugeno fuzzy system for nonlinear helicopter model and frequency sweep input used to excite the identification process. Section 5 shows how gradient method is used to estimates the unknown parameters of TS fuzzy model. Section 6 showed simulation results. Finally section 7 states the conclusion remarks.

2. System Dynamic Modeling

2.1. Coordinates and Frames

Equations of motion for model helicopter based on rigid body dynamics, basic aerodynamics and helicopter theory is derived [12-13]. Two frames (reference inertial frame and body-fixed frame) are considered as shown in Figure 1.

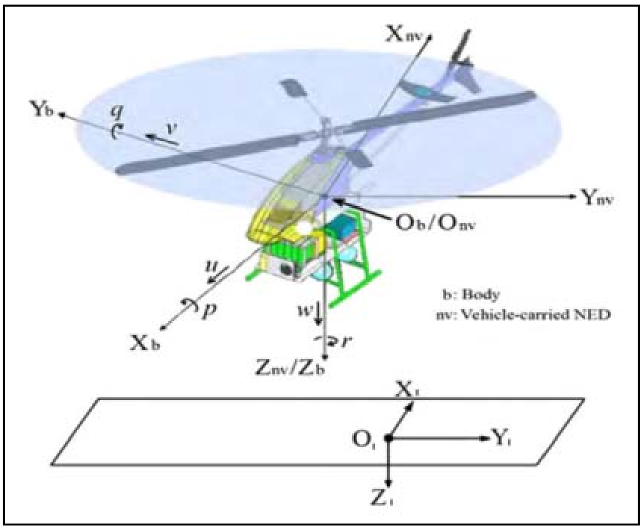


Figure 1. The body fixed frame and inertial frame coordinates

2.2. Rigid Body Equations

Rigid body equation of motion can be described in above two reference frames. Both frame are defined as $\mathcal{F}_I = (O_I, \vec{\imath}_I, \vec{\jmath}_I, \vec{k}_I)$ and $\mathcal{F}_B = (O_B, \vec{\imath}_B, \vec{\jmath}_B, \vec{k}_B)$. In Figure 1 $\vec{v}$ represents the linear velocity vector and its coordinate vector is $v^B = [u\, v\; w]^T$ w.r.t. body fixed frame and $\omega^B = [p\; q\; r]^T$ is the angular velocity w.r.t. body-fixed frame. The forces and moments ($\vec{f}$ and $\vec{\tau}$) acting externally on the fuselage are sum up as vectors. The $f^B = [X\, Y\, Z]^T$ represents force vector components and $\tau^B = [L\, M\, N]^T$ represents the component of the torque vector w.r.t. body-fixed frame. The Newton-Euler equation of motion becomes [14]:

$$\begin{bmatrix} mI_{3\times 3} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{v}^B \\ \dot{\omega}^B \end{bmatrix} + \begin{bmatrix} \omega^B \times mv^B \\ \omega^B \times I\omega^B \end{bmatrix} = \begin{bmatrix} f^B \\ \tau^B \end{bmatrix} \tag{1}$$

where "I" is the inertial matrix w.r.t. body-fixed reference frame and the mass of the helicopter is denoted by "m". Finally the external aerodynamic forces and moments are represented by the following equations and descriptions of variables are given in Table-1:

$$f^B = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_m + X_f \\ Y_m + Y_t + Y_v + Y_f \\ Z_m + Z_h + Z_f \end{bmatrix} + R_{IB}^T \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \quad (2)$$

$$\tau^B = \begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} L_m + L_t + L_v + L_f \\ M_m + M_h + M_f \\ N_m + N_t + N_v + N_f + Q_m \end{bmatrix} \quad (3)$$

The complete rigid body dynamic equations become:

$$\dot{v}^I = \frac{1}{m} R_{IB} f^B \quad (4)$$

$$I\,\dot{\omega}^B = -\,\omega^B \times (I\,\omega^B) + \tau^B \quad (5)$$

2.3. Complete Small Unmanned Helicopter Model

The main rotor thrust vector and tail rotor thrust vector, fuselage drug, damping forces of the stabilizers and gravitational force are considered as main force production for the helicopter. Four control inputs are observed and defined as $\delta = [\delta_{col}, \delta_{ped}, \delta_{lon}, \delta_{lat}]^T$, where δ_{col}, δ_{ped} is the main and tail rotor collective control. δ_{lon}, δ_{lat} is the longitudinal and lateral control of the helicopter. The TPP is defined by a and b angles which show the TPP tilt at the longitudinal axis and lateral axis as shown in Figure 2.

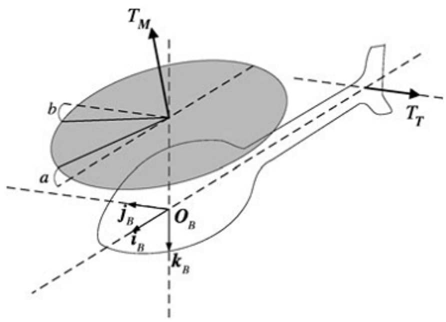


Figure 2. Tip-Path-Plane (TTP) angles & Thrust vectors T_M and T_T

The dynamic equations of TPP are given in [15] as:

$$a = -q - \frac{a}{\tau_f} + \frac{A_{lon}}{\tau_f}\delta_{lon} \quad (6)$$

$$b = -p - \frac{b}{\tau_f} + \frac{B_{lat}}{\tau_f}\delta_{lat} \quad (7)$$

where A_{lon} and B_{lat} are effective steady state lateral and longitudinal gains from the cyclic inputs to main rotor flap angles; δ_{lon} and δ_{lat} are the longitudinal and lateral control inputs. The term "τ_f" denotes main rotor time constant. The main rotor thrust and tail rotor thrust magnitude is proportional to the collective control commands (δ_{col}, δ_{ped}):

$$T_M = K_M\ \delta_{col} \text{ and } T_T = K_T\ \delta_{ped} \quad (8)$$

where T_M is a main rotor force and T_T is tail rotor force magnitude respectively. K_M & K_T are constant parameters. The main rotor thrust vector and tail rotor thrust vector is normal to the TPP, therefore thrusts of main and tail rotors can represented as:

$$T_M = \begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} = \begin{bmatrix} -S_a C_b \\ C_a S_b \\ -C_a C_b \end{bmatrix} T_M \approx \begin{bmatrix} -a \\ b \\ -1 \end{bmatrix} T_M \text{ and } T_T = \begin{bmatrix} 0 \\ Y_T \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} T_T \tag{9}$$

Longitudinal and lateral forces effects due to TPP tilt and effect of the tail rotor thrust on the translational dynamics are neglecting [16]. Therefore, force equation becomes:

$$f^B = \begin{bmatrix} 0 \\ 0 \\ -T_M \end{bmatrix} + R_{IB}^T \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \tag{10}$$

And complete torque vector will be:

$$\tau^B = \begin{bmatrix} R_M \\ M_M \\ N_M \end{bmatrix} + \begin{bmatrix} y_m Z_M - z_m Y_M + z_t Y_T \\ z_m X_M - x_m Z_M \\ x_m Y_M - y_m X_M + x_t Y_T \end{bmatrix} \tag{11}$$

substituting above equations, a more solid form of the torque could be achieved as:

$$\tau^B = A\, v_c + B\, \delta_{col} \tag{12}$$

where $v_c = (\delta_{lon}\delta_{col} \;\; \delta_{ped} \;\; \delta_{lat}\delta_{col})^T$ with $A \in \mathbb{R}^{3\times3}$ and $B \in \mathbb{R}^{3\times1}$. Therefore nonlinear equation of motion accompanied by a simplified model is expressed as:

$$\dot{v}^I = -\frac{1}{m} R_{IB} e_1\, T_M + g e_1 \text{ with } \{e_1 = [0\ 0\ 1]^T\} \tag{13}$$

$$I\, \dot{\omega}^B = -\,\omega^B \times (I\omega^B) + A\, v_c + B\, \delta_{col} \tag{14}$$

2.4. Nonlinear discretized Equation of Motion (EoM) for Translational Velocity

The nonlinear discretized EoM can be obtained using Euler's backward method, which stated as [$y_{n+1} = y_n + \Delta t\, f(y_{n+1}, t_{n+1})$]. Therefore, Equation-15 is obtained as:

$$v_{n+1}^I = v_n^I + \Delta t \left(-\frac{1}{m} R_{IB,n} e_1\, T_M\right) + \Delta t\,(g e_1) \tag{15}$$

By putting T_M, the above equations can simplify as:

$$v_{n+1}^I = v_n^I + \mathcal{A}_1 R_{IB,n} e_1\, \delta_{col,n} + \mathcal{A}_2 e_1 \tag{16}$$

where $\mathcal{A}_1 = -\Delta t\, \frac{K_M}{m}$ and $\mathcal{A}_2 = \Delta t\, g$.

3. Takagi-Sugeno (TS) Fuzzy Models

The model suggested by Takagi [17-18] has gained increasing interest in theoretical analysis and applications of fuzzy modeling and control. Takagi Sugeno (TS) fuzzy system is known as "functional fuzzy system" because output is a function rather than a fuzzy proposition. IF...THEN rules for reasoning is the base structure of TS models, in which antecedents (**If** part of the rule) are fuzzy sets and consequents (**Then** part of rule) are linear functions. Due to this arrangement, a complex affine nonlinear system could be approximated by TS fuzzy model. It can be used for best estimation $(y\,(k+1))$ of the state variable $y(k)$ at each time instant (k) given the inputs to the fuzzy system. Consider a TS fuzzy system with R rules of the form:
R_i: **If** $y(k)$ is $A_{i,1}$ and $y(k-1)$ is $A_{i,2}$ and ……. and $y(k-n+1)$ is $A_{i,n}$ and $u(k)$ is $B_{i,1}$ and $u(k-1)$ is $B_{i,2}$ and ……. and $u(k-m+1)$ is $B_{i,m}$

Then $y_i\,(k+1) = \alpha_{i,1}\,y(k) + \alpha_{i,2}\,y(k-1) + \ldots\ldots + \alpha_{i,n}\,y(k-n+1) + \beta_{i,1}\,u(k) + \beta_{i,2}\,u(k-1) + \ldots\ldots + \beta_{i,m}\,u(k-m+1)$
where $y(k)\ldots\ldots, y(k-n+1)$ and $u(k),\ldots\ldots, u(k-m+1)$ are present and past plant outputs and inputs. After center average de-fuzzification, the estimated output is:

$$y\,(k+1) = \frac{\sum_{i=1}^{R} y_i\,(k+1)\,\mu_i(y(k),\ldots\ldots,y(k-n+1),u(k),\ldots\ldots,u(k-m+1))}{\sum_{i=1}^{R} \mu_i(y(k),\ldots\ldots,y(k-n+1),u(k),\ldots\ldots,u(k-m+1))} \tag{17}$$

where $\mu_i(y(k),\ldots\ldots, y(k-n+1), u(k),\ldots\ldots, u(k-m+1))$ is the premise membership value of Rule i^{th} .
Therefore, estimated output can be re-written as:

$$y\,(k+1) = \xi^T(k)\,\theta_{TS} \tag{18}$$

where $\xi^T(k) = [(y(k), y(k-1)\ldots\ldots, y(k-n+1), u(k), u(k-1)\ldots\ldots, u(k-m+1)]$ and $\theta_{TS}^T = [\alpha_{1,1},\ldots.\alpha_{R,1},\alpha_{1,2},\ldots\alpha_{R,2},\ldots.\alpha_{1,n},\ldots.\alpha_{R,n},\beta_{1,1},\ldots.\beta_{R,1},\beta_{1,2},\ldots\beta_{R,2},\ldots.\beta_{1,m},\ldots.\beta_{R,m}]$.
$\xi(k) \in R^{n+m}$ is regression vector and θ_{TS} is the Takagi Sugeno parameter vector.

4. Proposed Takagi-Sugeno model for Translational Velocity dynamics

4.1. TS Fuzzy Rule Base

TS fuzzy model is developed using nonlinear velocity dynamic Equation-15, which represents the translational velocity of small helicopter system. Before developing the fuzzy model, some simplification is assumed to facilitate the identification process such as neglect the secondary forces produced by TTP tilt. The proposed TS fuzzy system for translational velocity dynamics is composed of R_1 rules, with i^{th} being as:

If (F_u^j and F_v^l and F_W^m) **Then**

$$\begin{aligned} u(k+1)i &= u(k) + a_1^i\,[\sin\phi(k)\sin\psi(k) + cos\phi(k)\sin\theta(k)cos\psi(k)]\mu_{col}(k) \\ v(k+1)i &= v(k) + a_1^i\,[\sin\phi(k)\cos\psi(k) - cos\phi(k)\sin\theta(k)sin\psi(k)]\mu_{col}(k) \\ w(k+1)i &= w(k) + a_1^i\,[\cos\phi(k)\,cos\theta(k)]\mu_{col}(k) + a_2^i \end{aligned} \tag{19}$$

where F_u^j, F_v^l and F_w^m are fuzzy set representing the linguistic variables values. The parameters of TS fuzzy model are unknown and can be evaluate through gradient based algorithm.

4.2. TS Fuzzy Membership Function

Consider the Gaussian membership function equally distributed with adjustable centers "c^i" and spreads "σ^i" on each input universe is given by Table-1.

Table 1. Initial Gaussian Membership function

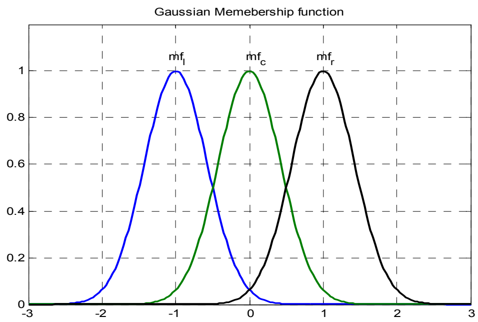
<table>
<tr><th>Mathematical Presentation</th><th>Graphical Presentation</th></tr>
<tr><td>

$$\mu_r^i(x_r) = \exp\left[-\frac{1}{2}\left(\frac{x_r - c_r^i}{\sigma_r^i}\right)^2\right]$$

$$\mu_l^i(x_l) = \exp\left[-\frac{1}{2}\left(\frac{x_l - c_l^i}{\sigma_l^i}\right)^2\right]$$

$$\mu_c^i(x_c) = \exp\left[-\frac{1}{2}\left(\frac{x_c - c_c^i}{\sigma_c^i}\right)^2\right]$$

where subscript r, l and c represents right, left and centre respectively.
</td><td>



</td></tr>
</table>

4.3. Frequency Sweep Input

This is the crucial part of identification method because input signal must have the capability of exciting the system modes that are required to appear in the identified model and secondly design of the input signal is important for collecting the experimental data. A wide variety of frequency excitation signal for aircraft system identification can be found in [19]. A frequency sweep signal is selected for this identification process, which is simply a sinusoidal signal with variable frequency, increases logarithmically with time. The amplitude of frequency sweeps are not kept constant due to position sustainability of helicopter around at certain operating condition. The frequency sweep input signal requires prior knowledge of frequency bandwidth. According to [20], rotorcraft identification requires frequency bandwidth lies between 0.3-12 rad/sec. The frequency excitation input signal is given by:

$$u = A\,sin[\varphi(t)],\ \varphi(t) = \int_0^{T_{rec}}[\omega_{min} + K(t)(\omega_{max} - \omega_{min})]\,dt\ ,\ K(t) = C_2\left[exp\left(\frac{C_1 t}{T_{rec}}\right) - 1\right]$$

The suggested value for $C_1 = 4.0$ and $C_2 = 0.0187$ are found in [20].

5. Gradient Method for Parameter Estimation

A gradient descent method proves better adequacy in terms of determining the unknown parameter of TS fuzzy system [21]. Gradient method is used to find the minimum of the objective function by training the parameters of the TS fuzzy system [22].

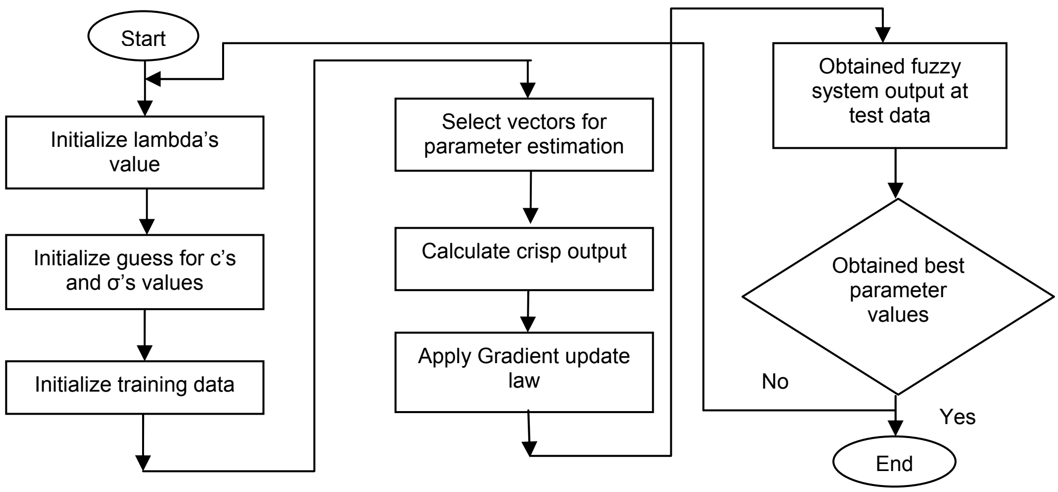


Figure 3. Gradient based training algorithm for Parameter Estimation

Using above gradient based algorithm, the final membership function for translational velocities (u, v and w) are found and shown by the Figure 4.

6. Simulation Results

X-plane simulator is used for this experimental work due to its realistic and powerful approach. The helicopter in X-Plane© simulator shows the resemblance to an actual small model excepting in yaw dynamics because a gyro mechanism is not included in computer model. The frequency sweep signals initialized when the small helicopter is set at hover mode.

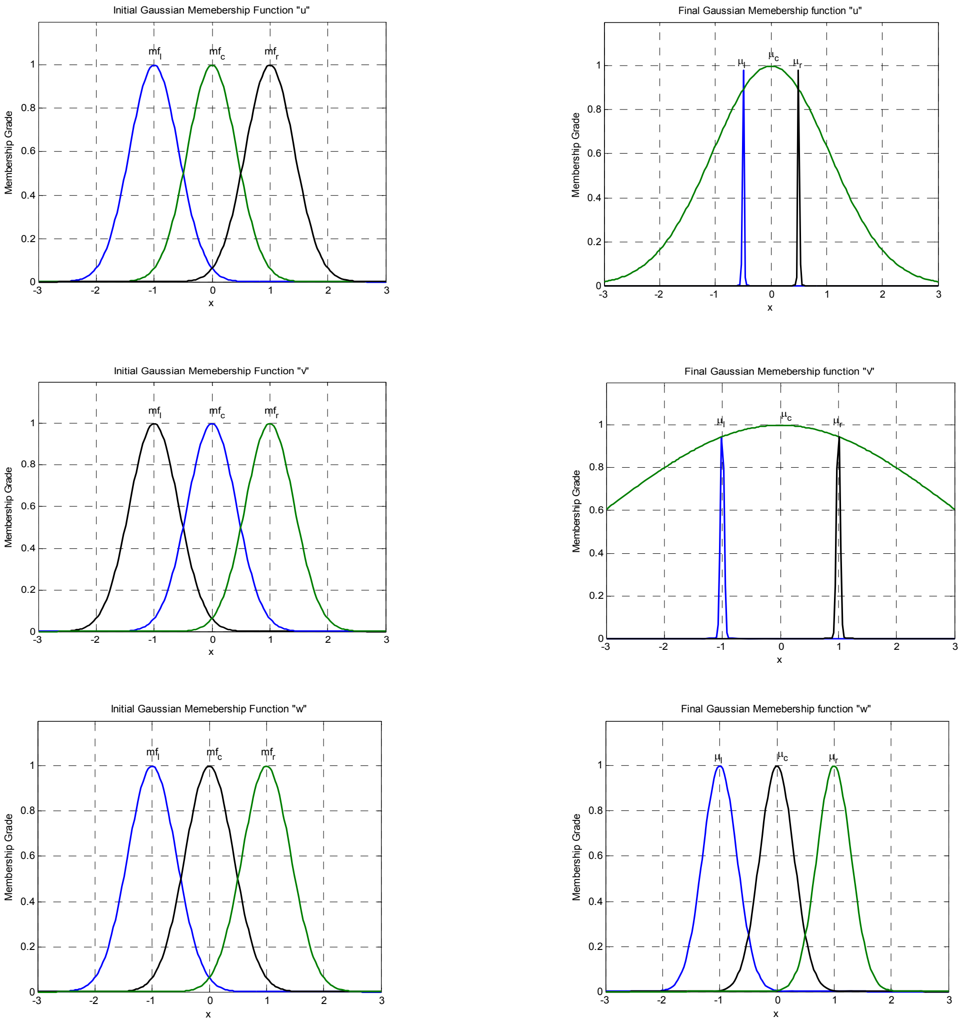


Figure 4. Initial and Final Membership function for Translational velocities

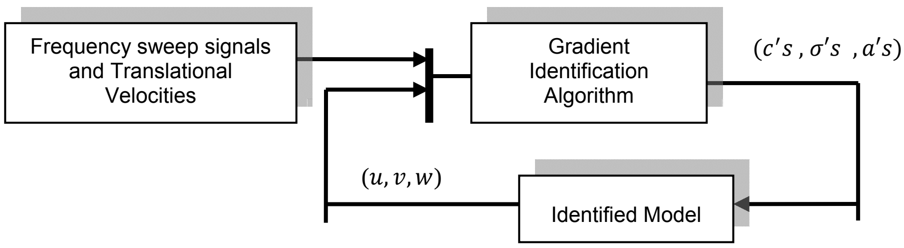


Figure 5. Simulation loop for Identification Process

Figure 5 showing the simulation loop for system, where identification algorithm is programmed in Matlab. The model is built in Simulink and executed in real time. For verification of model, the helicopter is set at hover state, then frequency sweep signals (control commands) are applying periodically that perturb the helicopter to a new hover position until a new input is excited. The estimated and actual translational velocities can be observed by Figure 6. The solid line shows actual model whereas dotted line represents the estimated model.

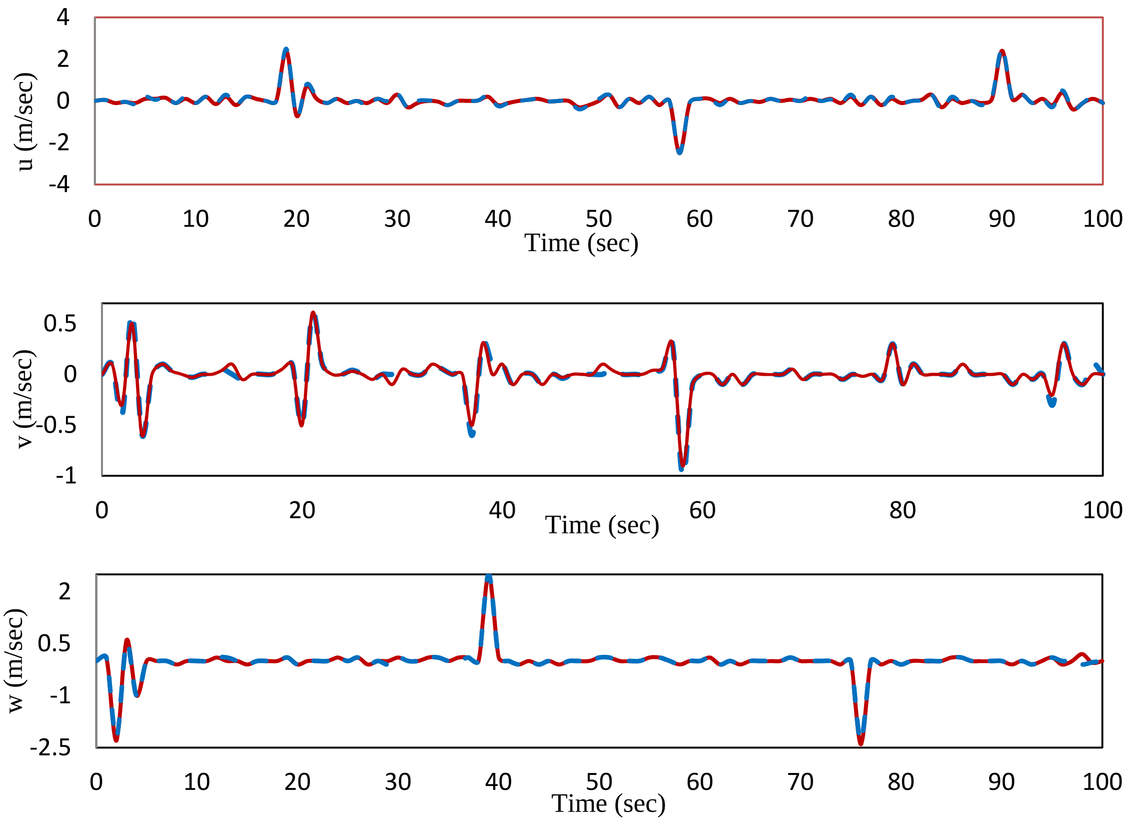


Figure 6. Comparison between Actual and Estimated Translational Velocities

The above simulation results show the successful applicability of the proposed identification method. The identified model using TS fuzzy system and actual model values matched better for translational velocities. It is clear from figures that the proposed identification process is very effective and quick.

7. Conclusion

The main contribution of this paper is to identify the nonlinear dynamic behavior of small unmanned helicopter using Takagi Sugeno fuzzy model. The input signal selected for this proposed method is frequency sweep signals. The proposed TS model proves better to encapsulate the dynamical behavior of SUH at hover state. The produced TS model is relatively simple nonlinear discrete system, which further facilitates controller design. The proposed modeling method has simple structure therefore it can easily be implemented. The simulation results show the effectiveness and accuracy of the modeling method. Derivate based gradient method is used for parameter estimation because in gradient adaption, both input and output fuzzy sets could be adjusted as desired. Future research work will address the design of fuzzy controller which stabilizes the identified model at hovering and at low velocities.

References

[1] B Song, JK Mills, Y Liu, C Fan. Nonlinear Dynamic Modeling and Control of a Small-Scale Helicopter. *Int. J. Control Automation System*. 2010; 8(3): 534-543.
[2] de Voogt AJ, Uitdewilligen S, Eremenko N. Safety in high-risk helicopter operations: The role of additional crew in accident prevention. *Safety Science*. 2009; 47: 717-721.
[3] Marconi L, Naldi R Robust. Full degree-of-freedom tracking control of a helicopter. *Automatica*. 2007; 43: 1909-1920.

[4] Avila JC, Brogliato B, Dzul A, Lozano R. Nonlinear modeling and control of helicopters. *Automatica*. 2003; 39: 1584-1596.
[5] R Heffley and M Mnich. Minimum complexity helicopter simulation mathematical model. *Technical Report 87-A-7, NASA, Moffett Field, CA*, 1987.
[6] Yidong Yang. Flight Control for Helicopter. *National Defense Industry Press*. 2007.
[7] Bernard Mettler, Tischler Mark B, Kanade Takeo. System identification modeling of a small scale unmanned rotorcraft for flight control design. *Journal of the American Helicopter Society.* 2002.
[8] Park D, Park MS, Hong SK. *A study on the 3-DOF attitude control of free-flying vehicle*. IEEE International Symposium on Industrial Electronics. Pusan, Korea. 2001; 2: 1260-1265.
[9] De Jesus Mota, Sandrine, Botez, Ruxandra Mihaela. *New identification method based on neural network for helicopters from flight test data*. AIAA Atmosphere Flight Mechanics Conference, 2009.
[10] Lin Niu, Junsheng Li. Adaptive Neural Network Generalized Predictive Control for Unknown Nonlinear System. *TELKOMNIKA Indonesian Journal of Electrical Engineering.* 2013; 11(7): 3611-3617
[11] You J, Yang QM, Lu JG. Nonlinear MPC Design for Identified Nonlinear Parameter Varying Model. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2012; 10(3): 514-523.
[12] Murray RM, Zexiang L, Sastry S. A Mathematical Introduction to Robotic Manipulation. CRC, Boca Raton, 1994.
[13] G Padfield. Helicopter flight dynamics. Blackwell Science, Oxford, 1996.
[14] W Johnson. Helicopter Theory. Dover, New York, 1980.
[15] V Gavrilets. Autonomous aerobatic maneuvering of miniature helicopter. Ph.D. Dissertation: MIT. USA: Boston, 2003.
[16] A Isidori, L Marconi, A Serrani. Robust nonlinear motion control of a helicopter. *IEEE Transactions on Automatic Control.* 2003; 48: 413–426.
[17] M Sugeno, K Tanaka. Successive identification of a fuzzy model and its application to prediction of a complex system. *Fuzzy Sets and Systems*. 1991; 42: 315–334.
[18] T Takagi and M Sugeno. Fuzzy identification of systems and its application to modeling and control. *IEEE transaction on Man, Cybernetics*. 1985; 15: 116-132.
[19] Klein V, Moreli EA. Aircraft System Identification Theory and Practice. AIAA Education Series. American Institute of Aeronautics and Astronautics, New York, 2006.
[20] Tischler MB, Remple RK. Aircraft and Rotorcraft System Identification. AIAA Education Series. American Institute of Aeronautics and Astronautics, New York, 2006.
[21] Passino KM, Yurkovich S. Fuzzy Control. Prentice Hall. Englewood Cliffs.1998.
[22] Madsen K, HB Nielsen and O Tingleff. Methods for non-linear least square problems. 2nd Edn. 2004: 5-24.