

Nonlinear Robust Control for Spacecraft Attitude

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Abstract

This paper proposed a nonlinear robust control for spacecraft attitude based on passivity and disturbance suppression vector. The spacecraft model was described using quaternion. The control law introduced the suppression vector of external disturbances and had no information related to the system parameters. The desired performance of spacecraft attitude control could be achieved using the designed control law. And stability conditions of the nonlinear robust control for spacecraft attitude were given. The stability could be proved by applying Lyapunov approach. The verification of the proposed attitude control method was performed through a series of simulations. The numerical results showed the effectiveness of the proposed control method in controlling the spacecraft attitude in the presence of external disturbances. The main benefit of the proposed attitude control method does not need angular velocity measurement and has its robustness against model uncertainties and external disturbances.

Keywords: spacecraft attitude, external disturbance, robust control, quaternion, stability

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1. Introduction

Attitude control is a particularly important component for spacecrafts. A spacecraft must maintain a certain attitude while in orbit. Nowadays, attitude control of spacecrafts demand better performance. The spacecraft attitude can be expressed by matrix, Euler angle, or quaternion. The method of matrix representation is complicated in calculation; Euler angle also exist some limitations. For example, the rotation matrix is not interchangeable, Euler angle rotation must be in a particular order, and equivalent to Euler angle change may not cause equal rotation, which leads to a rotating unevenness. When Euler angle is equal to $\pm \pi/2$, there will be a singular point, leading to the loss of degrees of freedom, which is called as the phenomenon of gimbal lock. But expressing 3D rotation with quaternion can avoid these limitations, and also has clear geometric meaning and simple calculation. In the past several decades, researchers have devoted to the problem of spacecraft attitude stabilization based on quaternion representation. Some control methods have been developed to treat this problem, such as robust control approach [1, 2], Lyapunov-based approach [3-5], adaptive control approach [6-9], variable structure control approach [10-14].

In general, angular velocity and quaternion, are used to deal with the stability of feedback control. However, the angular velocity measurement is not necessary in some of the previous works. For example, in [10], a design criterion for a class of proportional-derivative (PD) controllers was firstly proposed by using the Lyapunov-based approach, and then a design criterion of controller without angular velocity measurement was presented based on passivity. The approach proposed in [10] was further extended to the system described by the Rodrigues and modified Rodrigues parameters [11].

However, the external disturbances, which inevitably affect the motion of the spacecraft in its attitude, are ignored in the above-mentioned literatures. In this paper, we focus on the stability of spacecraft attitude in the presence of the bounded external disturbances and propose a nonlinear robust control method for spacecraft attitude. The spacecraft attitude is represented by quaternion. The suppression vector of external disturbances, which is independent of angular velocity size, is introduced into the control law. In addition, the control law has no information related to the system parameters so that the robustness is guaranteed. To demonstrate the performance of the proposed attitude control method in suppressing disturbances and maintaining stability, the numerical simulations are carried out using MATLAB.

2. Proposed Attitude Control Method

2.1. Spacecraft Model

The motion of spacecraft attitude can be described by kinematic and dynamic equations.

We use the unit quaternion to represent spacecraft attitude in order to avoid singularity. Define the unit quaternion as in Eq. (1).

$$\bar{q} = \begin{pmatrix} q \\ q_0 \end{pmatrix} = \begin{pmatrix} \sin(\theta/2)\hat{n} \\ \cos(\theta/2) \end{pmatrix} \quad (1)$$

where $\hat{n} \in R^3$ is the rotation axis represented by unit vector, θ is the rotation angular, $q \in R^3$ and $q_0 \in R$ are the components of the unit quaternion, which subject to the following constraint:

$$q^T q + q_0^2 = 1 \quad (2)$$

The kinematic equation represented by the unit quaternion is given by Eq. (3).

$$\begin{aligned} \dot{q} &= \frac{1}{2} E(q)\omega = \frac{1}{2}(q_0 I + q^\times)\omega \\ \dot{q}_0 &= -\frac{1}{2} q^T \omega \end{aligned} \quad (3)$$

where $\omega = [\omega_1 \ \omega_2 \ \omega_3]^T$ is the spacecraft angular velocity vector with respect to the inertial reference frame, expressed in the spacecraft body-fixed reference frame, I is the 3×3 unit matrix, q^\times is the skew symmetric matrix which is defined by Eq. (4).

$$q^\times = \begin{pmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{pmatrix} \quad (4)$$

The dynamic model of the spacecraft attitude control system is described by the differential equation, as in Eq. (5).

$$J\dot{\omega} = -\omega^\times J\omega + u + d \quad (5)$$

where $J = J^T \in R^{3 \times 3}$ is the inertia matrix which is a symmetric and positive definite matrix, $u \in R^3$ is the vector of control torque, $d = [d_1 \ d_2 \ d_3]^T$ is the vector of external disturbance which is bounded as $|d_i| \leq \delta_i$, where δ_i is a positive constant, for $i=1, 2, 3$.

2.2. Passivity and Disturbance Suppression Based Attitude Control

First, we consider attitude control with angular velocity measurement. The nonlinear control law is given in Eq. (6).

$$u = -k_1 \omega - k_2 E^T q \quad (6)$$

where k_1 and k_2 are positive constants.

Consider the Lyapunov function candidate

$$V = \frac{1}{2} \omega^T J \omega + k_2 q^T q \quad (7)$$

Using Eqs. (3), (5) and (6), the time derivate of V can be computed to

$$\begin{aligned}\dot{V} &= \omega^T J \omega + 2k_2 q^T \dot{q} = \omega^T (-\omega^\times J \omega + u + d) + k_2 q^T E \omega \\ &= \omega^T (-\omega^\times J \omega + u + d + k_2 E^T q) = -k_1 \omega^T \omega + \omega^T d\end{aligned}\quad (8)$$

When $d=0$, Eq. (8) can be simplified as $\dot{V} = -k_1 \omega^T \omega \leq 0$. Since the Lyapunov function candidate V is positive definite and radially unbounded. By LaSalle invariance principle, all trajectories converge to the largest invariant set $\psi = \{(\omega, q) : \dot{V} = 0\} = \{(\omega, q) : \omega = 0\}$, which implies that $\dot{\omega} = 0$. Since $\omega = 0$, then $\dot{q} = 0$, $\ddot{q} = 0$ from Eq.(3). From Eq. (5), we have that $J\dot{\omega} = -\omega^\times J \omega + u$ ($d \neq 0$) or $u = J\dot{\omega} + \omega^\times J \omega = 0$. From Eq. (6), we obtain that $k_2 E^T q = -u + k_1 \omega = 0$. So $q = 0$. The largest invariant set is $\psi = \{(\omega, q) : \omega = 0, q = 0\}$, which corresponds to the stable equilibrium.

When $d \neq 0$, we will present a controller for the system.

First of all, we introduce a result about Input-to-State Stability [15].

Lemma 1 Let $V : [0, \infty) \times R^n \rightarrow R$ be a continuously differentiable function that satisfies the following properties:

$$\alpha_1(\|x\|) \leq V(t, x) \leq \alpha_2(\|x\|) \quad (9)$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x, u) \leq -W_3(x), \quad \forall \|x\| \geq \rho(\|u\|) > 0 \quad (10)$$

$\forall (t, x, u) \in [0, \infty) \times R^n \times R^m$, where α_1 and α_2 are class \mathcal{K}_∞ functions, ρ is a class \mathcal{K} function, and $W_3(x)$ is a continuous positive definite function on R^n . Then, the system is input-to-state stable with $\gamma = \alpha_1^{-1} \circ \alpha_2 \circ \rho$.

Now Eq. (8) can be rewritten as follows

$$\begin{aligned}\dot{V} &= -(k_1 - \theta) \omega^T \omega - \theta \omega^T \omega + \omega^T d \leq -(k_1 - \theta) \omega^T \omega - \theta \sum_{i=1}^3 |\omega_i|^2 + \sum_{i=1}^3 |\omega_i| |d_i| \\ &= -(k_1 - \theta) \omega^T \omega - \sum_{i=1}^3 |\omega_i| (\theta |\omega_i| - |d_i|)\end{aligned}\quad (11)$$

Therefore, when ω satisfies $|\omega_i| \geq |d_i|/\theta$ for $i=1,2,3$, we have that $\dot{V} \leq -(k_1 - \theta) \omega^T \omega$ where $0 < \theta < k_1$. By Lemma 1, the proposed controller can make the closed-loop system achieve input-to-state stable.

Now we will provide an improved controller. In order to suppress the effect of external disturbances, we introduce the suppression vector v of external disturbances into the control law. Let $u = -k_1 \omega - k_2 E^T q - v$, where $v = [v_1 \ v_2 \ v_3]^T$, $v_i = \delta_i \text{sgn}(\omega_i)$ for $i=1, 2, 3$. The symbolic function $\text{sgn}(x)$ is defined by

$$\text{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} \quad (12)$$

Then the time derivative of Lyapunov function candidate V can be computed to

$$\begin{aligned}\dot{V} &= \omega^T(-\omega^\times J\omega + u + d + k_2 E^T q) = \omega^T(-k_1 \omega - v + d) \\ &= -k_1 \omega^T \omega - \sum_{i=1}^3 \delta_i |\omega| + \omega^T d = -k_1 \omega^T \omega - \sum_{i=1}^3 \delta_i |\omega| + \sum_{i=1}^3 |d_i| |\omega_i|\end{aligned}\quad (13)$$

Therefore, when ω satisfies $|\omega_i| \geq |d_i|/\theta$ for $i=1,2,3$, $V \leq -\sum_{i=1}^3 \delta_i |\omega_i|$. According to Lemma 1, we know that the proposed attitude controller can make the closed-loop system input-to-state stable.

Second, we consider nonlinear attitude control without angular velocity measurement. In [10], a controller without angular velocity measurement was proposed using passivity-based approach. Along the line of [10], we construct a controller as follows.

$$\begin{cases} \dot{x} = Ax + Bq \\ y = B^T P(Ax + Bq) = B^T P \dot{x} \\ u = -k_1 E^T y - k_2 E^T q - v \end{cases}\quad (14)$$

where B is a full rank matrix. There exist positive definite matrices P and Q which can make matrix A satisfy the following Lyapunov equation.

$$A^T P + PA = -Q \quad (15)$$

It can be seen that the disturbance suppression vector v is related to the angular velocity in the control law. The disturbance suppression vector can be determined only if the direction of the angular velocity is known. While the size of the angular velocity is not necessary. Consider the following Lyapunov function candidate

$$V = \frac{1}{2} \omega^T J \omega + k_2 q^T q + k_1 (Ax + Bq)^T P (Ax + Bq) \quad (16)$$

Using Eqs. (3), (5), (10) and (14), the time derivate of V is can be computed to

$$\begin{aligned}\dot{V} &= \omega^T J \dot{\omega} + 2k_2 q^T \dot{q} + k_1 (A\dot{x} + B\dot{q})^T P \dot{x} + k_1 \dot{x}^T P (A\dot{x} + B\dot{q}) \\ &= -k_1 \dot{x}^T Q \dot{x} - \sum_{i=1}^3 \delta_i |\dot{\omega}_i| + \omega^T d\end{aligned}\quad (17)$$

When $d=0$, Eq. (17) can be simplified as $\dot{V} = -k_1 \dot{x}^T Q \dot{x} - \sum_{i=1}^3 \delta_i |\dot{\omega}_i| \leq 0$. Since the Lyapunov function candidate V is positive definite and radially unbounded. By LaSalle invariance principle, all trajectories converge to the largest invariant set $\psi = \{(\omega, q, x) : \dot{V} = 0\} = \{(\omega, q, x) : \omega = 0, \dot{x} = 0\}$ which implies that $\dot{\omega} = 0$. Since $\omega = 0$, then $\dot{q} = 0$, $\dot{q}_0 = 0$ from Eq.(3). It is easy to have $y = B^T P \dot{x}$. At the same time, we have that $J\dot{\omega} = -\omega^\times J\omega + u$ ($d=0$) or $u = J\dot{\omega} + \omega^\times J\omega = 0$ from Eq. (5). From Eq. (14), we have that $k_2 E^T q = -u + k_1 E^T y + v = 0$. So $q = 0$. Consequently, the largest invariant set is $\psi = \{(\omega, q) : \omega = 0, q = 0\}$, which corresponds to the stable equilibrium.

When $d \neq 0$, Eq. (17) can be rewritten as follows.

$$\begin{aligned}
\dot{V} &= -k_1 \dot{x}^T Q \dot{x} - \sum_{i=1}^3 \delta_i |\omega_i| + \omega^T d \leq -k_1 \dot{x}^T Q \dot{x} - \sum_{i=1}^3 \delta_i |\omega_i| + \sum_{i=1}^3 |d_i| |\omega_i| \\
&\leq -k_1 \lambda_{\min} \dot{x}^T \dot{x} + \sum_{i=1}^3 |\omega_i|^2 - \sum_{i=1}^3 |\omega_i|^2 - \sum_{i=1}^3 \delta_i |\omega_i| + \sum_{i=1}^3 |d_i| |\omega_i| \\
&= -k_1 \lambda_{\min} \dot{x}^T \dot{x} + \sum_{i=1}^3 |\omega_i|^2 - \sum_{i=1}^3 \delta_i |\omega_i| - \sum_{i=1}^3 |\omega_i| (|\omega_i| - |d_i|)
\end{aligned} \tag{18}$$

Therefore, when ω satisfies $|\omega_i| \geq |d_i|$ and $\left| \frac{\dot{\omega}_i}{x} \right| \geq \frac{|\omega_i|}{k_1 \lambda_{\min}}$ for $i=1,2,3$, we have that $\dot{V} \leq 0$,

where λ_{\min} is the minimum eigenvalue of Q . According to Lemma 1, we know that the proposed control method can make the closed-loop system described by Eqs.(3), (5) and (14) input-to-state stable.

3. Research Method

In order to demonstrate and verify the effectiveness and robustness of the proposed attitude control method for spacecraft, several numerical simulations are carried out using MATLAB.

A spacecraft with the following inertia matrix $J = \begin{pmatrix} 15 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 10 \end{pmatrix}$ (kg·m²) is considered.

The other main parameters are $k_1=8$, $k_2=4$, $-A=P=B=I$. And the initial states are $\hat{n} = \begin{pmatrix} 0.5345 \\ 0.2673 \\ 0.8018 \end{pmatrix}$,

$\theta = 11\pi/6$, $\omega(0) = 0$. Then the corresponding quaternion representations are $q(0) = \begin{pmatrix} 0.1383 \\ 0.0692 \\ 0.2075 \end{pmatrix}$,

$q_0(0) = -0.9659$. At this time, yaw angle, roll angle and pitch angle are 23.56°, 17.21° and 4.98°, respectively.

Consider the following four cases.

Case 1: Choose the external disturbance $d_1 = \begin{pmatrix} 0.01 \sin(0.2t) \\ -0.006 \sin(0.3t) \\ 0.014 \sin(0.4t) \end{pmatrix}$ (N·m), and the

components of the disturbance suppression vector $\delta_1 = \delta_2 = \delta_3 = 0$.

Case 2: Choose the external disturbance $d_1 = \begin{pmatrix} 0.01 \sin(0.2t) \\ -0.006 \sin(0.3t) \\ 0.014 \sin(0.4t) \end{pmatrix}$ (N·m), and $\delta_1 = 0.01$,

$\delta_2 = 0.006$, $\delta_3 = 0.014$.

Case 3: Choose the external disturbance $d_2 = \begin{pmatrix} 0.1 \sin(0.2t) \\ -0.06 \sin(0.3t) \\ 0.14 \sin(0.4t) \end{pmatrix}$ (N·m), and $\delta_1 = 0.1$,

$\delta_2 = 0.06$, $\delta_3 = 0.14$.

Case 4: Choose the external disturbance $d_1 = \begin{pmatrix} 0.01\sin(0.2t) \\ -0.006\sin(0.3t) \\ 0.014\sin(0.4t) \end{pmatrix}$ (N·m), and $\delta_1 = 0.01$,

$\delta_2 = 0.006$, $\delta_3 = 0.014$. And assume that there exist model error and model parameter

uncertainty. That is, $J = \begin{pmatrix} 15+0.5 & 0 & 0 \\ 0 & 20+0.4 & 0 \\ 0 & 0 & 10+0.6 \end{pmatrix}$ (kg·m²).

4. Results and Analysis

When there exists the external disturbance d_1 and the control law does not include the disturbance suppression vector v (Condition: Case 1), the performance of the attitude controller without angular velocity measurement is shown in Figure 1. When the condition is changed to Case 2, the performance of the proposed attitude controller without angular velocity measurement is given in Figure 2. By comparing Figure 1 with Figure 2, we can see that the controller without the disturbance suppression vector v can not converge to the equilibrium point and be not any more stable. While the proposed controller with the disturbance suppression vector v can make the closed-loop system which is described by Eqs. (3), (5) and (14) achieve the input state stability. It proves that the disturbance suppression vector can suppress the effect which external disturbances have on the closed-loop system.

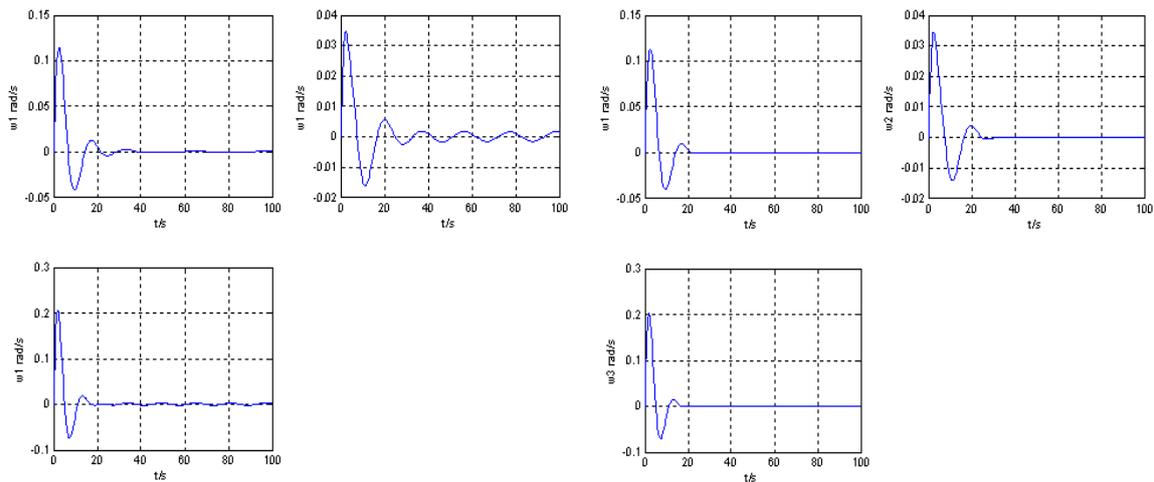


Figure 1. The angular velocity curve without angular velocity measurement (Condition: Case 1)

Figure 2. The angular velocity curve without angular velocity measurement (Condition: Case 2)

Figure 3 shows the convergence of the proposed controller without angular velocity measurement under the condition of Case 2. Compared with the controller with angular velocity measurement under the same condition, whose quaternion curve is shown in Figure 4, the proposed controller in this paper can converge more fastly to the equilibrium point. It illustrates the effectiveness of the proposed attitude control method.

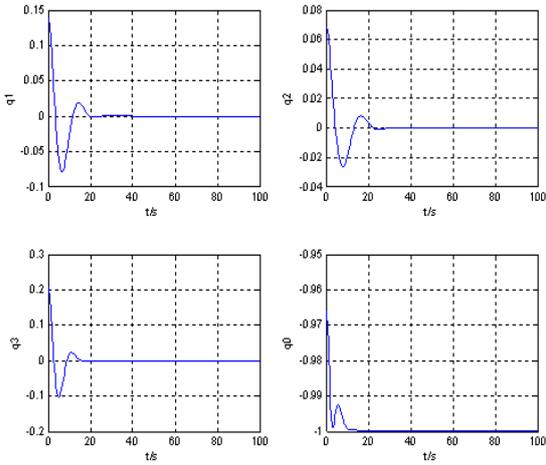


Figure 3. The quaternion curve without angular velocity measurement (Condition: Case 2)

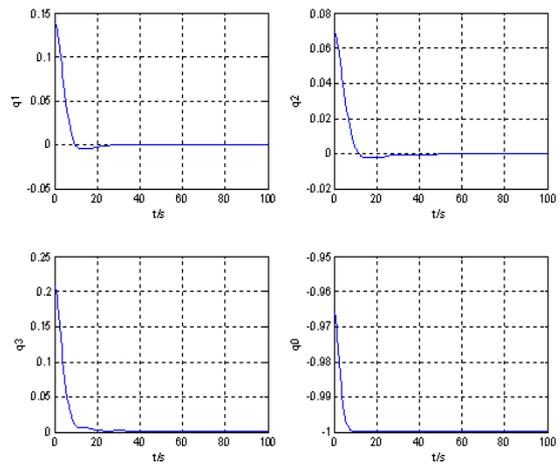


Figure 4. The quaternion curve with angular velocity measurement (Condition: Case 2)

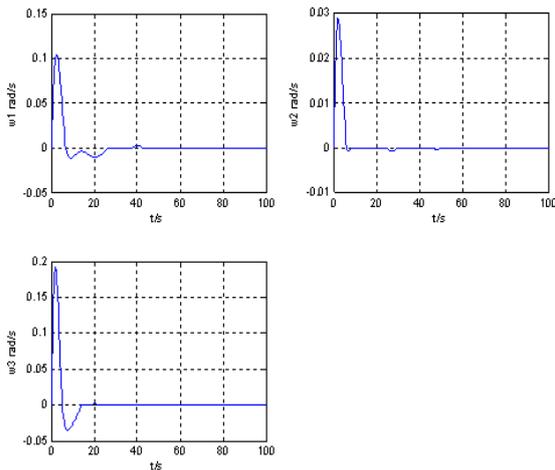


Figure 5. The angular velocity curve without angular velocity measurement (Condition: Case 3)

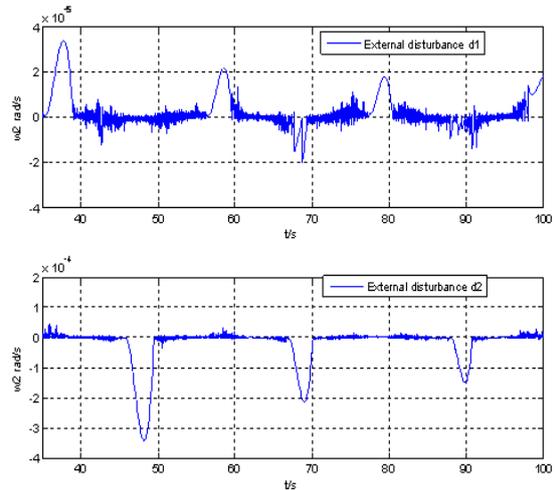


Figure 6. The enlarged figure of ω_2 in Figure 2 and Figure 5 for $t \in [35s, 100s]$

When the external disturbance increases from d_1 to d_2 , that is the condition of Case 3, the result is depicted in Figure 5. And Figure 6 gives the enlarged part of angular velocity curve shown in Figure 2 and Figure 5 for $t \in [35s, 100s]$. From Figure 2, Figure 5 and Figure 6, it can be observed that the area which the angular velocity converges to is related to the external disturbances; that is to say, the bigger the amplitude of external disturbance, the larger the convergence area of angular velocity and the less effective the proposed attitude control method.

When there exist model error and model parameter uncertainty, the performance of the closed-loop system under control torque is found in Figure 7. Obviously, the system can be still stable at equilibrium point. It shows that the proposed attitude control method is robust to model error and model parameter uncertainty.

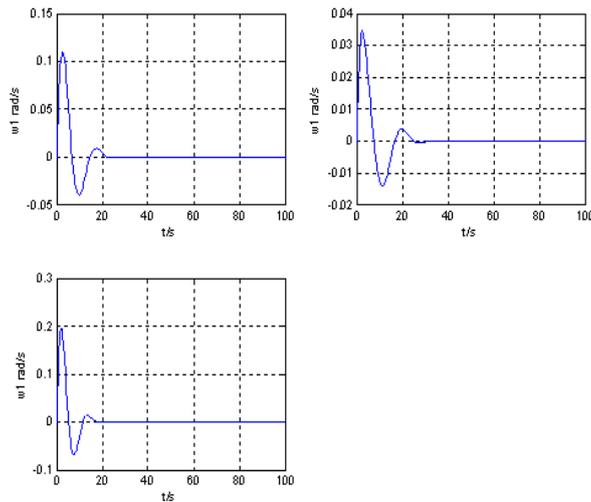


Figure 7. The angular velocity curve with the existence of model error and model parameter uncertainty (Condition: Case 4)

5. Conclusion

We considered the stability of spacecraft attitude in the presence of external disturbances and model uncertainties in this paper. A nonlinear robust controller is proposed by using passivity-based approach and introducing the suppression vector of external disturbance into the control law. The proposed controller does not need the angular velocity measurement and can suppress the effect of external disturbance to a certain extent. In addition, the control law doesn't contain information related to the system parameters, which makes the spacecraft attitude control system robust to model error and model parameter uncertainty. The stability of the proposed controller is proved theoretically and the numerical simulation results illustrated the effectiveness and robustness of the spacecraft attitude control method.

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