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Nonlinear Direct Robust Adaptive Control Using Control Lyapunov Method

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Abstract

The problem of robust adaptive stabilization of a class of multi-input multi-output nonlinear systems with constant linearly parameterized uncertainty and unknown structure of bounded variation have been considered. With the aid of direct adaptive technique and control Lyapunov function method, a robust adaptive controller is designed to complete the globally adaptive stability of the closed-loop system. By employing our results, a kind of nonlinear system is analyzed, the concrete form of the control law is given and the meaningful quadratic control Lyapunov function for the system is constructed. Simulation of parallel manipulator is provided to illustrate the effectiveness of the proposed method.

Keywords: *robust adaptive control, control Lyapunov function, nonlinear system, globally adaptive stability*

1. Introduction

In the applications, dynamics of the plant are usually partially known, estimation is the key in designing a successful control. Adaptive control, represents the means of achieving online estimation, and the estimates are used to synthesize a stabilizing control. Robust control, focuses on the stability and performance under a fixed controller is guaranteed for a specific class of uncertainties (such as unknown parameter variations, unknown structures, disturbances, etc).

Robust adaptive stabilization of the nonlinear uncertain system has widely been investigated ([1]-[6]). In [1], the purpose of this direct robust adaptive fuzzy controller was to deal with a class of nonlinear systems containing both unconstructed state-dependent unknown nonlinear uncertain and gain functions. Bartolini [2] suggested the second-order sliding mode controller to cope with the uncertain system non-affine in the control law and the presence of the unmodeled dynamic actuator. A solution for robust adaptive stabilization of non-minimum phase systems was proposed in [3], and an output feedback robust adaptive controller was designed in [4] by the internal model design method. Neural network is also a useful method to identify unknown nonlinearities [5]-[6].

Lyapunov-based direct adaptive control framework was developed to guarantee globally adaptive stability of the closed-loop system by Lyapunov method. In [7], the robust adaptive controller for SISO nonlinear uncertain system was presented by the input/output linearization approach. In the case where the nonlinear uncertain systems include constant linearly parameterized uncertainty and nonlinear state-dependent parametric uncertainty, the direct robust adaptive control framework was developed in [8] where the Hamilton-Jacobi-Bellman(HJB) equation had to be solved to find the Lyapunov function, however, to find the solution of the HJB equation is not a feasible task. [9] tried to construct a controller for the system in [8] without solve the HJB equation by control Lyapunov function method, but the structural uncertainty was not considered. The structural uncertainty was further considered in [10] to make the conclusion in [9] more perfect.

In this note, we continue to explore the benefits of adaptive and robust controls by Lyapunov method. The technical problem addressed in the note is how to design a stabilizing control for a class of multi-input nonlinear systems own arbitrary unknown parameter matrix and unknown structure with bounded variation. Adaptive robust controller can be designed to guarantee globally adaptive stability of the system. Furthermore, to further illustrate our result, we give a concrete adaptive controller realization method to a kind of nonlinear system, in which

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the control Lyapunov function is also constructed. Some of our results are related to the work of direct adaptive control in [10], which controller relays on the input function.

2. The Proposed Algorithm

2.1. Preliminary knowledge

Definition 1 [11] [12] Consider the following system

$$\dot{x} = f(x) + g(x)u(x) \tag{1}$$

where $x \in R^n$, $u \in R^m$, and $f(0) = 0$. A C^1 function $V(x)$ is called a control Lyapunov function (CLF) of (1) if the following hold:

1) $V(x)$ is a smooth, positive definite, and radially unbounded function.

2) For all $x \neq 0$, it satisfies

$$L_g V = 0 \Rightarrow L_f V < 0 \tag{2}$$

where $L_f V = \frac{\partial V}{\partial x} f$ and $L_g V = \frac{\partial V}{\partial x} g$ are the lie derivative of $g(x)$ and $f(x)$ respectively. We see that the set where $L_g V = 0$ is significant, because in this set the uncontrolled system has the property $L_f V < 0$ When a CLF is known, the asymptotically stable control law for the certain system can be obtained directly. [13] [14]

2.2. Nonlinear Robust Adaptive Control

Consider the following nonlinear uncertain system:

$$\dot{x} = f(x) + \Theta F(x) + \Delta f(x) + g(x)u \tag{3}$$

where, $x \in R^n$ and $u \in R^m$ are the states and the inputs of the system, respectively. The mappings:

$f: R^n \to R^n$, $F: R^n \to R^p$, $g: R^n \to R^{n\times m}$

with $f(0) = 0$, $F(0) = 0$. $\Theta = \begin{bmatrix} \theta_{11} & \cdots & \theta_{1p} \\ & \cdots & \\ \theta_{n1} & \cdots & \theta_{np} \end{bmatrix}$ is an uncertain parameter matrix. $\Delta f: R^n \to R^n$ represents structural uncertainty with $\Delta f(0) = 0$, and characterized by

$\Delta f(x) = e(x)\delta(x)$,

where $e(x): R^n \to R^{n\times n}$, and $\delta(x): R^n \to R^n$ is an unknown function. It is assumed that $\delta(x)$ is constrained to a given function $n(x): R^n \to R^n$ with $n(0) = 0$, i.e.

$$S = \left\{\delta(x) : \delta^T(x)\delta(x) \le n^T(x)n(x)\right\} \tag{4}$$

$f(x), F(x), g(x), e(x), n(x)$ are all smooth.

Assumption 1 According to system (3), there exist

1) Function matrixes $g_n(x), h(x), f_s(x)$ and matrix K_θ which satisfied $g_n(x)K_\theta h(x): R^n \to R^{n\times 1}$, K_θ is a constant matrix which include the whole knowledge of the uncertain parameters $\theta_{ij} (i = 1,\cdots,n,\ \ j = 1,\cdots,p)$.

2) A positive definite and proper C^1 function $V(x)$, such that for all x,

$$\frac{\partial V}{\partial x} f_s(x) \le 0 \tag{5}$$

holds, where

$$f_s(x) = f(x) + \Theta F(x) + e(x)n(x) - g_n(x)K_\theta h(x) \tag{6}$$

with the properties:

$$L_e V\delta(x) \le L_e Vn(x) \tag{7}$$

and

$$L_g V = 0 \Rightarrow L_{g_n} VK_\theta h < 0 \tag{8}$$

Theorem 1 Let $V(x)$ be a control Lyapunov function of the system

$$\dot{x} = f(x) + \Theta F(x) + e(x)n(x) + g(x)u \ , \tag{9}$$

for all $g_n(x), h(x), K_\theta$ which satisfy Assumption 1, there always exists a feedback law $u\left(x, K_{\hat{\theta}}(t)\right)$ which is smooth on $R^n / \{0\} \times R^{n\times p}$,

$$u = -p_s(x)\left[L_g V\right]^T \tag{10}$$

$$p_s(x) = \begin{cases} 0 & \beta = 0 \\ \dfrac{a(x) + \sqrt{a^2(x) + \beta^2(x)}}{\beta(x)} & \beta \ne 0 \end{cases} \tag{11}$$

and update law

$$\dot{\mathrm{K}}_{\hat{\theta}}(t) = \left(h(x)L_{g_n}V\right)^T \tag{12}$$

guarantees that all the states of the close system (3), (10)-(12) globally stable and $x(t) \to 0$ as $t \to \infty$. Where $\beta(x) = L_g V\left(L_g V\right)^T$, $a = \frac{\partial V}{\partial x}\left(g_n(x)\mathrm{K}_{\hat{\theta}}(t)h(x)\right)$, $\mathrm{K}_{\hat{\theta}}(t)$ is the parameter estimation matrix of constant matrix K_θ.

Proof: Construct the following candidate Lyapunov function

$$W(x, K_{\hat{\theta}}) = V(x) + \frac{1}{2} tr\left(\mathrm{K}_{\hat{\theta}}(t) - K_\theta\right)\left(\mathrm{K}_{\hat{\theta}}(t) - K_\theta\right)^T \tag{13}$$

The derivative of W along the system (3) is given by

$$\dot{W} = \frac{\partial V}{\partial x}\left(f(x) + \Theta F(x) + \Delta f(x) + g(x)u\right) + tr\left[\left(\mathrm{K}_{\hat{\theta}}(t) - K_\theta\right)\dot{\mathrm{K}}_{\hat{\theta}}^T(t)\right].$$

Substitute (12) into $\dot{W}$, we have

$$\dot{W} = \frac{\partial V}{\partial x}\left(f(x) + \Theta F(x) + \Delta f(x) + g(x)u\right) + tr\left[\left(\mathrm{K}_{\hat{\theta}}(t) - K_\theta\right)h(x)L_{g_n}V\right].$$

It can easily deduce that $tr\left[\left(K(t)-K_\theta\right)h(x)L_{g_n}V\right]=L_{g_n}V\left(\mathrm{K}(t)-K_\theta\right)h(x)$.Furthermore, with (6) and property (7) we have

$$\begin{aligned}\dot{W}&\le\frac{\partial V}{\partial x}\left(f(x)+\Theta F(x)+e(x)n(x)+g(x)u\right)+L_{g_n}V\left(\mathrm{K}_{\hat\theta}(t)-K_\theta\right)h(x)\\&=\frac{\partial V}{\partial x}\left(f_s(x)+g_n(x)K_\theta h(x)+g(x)u\right)+L_{g_n}V\left(\mathrm{K}_{\hat\theta}(t)-K_\theta\right)h(x)\\&\le L_gVu+L_{g_n}V\mathrm{K}_{\hat\theta}(t)h(x)\end{aligned}.$$

1) if $x\ne 0$, $L_gV=0$, we can obtain $u=0$ from (10). According to Assumption 1, it holds $L_gV=0\Rightarrow L_{g_n}VK_\theta h\le 0$. Furthermore, L_gV doesn't contain any information of the uncertain parameters $\theta_{ij}(i=1,\cdots,n,\ j=1,\cdots,p)$ and θ_{ij} can take any value, thus, when take $\theta_{ij}=\hat\theta_{ij}$, (8) can also be expressed as

$$L_gV=0\Rightarrow L_{g_n}VK_{\hat\theta}(t)h(x)<0 \tag{14}$$

Thus, we can conclude that $\dot W<0$ as $x\ne 0$, $L_gV=0$.

2) if $x\ne 0$, $L_gV\ne 0$,then, according with the control law(10) and (11),

$$\dot W\le L_gVu+L_{g_n}V\mathrm{K}_{\hat\theta}(t)h(x)=-a-\sqrt{a^2+\beta^2}+L_{g_n}V\mathrm{K}_{\hat\theta}(t)h(x)=-\sqrt{a^2+\beta^2}<0$$

3) if $x=0$, $\dot W=0$.

Thus, the equilibrium $x=0$, $\mathrm{K}_{\hat\theta}(t)=K_\theta$ is globally stable, and by LaSalle's theorem, we can obtain $\lim_{t\to\infty}x(t)=0$.

Remark 1 It is important to note that the adaptive control law (12) does not require explicit knowledge of the gain matrix K_θ, even though Theorem 1 requires the existence of $g_n(x), h(x), K_\theta$ to satisfy the assumption 1.

Remark 2 No specific structure on the nonlinear dynamics $g_n(x), h(x), K_\theta$ is required only with $g_n(x)K_\theta(t)h(x): R^n\to R^{n\times 1}$.

Remark 3 Even if $f_s(x)$ includes some information of the uncertain parameters, the result of the theorem 1 will not be influenced.

Proposition 1 Let $V(x)$ be a CLF of (9), then there must be a feedback law (15) and adaptive law (16) smooth on $R^n/\{0\}\times R^{n\times p}$ guarantee the system (3) globally adaptive stable.

$$u=\begin{cases}0 & \beta=0\\ -\left(L_gV\right)^T\left(\dfrac{L_fV+L_{\hat\Theta F}V+L_eVn(x)+\sqrt{\left(L_fV+L_{\hat\Theta F}V+L_eVn(x)\right)^2+\beta^2}}{\beta}\right) & \beta\ne 0\end{cases} \tag{15}$$

$$\dot{\hat\Theta}=\left(F(x)\frac{\partial V}{\partial x}\right)^T \tag{16}$$

where $\beta(x)=L_gV\left(L_gV\right)^T$.

proof: Make

$$g_n(x)=g\frac{\left(L_gV\right)^T}{\beta}\frac{\partial V}{\partial x},\ h(x)=\left(f(x)\ \ F(x)\ \ e(x)n(x)\right)^T,\ K_\theta=\left(I_{n\times n}\ \Theta\ \ I_{n\times n}\right),$$

then we have $\frac{\partial V}{\partial x} f_s(x)=0$. Furthermore, $V(x)$ be the control Lyapuov function for system (9), which implies that

$$L_g V=0 \Rightarrow \frac{\partial V}{\partial x}\left(f(x)+\Theta F(x)+e(x)n(x)\right)<0 .$$

With $\frac{\partial V}{\partial x} f_s(x)=0$, we have

$$L_g V=0 \Rightarrow L_{g_n} V K_\theta h<0 .$$

Thus, Assumption 1 is satisfied. Substitute the above variable into (10), (11),（12）, the formula (15) and (16) can be obtained directly.

3. Research Method

Sometimes it is difficult to find the CLF for a given system. By employing our result, we will analysis some control systems, give the concrete form of the control law, and at the same time, find the meaningful quadratic control Lyapunov function for the systems.

We assume the uncertain of the construction $\Delta f(x)=e(x)\delta(x)$ have the form as follows:

$$e(x)=E_e\,,\quad \delta(x)=\delta x \tag{17}$$

where $E_e \in R^{n\times n}$ is a positive-definite diagonal matrix , $\delta=diag\begin{bmatrix}\delta_1 & \delta_2 & \cdots & \delta_n\end{bmatrix}$ is a diagonal matrix, and $|\delta_i|\le n_i\ (i=1,2,\cdots,n)$, where $N=diag\begin{bmatrix}n_1 & n_2 & \cdots & n_n\end{bmatrix}$ is a known diagonal matrix.

Consider $f(x), F(x), \Theta$ and $g(x)$ in (3) have the form as the follows:

$$f(x)=A^* x+\begin{pmatrix} C(x)X_2 \\ \tilde{f}(x)\end{pmatrix},\ \Theta=\begin{pmatrix} 0_{(n-m)\times p} \\ \tilde{\Theta}_{m\times p}\end{pmatrix},\ g(x)=\begin{pmatrix} 0_{(n-m)\times m} \\ \tilde{g}_{m\times m}\end{pmatrix},\ A^*=A+E_e N \tag{18}$$

Then, we rewrite the system as

$$\begin{pmatrix}\dot{X}_1 \\ \dot{X}_2\end{pmatrix}=\begin{pmatrix}A_1^* & A_2^* \\ A_3^* & A_4^*\end{pmatrix}\begin{pmatrix}X_1 \\ X_2\end{pmatrix}+\begin{pmatrix}C(x)X_2 \\ \tilde{f}(x)\end{pmatrix}+\begin{pmatrix}0 \\ \tilde{\Theta}F(x)\end{pmatrix}+\begin{pmatrix}0 \\ \tilde{g}\end{pmatrix}u \tag{19}$$

where $X_1=(x_1,\cdots x_{n-m})$, $X_2=(x_{n-m+1},\cdots,x_n)^T$, and $C(x)\in R^{(n-m)\times m}$ are function matrixes.

Proposition 2 Assume the variables in system (3) have the form in (17) and (18) with $\det\tilde{g}(x)\ne 0$, if $\dot{X}_1=A_1^* X_1$ is asymptotically stable, there always exists a positive-definite matrix P and a feedback law $u\left(x,\hat{\Theta}\right)$ which is smooth on $R^n/\{0\}\times R^{n\times p}$

$$u=-p_s(x)g(x)^T P^* x \tag{20}$$

$$p_s(x)=\begin{cases} 0 & \left\|x^T P^* g(x)\right\|=0 \\ \dfrac{a+\sqrt{a^2+\left\|x^T P^* g(x)\right\|^4}}{\left\|x^T P^* g(x)\right\|^2} & \left\|x^T P^* g(x)\right\|\ne 0\end{cases} \qquad \dot{\hat{\Theta}}=\left(F(x)x^T P^*\right)^T \tag{21}$$

guarantees that all the states of the close system (3),(20)-(21) globally stable and $\lim_{t\to\infty} x(t) = 0$, where $a = x^T P^*\left(f^*(x) + \hat{\Theta} F(x)\right)$, $P^* = \begin{pmatrix} P & 0 \\ 0 & I \end{pmatrix}$.

Proof: With the stability of $\dot{X}_1 = A_1^* X_1$, for a given positive-definite Q, we can find a positive-definite P, such that

$$\left(A_1^*\right)^T P + PA_1^* = -Q \tag{22}$$

Write

$$f^*(x) = \begin{pmatrix} A_1^* & A_2^* \\ A_3^* & A_4^* + B \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \begin{pmatrix} C(x)X_2 \\ \tilde{f}(x) \end{pmatrix} \tag{23}$$

where $B \in R^{m\times m}$ is an arbitrary positive-definite matrix. When take

$$g_n = I_{n\times n},\ K_\theta = \begin{pmatrix} I_{n\times n} & \Theta \end{pmatrix},\ h(x) = \left(\left(f^*\right)^T(x) \quad F^T(x)\right)^T \tag{24}$$

and the following positive-definite

$$V(x) = x^T \begin{pmatrix} P & 0 \\ 0 & I \end{pmatrix} x \tag{25}$$

we get

$$\frac{\partial V}{\partial x}\left(f(x) + F(x)\theta + e(x)n(x) - g_n(x)K_\theta h(x)\right) = 2x^T \begin{pmatrix} P & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & -B \end{pmatrix} x \le -2x_2^T B x_2 < 0 \tag{26}$$

Furthermore, with $\det \tilde{g}(x) \ne 0$ we can conclude that

$$X \ne 0, L_g V = 0 \Rightarrow X_2 = 0 \Rightarrow L_{g_n} K_\theta h = -X_1 Q X_1^T < 0 \tag{27}$$

It is easy to verify that

$$L_e V\delta(x) \le L_e V n(x) \tag{28}$$

With (26)-(28), assumption 1 is satisfied. Moreover, we can also conclude that

$$X \ne 0, L_g V = 0 \Rightarrow X_2 = 0 \Rightarrow \frac{\partial V}{\partial x}\left(f(x) + F(x)\theta + e(x)n(x)\right) = -X_1 Q X_1^T < 0 \tag{29}$$

which means $V(x)$ in (25) is the control Lyapunov function of system (9). Substitute the above variable (23) - (25) into (10)- (12), the formula (20) and (21) can be obtained directly.

4. Results and Analysis

The dynamic model of the parallel manipulator is shown as follows [15]

$$M\ddot{q} + C\dot{q} + J^T f = J^T \tau + \delta(\dot{q}) \tag{30}$$

Where, $q=(x\ y)^T$ is the position coordinates of the end-effecter; $\tau=[\tau_1 \quad \tau_2]$ is the actuator torque vector of the active joints; f is the friction torque vector of the active joints; J is the velocity Jacobian matrix between the end-effecter and the three active joints of the parallel manipulator; $M=diag(m_1,m_2)$ is the inertia matrix in the task space, and C is the coriolis and centrifugal force matrix in the task space. $\delta(\dot{q})=diag(\delta_1\ \delta_2)\dot{q}$ is structural uncertainty with $\delta_1<N_1,\delta_2<N_2$ The detailed definition of the above symbols can be found in [15].

When choose $X_1=q,\ \ X_2=\dot{q}$, (30) can be rewriten as

$$\begin{aligned}&\dot{X}_1=X_2\\&\dot{X}_2=-M^{-1}CX_2-M^{-1}J^Tf+M^{-1}J^T\tau+M^{-1}\delta(X_2)\end{aligned} \tag{31}$$

We assume C is an uncertain constant matrix here.

In this system, we can write $f(X)=\begin{pmatrix}0 & I_{2\times2}\\0 & 0\end{pmatrix}\begin{pmatrix}X_1\\X_2\end{pmatrix}-\begin{pmatrix}0\\M^{-1}J^Tf\end{pmatrix}$, $\Theta=\begin{bmatrix}0 & 0\\0 & -M^{-1}C\end{bmatrix}$, $F(X)=X$, $\Delta f=diag(0\quad 0\quad \delta_1\quad \delta_2)X$, $g=M^{-1}\begin{pmatrix}0 & J^T\end{pmatrix}^T$, $u=\tau$.

Construct $f^*(X)=f(X)+M^{-1}diag(0\quad 0\quad N_1\quad N_2)X+BX$,note that $\det\left(M^{-1}J^T\right)\neq 0$, let $K_\theta=\begin{pmatrix}I_{2\times2} & 0 & 0 & 0\\0 & I_{2\times2} & 0 & M^{-1}C_{2\times2}\end{pmatrix}$, $h(X)=\left(\left(f^*(X)\right)^T\quad X^T\right)^T$, $g_n(X)=I_{4\times4}$,thus,

$$f(x)+\Theta F(x)+e(x)n(x)-g_n(x)K_\theta h(x)=-BX$$

when choose $B=\begin{pmatrix}3I & -I\\-I & 2I\end{pmatrix}$, $\dot{X}=-BX$ is global asymptotically stable, and with $Q=\begin{pmatrix}10I & 0\\0 & 10I\end{pmatrix}$, we can get a positive-definite $P=\begin{pmatrix}2I & I\\I & 3I\end{pmatrix}$ satisfies the Lyapunov function $(-B)^TP+P(-B)=-Q$, furthermore, we have

$$X^TPg(X)=0\Rightarrow X_1+3X_2=0\Rightarrow X^TPg_n(X)K_\theta h=-3X_1^TX_1<0.$$

and

$$X^TPg(X)=0\Rightarrow X_1+3X_2=0\Rightarrow X^TP\left(f(x)+\Theta F(x)+e(x)n(x)\right)=-5X_1^TX_1<0$$

Thus, $V=X^TPX$ is the control Lyapunov function for (31), and the control laws according to (11) - (13) are the following

$$\tau=-JM^{-1}\left(X_1+3X_2\right)\left(\frac{X^Tf^*(X)+X_2^TM^{-1}\hat{C}X_2+\sqrt{\left(X^Tf^*(X)+X_2^TM^{-1}\hat{C}X_2\right)^2+\left\|X_2^TM^{-1}J^T\right\|^2}}{\left\|\left(X_1^T+3X_2^T\right)M^{-1}J^T\right\|}\right) \tag{32}$$

$$\dot{\hat{C}}=X_2^T\left(X_1+3X_2\right) \tag{33}$$

For simulation uses, we make initial conditions as $X=\left(x,y,u_x,v_y\right)=(5,-2,0,0)$, and pick the parameters of the manipulator as the following: $m_1=m_2=1$, $N_1=0.5$. $N_2=1$, $J=\begin{bmatrix}0.5 & 0.2\\0.3 & 0.5\end{bmatrix}$. the initial parameter estimation is $\hat{C}=[0]_{2\times2}$.

1）Take the different parameter matrix: a) $C_1=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$, b) $C_2=\begin{bmatrix}5 & 10\\15 & 20\end{bmatrix}$. The simulation results under the different parameter matrix are shown in Figure1 and Figure 2 respectively, each results include system states of the position and the velocity and the parameter estiamation values.

We can conclud from Figure1 and Figure 2 that the system states are all converge to the expected values, that is the system states are asymptotic stable even though the constant matrix C is uncertain. The parameter estimation values can not converge to the real values, they are Lyapunov stable. It is obviously that the convergence rate in Figure 1 is faster than the rate in Figure 2 because of the initial estiamation errors in a) is smaller than the errors in b).

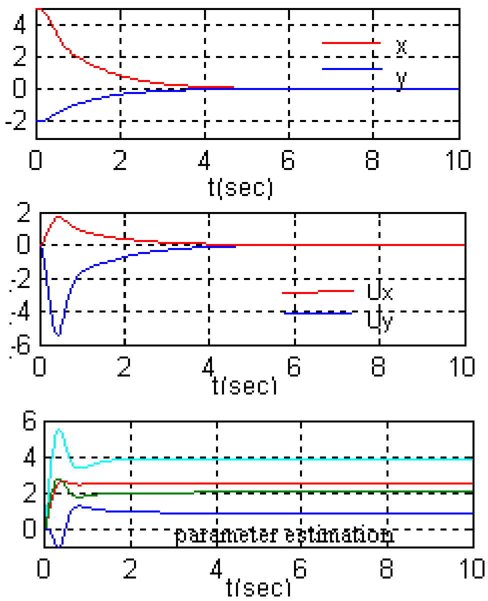

Figure 1. Simulation result of the system with the parameter matrix C_1

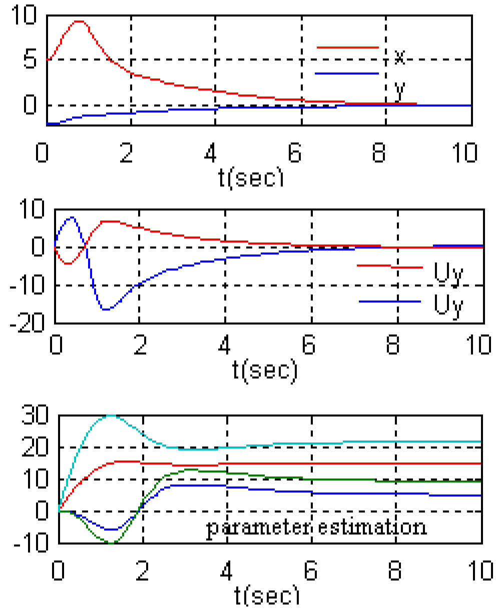

Figure 2. Simulation result of the system with the parameter matrix C_2

2) For further comparison, we construct another controller introduced in [10] with the same initial conditions and parameters, and take $C=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, $g_n(X)=\begin{pmatrix}\Lambda_1 X_1 & 0\\ \Lambda_2 X_1+\Lambda_3 X_2 & -M^{-1}\bar{X}_2\end{pmatrix}$,

$$h(X)=1,\ K_\theta=\begin{pmatrix}I_{2\times 2} & 0 & 0 & 0\\ 0 & I_{2\times 2} & 0 & M^{-1}C_{2\times 2}\end{pmatrix},\ P=\begin{pmatrix}14I & 5I\\ 5I & 3I\end{pmatrix}.$$

The simulation is shown in figure 3, Clearly, the controller can also make the system adaptive stable. However, this controller relays on the input function and a bit more complex than the controller in (31), the convergence rate in Figure 1 is faster than the rate in Figure 3.

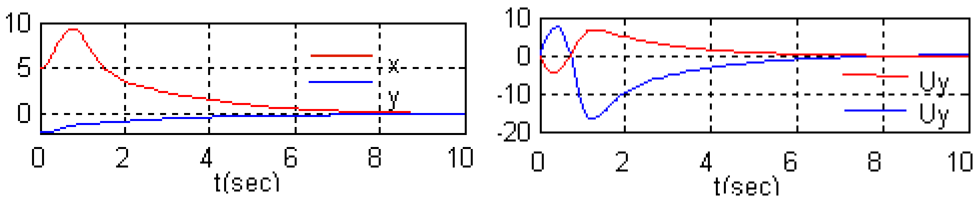

Figure 3. Simulation result of the system with controller in 2)

5. Conclusion

According to a class of parameter and structural uncertainty nonlinear system, an adaptive robust control scheme is development with the aid of control Lyapunov function, the controller can guarantee globally adaptive stability of the close-loop system.

Future research will address the extension of this design method to the output feedback adaptive control system.

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