

Stabilizing Planar Inverted Pendulum System Based on Fuzzy Nine-point Controller

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Abstract

In order to stabilize planar inverted pendulum, after analyzing the physical characteristics of the planar inverted pendulum system, a pendulum nine-point controller and a car nine-point controller for X-axis and Y-axis were designed respectively. Then a fuzzy coordinator was designed using the fuzzy control theory based on the priority of those two controllers, and the priority level of the pendulum is higher than the car. Thus, the control tasks of each controller in each axis were harmonized successfully. The designed control strategy did not depend on mathematical model of the system; it depended on the control experience of people or the control experts. The compared experiments showed that the control strategy was easy and effective, what was's more; it had a very good robust feature.

Keywords: planar inverted pendulum; nine-point controller; fuzzy coordinator

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1. Introduction

Inverted pendulum is an absolute instable, high-level, multi-variable, less-driven and strong coupling nonlinear system. These characteristics make it an ideal device to examine if a control method is available. Basing on the base motion forms, inverted pendulum is divided into linear inverted pendulum, circular inverted pendulum and planar inverted pendulum, among these forms of inverted pendulum, planar inverted pendulum has a stronger features of high-level, multi-variable and strong coupling. So, there are more challenges to research the planar inverted pendulum, and the control technologies basing on the planar inverted pendulum have been widely used in rocket launching, robot control and other field in industry control. In the literature [1-4], the research basing on planar inverted pendulum system, specifically, in [1-2], sliding mode control is applied to stabilize the planar inverted pendulum; in [3], sliding mode control theory is used to control the circular movement and the stabilization; in [4], the LQR control method is used to control the stabilization of planar inverted pendulum basing on the mathematical model of the system; in [1-4], all the control behavior is based on the mathematic model of the system in different degrees. In [5-11], the control behavior is based on linear inverted pendulum; in [5], the variable universe adaptive fuzzy control is used to stabilize the quadruple inverted pendulum, and the mathematical model parameters of high degree accuracy is applied in deducing the deviation fusion function; in [6], the cloud control method is used to stabilize the triple inverted pendulum system; in [7-8], the human simulated intelligent control theory is used to stabilize the linear multi-pendulum system, and the LQR theory is used to steady the pendulum, which is also based on mathematical model; in [9], the fuzzy control theory is used in swinging up and the LQR theory is used to steady the pendulum. Among the present intelligent control theory in inverted pendulum control, in [10], it's adaptive fuzzy control method; in [11], it's the single rule of fuzzy control method; in [10-11], there's no mathematical model, but they are all based on linear inverted pendulum and there are more challenges for stabilizing the planar inverted pendulum.

Nine-point controller [12] is a new intelligent control theory and it's based on pan-Boolean algebra theory. In which, a system is divided into plus-minus nine control effect with strong and weak differ. Basing on the change of the error and its derivative in the system, the control effect is changing over time. In [13], a fuzzy nine-point controller for linear inverted pendulum is designed for the first time.

In this paper, the fuzzy nine-point controller is designed for planar inverted pendulum system, compared to the control strategy in [1-9], there is no mathematical model and it has a good robust feature. It is a good control strategy for a single input multi-output (SIMO) complicated system with strong nonlinear feature.

2. System Model Description

The planar single inverted pendulum comprises two rails in X-axis and Y-axis, a cart, and a pendulum; as is shown in Figure1. The cart can move along the rail of Y-axis, while the whole rail of Y-axis can move along the rail of X-axis, in this way, the cart can move within the planar plane. The pendulum, which is set on the cart by a Hooke's joint, can rotate around X-axis and Y-axis. At any instant, the pendulum deviate from OZ direction θ_x within XOZ plane, as well as θ_y within YOZ plane; as is shown in Figure1.

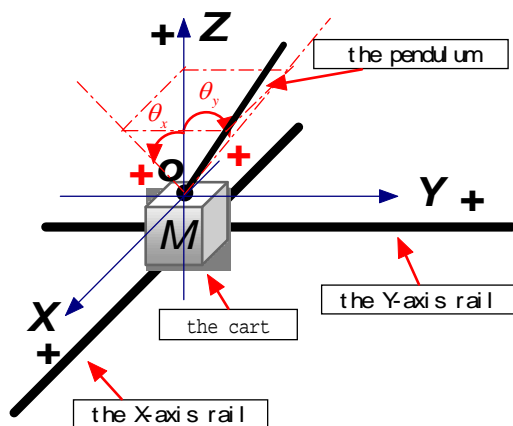


Figure 1. The setup of planar inverted pendulum system

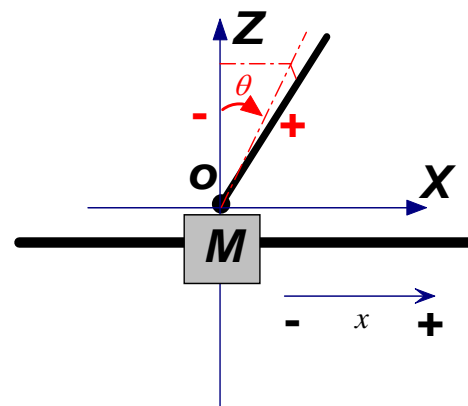


Figure 2. The setup of planar inverted pendulum in XOZ plane

Take the motion behavior in XOZ plane as an example, the pendulum, which is on the base cart, slide along the X-axis guide rail; as is shown in Figure 2.

In Figure 2, x is the the cart's displacement in X-axis with meter unit (To the left is the positive direction and to the right is the negative direction). θ represents the angle of the rod deviating from the upright position in XOZ plane (Clockwise direction is positive). According to the control experience, if the pendulum is deviating from the upright position clockwise, the pendulum controller should control the cart move to the right direction, in this way, the pendulum will go back to its equilibrium position by rotating anticlockwise. On the contrary, if the pendulum is deviating from the upright position anticlockwise, the pendulum controller should control the cart move to the left direction, and the pendulum will go back to its equilibrium position by rotating clockwise.

The cart controller will control the cart indirectly. If the cart is leaving the original point to its right direction, the cart controller should keep its move to the right direction, and this behavior will lead the pendulum deviate from the upright position clockwise, and at this moment the pendulum controller will have the cart move to the left direction, so the pendulum will rotate clockwise and go back to its equilibrium position, of course, the cart will go back to its original position automatically. If the cart is leaving the original point to its left direction, the movement behavior will be on the contrary as is described above.

According to the analysis above, in order to stabilize the planar inverted pendulum system, a strong coordinator is needed to harmonize the behavior between the pendulum controller and the cart controller. Basing on the physical understanding described above, the control stratege for the planar inverted pendulum system is designed, as is shown in Figure 3.

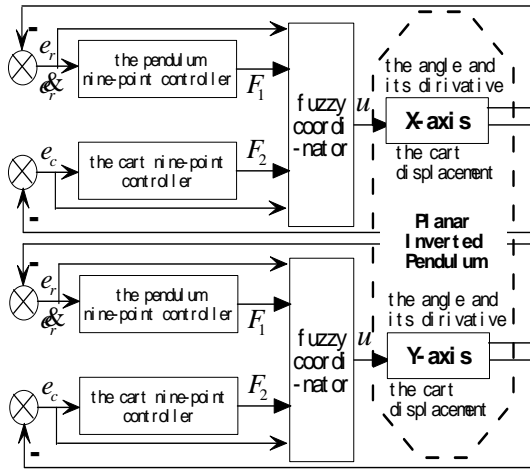


Figure 3. The scheme of the control system

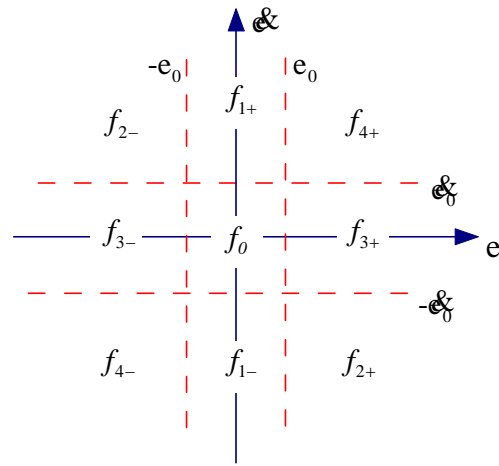


Figure 4. The theory of nine-point controller

The motion behavior in YOZ plane is similar with XOZ plane. Here no longer repeat them.

3. Design the Controller

The controller design includes designing the pendulum nine-point controller and the cart nine-point controller of the planar inverted pendulum shown in Figure 3, as well as the fuzzy coordinator in Figure 3.

3.1. The Nine-point Controller

The nine-point controller^[12] theory, as is shown in Figure 4, it divide the phase plane into nine control zones, basing on the zero zone of deviation $|e| \leq e_0$ and the deviation derivative $|\dot{e}| \leq \dot{e}_0$. Then the control effect of the whole system is divided into nine part, with plus-minus differ, as well as strong and weak differ.

For example, if the system state is $e > e_0$ and $\dot{e} > \dot{e}_0$, it means the error is very big in plus direction, so the system need the strongest control effect f_{4+} with plus direction, in this way to enlarge the output and reduce the deviation; the other eight zones can be done in the same manner, if the system state is $e > e_0$ and $|\dot{e}| \leq \dot{e}_0$, apply the strong control effect f_{3+} with plus direction to the system; if the system state is $e > e_0$ and $\dot{e} < -\dot{e}_0$, apply the weak control effect f_{2+} with plus direction to the system; if the system state is $|e| \leq e_0$ and $\dot{e} > \dot{e}_0$, apply the weakest control effect f_{1+} with plus direction to the system; if the system state is $e < -e_0$ and $\dot{e} < -\dot{e}_0$, apply the strongest control effect f_{4-} with minus direction to the system; if the system state is $e < -e_0$ and $|\dot{e}| \leq \dot{e}_0$, apply the strong control effect f_{3-} with minus direction to the system; if the system state is $e < -e_0$ and $\dot{e} > \dot{e}_0$, apply the weak control effect f_{2-} with minus direction to the system; if the system state is $|e| \leq e_0$ and $\dot{e} < -\dot{e}_0$, apply the weakest control effect f_{1-} with minus direction to the system; if the system state is $|e| \leq e_0$ and $|\dot{e}| \leq \dot{e}_0$, apply the maintaining control effect f_0 .

Basing on the pendulum physical knowledges within the whole system and the nine-point controller theory mentioned above, if the pendulum is deviating from the upright equilibrium position with clockwise with a high angular rate, then the system state should be

$e > e_0$ and $\dot{e} > \dot{e}_0$, at this time the cart should move fastly to the right direction to help the pendulum back to its equilibrium position, so it should be the strongest control effect with plus direction. If the pendulum's behavior is within other control effect zone of the nine-point controller theory, the control effect is given in table 1, and table 1 is the pendulum nine-point controller (the control effect in the parentheses is for the Y-axis, which is different from the X-axis). Here e_r and \dot{e}_r is the pendulum error and its changing rate, and their zero boundary is $e_0=0.0087rad$ and $\dot{e}_0=0.01rad / s$ respectively.

Table 1. The Rod Nine-point Controller

	$\dot{e}_r > \dot{e}_0$	$ \dot{e}_r \leq \dot{e}_0$	$\dot{e}_r < -\dot{e}_0$
$e_r > e_0$	-19(21)	-10	-7
$ e_r \leq e_0$	-2.5	0	2.7(2.4)
$e_r < -e_0$	7.3(6.5)	10.5(9.5)	21(20)

Table 2. The Cart Nine-point Controller

	$\dot{e}_c > \dot{e}_0$	$ \dot{e}_c \leq \dot{e}_0$	$\dot{e}_c < -\dot{e}_0$
$e_c > e_0$	-2.5	-1	-0.7
$e_c \leq e_0$	-0.5	0	0.7
$e_c < -e_0$	0.9	1.8	3(2.5)

Basing on the analysis about the cart motion behavior and the nine-point controller theory, if the cart is leaving the original point to its left direction and keep enlarging the displacement, the system state is $e_c > e_0$ and $\dot{e}_c > \dot{e}_0$, the cart controller should help keep the cart moving to its left direction, in this way, the pendulum controller will be activated, so the control effect should be the strongest with minus dirction. Here table 2 shows the cart nine-point controller (the control effect in the parentheses is for the Y-axis, which is different from the X-axis). Here e_c and \dot{e}_c represents the cart's displacement error and its speed, and their zero boundary is $e_0 = 0.02cm$ and $\dot{e}_0=0.04m / s$ respectively.

3.2. The Fuzzy Coordinator

While applying the pendulum nine-point controller and the cart nine-point controller to the planar invereted pendulum system synchronously, a coordinator is needed. Take c_1 and c_2 as the coordinator of the pendulum nine-point controller output F_1 and the cart nine-point controller output F_2 respectively. The final control effect for each axis is $u = c_1F_1 + c_2F_2$. In order to coordinate the control effect on-line, the c_1 and c_2 is adjusted on-line with fuzzy control methods.

The priority task is steady the pendulum, then it is the cart's going back to original position; so the pendulum has a higher priority than the cart. Here choosing the absolute value of the pendulum deviation $|e_r|$ and its changing rate $|\dot{e}_r|$ as the leading variable of the fuzzy coordinator, and the universe for the $|e_r|$ and $|\dot{e}_r|$ is $[0, 15^\circ]$ and $[0, 1]$ respectively; the fuzzy set for $|e_r|$ and $|\dot{e}_r|$ is designed in Figure 5 and Figure 6 respectively. The outcome variables c_1 and c_2 , their universe are the same and it is $[0,1]$, and the fuzzy set for the coordinators is shown in Figure 7.

As the pendulum has a higher priority than the cart, so if the pendulum error and its changing rate is big, the pendulume controller should be the main control effect to help the pendulum go back to its upright equilibrium position as soon as possible. On the contrary,if the pendulum error and its changing rate is very small, the cart nine-point controller will be the main controller to help the cart go back to its original position. Basing on these knowledges, 25 fuzzy rules are designed in table 3. For example, if $|e_r|$ is A_1 and $|\dot{e}_r|$ is A_1 , so c_1 should be B_1 and

c_2 shall be B_9 ; and the rest can be done in the same manner. Table 3 shows all the fuzzy rules of c_1 and c_2 .

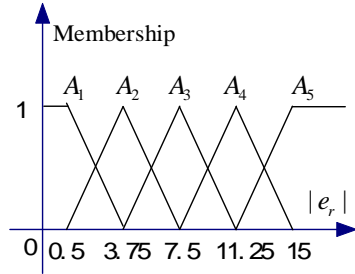


Figure 5. The fuzzy set of pendulum error

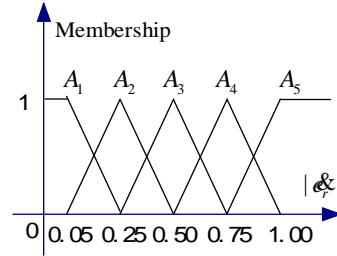


Figure 6. The fuzzy set of changing rate of pendulum error

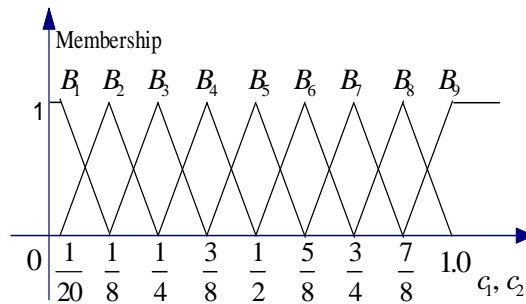


Figure 7. The fuzzy set of coordinator

Table 3. The Fuzzy Rules of Coordinator

	$ e_r = A_1$		$ e_r = A_2$		$ e_r = A_3$		$ e_r = A_4$		$ e_r = A_5$	
	c_1	c_2	c_1	c_2	c_1	c_2	c_1	c_2	c_1	c_2
$ e_r = A_1$	B_1	B_9	B_2	B_8	B_3	B_7	B_4	B_6	B_5	B_5
$ e_r = A_2$	B_2	B_8	B_3	B_7	B_4	B_6	B_5	B_5	B_6	B_4
$ e_r = A_3$	B_3	B_7	B_4	B_6	B_5	B_5	B_6	B_4	B_7	B_3
$ e_r = A_4$	B_4	B_6	B_5	B_5	B_6	B_4	B_7	B_3	B_8	B_2
$ e_r = A_5$	B_5	B_5	B_6	B_4	B_7	B_3	B_8	B_2	B_9	B_1

Basing on product inference engine, singleton fuzzifier and center average defuzzifier in fuzzy control theory^[14], the function of c_1 and c_2 are shown as function(1).

$$c_1 = k_1 \frac{\sum_{i=1, j=1}^{i=5, j=5} b_{(i+j-1)} u_{A_i}(|e_r|) u_{A_j}(|e_r|)}{u_{A_i}(|e_r|) u_{A_j}(|e_r|)}, \quad c_2 = k_2 \frac{\sum_{i=1, j=1}^{i=5, j=5} b_{(10-i-j)} u_{A_i}(|e_r|) u_{A_j}(|e_r|)}{u_{A_i}(|e_r|) u_{A_j}(|e_r|)} \quad (1)$$

In function (1), k_1 and k_2 are scaling factors to adjust the weight of c_1 and c_2 respectively, $b_{(i+j-1)}$ represents the central point of fuzzy set $B_{(i+j-1)}$ ($i, j = 1, 2, \dots, 5$), $u_{A_i}(|e_r|)$ and $u_{A_j}(|\dot{e}_r|)$ are the membership function of the fuzzy set A_i and A_j respectively.

4. Robust Control Experiment

Set up the simulation control system of planar inverted pendulum in MATLAB software basing on the designed control strategy in Figure 3. Choose 5 millisecond as the control cycle to stabilize the planar inverted pendulum. Set the deviation from the upright with $+5^\circ$ to start the experiment, and choose the scaling factor in X-axis with $k_1=3.4$ and $k_2=3.4$, and in Y-axis they are $k_1=3.4$ and $k_2=3.5$.

For the first experiment, the pendulum is a long rod with the length is 50 centimeters and the weight is 110 gram (Here, the “long” rod is differ from the “short” one with the length is 40 centimeters, which will be used in the second experiment). The outcome of this experiment are shown from Figure 8 to Figure 14.

Figure 8 shows that the pendulum will overcome the initial deviation in X-axis and Y-axis and keep a fine adjustment at last. In Figure 9, the biggest displacement in X-axis is 16 centimeter, while in Y-axis is 12 centimeter; the displacement in X-axis is a little bigger than in Y-axis, because the whole Y-rail is moving along the X-rail. Figure 10 and Figure 11 show that the pendulum/cart controller act as the pendulum/cart nine-point controller designed in table 1 and table 2; as time goes on, the error is smaller and smaller, so the changing over the nine zones more frequently and the curves are more dense.

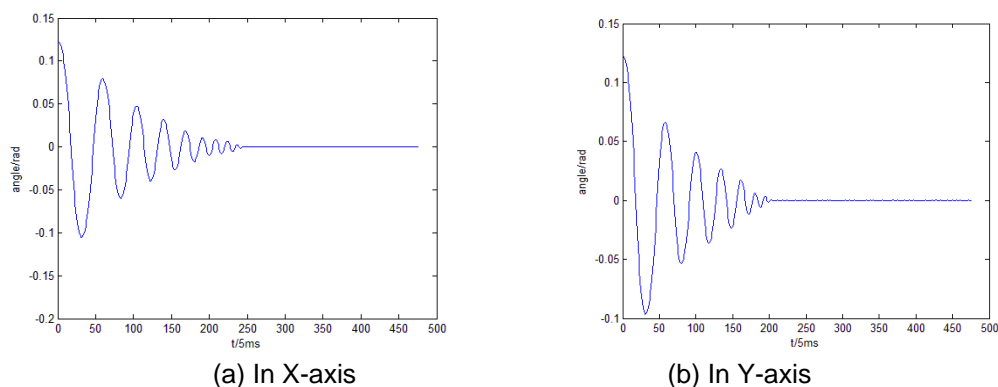


Figure 8. The pendulum error of long rod

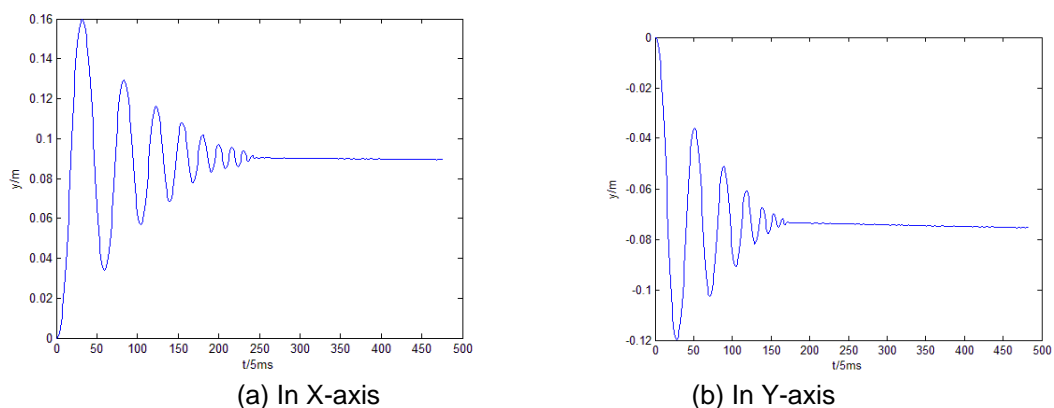


Figure 9. The cart displacement of long rod

Figure 12 and Figure 13 show the changes of c_1 and c_2 , in the beginning, the pendulum error is much bigger than the cart error, so c_1 has a bigger value than c_2 , later the pendulum error becomes smaller, and at this time c_2 has a bigger value than c_1 ; the curves show very well of the changing process, it also confirm that the priority of the pendulum controller is higher than that of the cart. Figure 14 gives the change tendency of final control effect, the output ranges from -60 to 70 in X-axis, and in Y-axis it is from -70 to 68, and they always keep changing slightly.

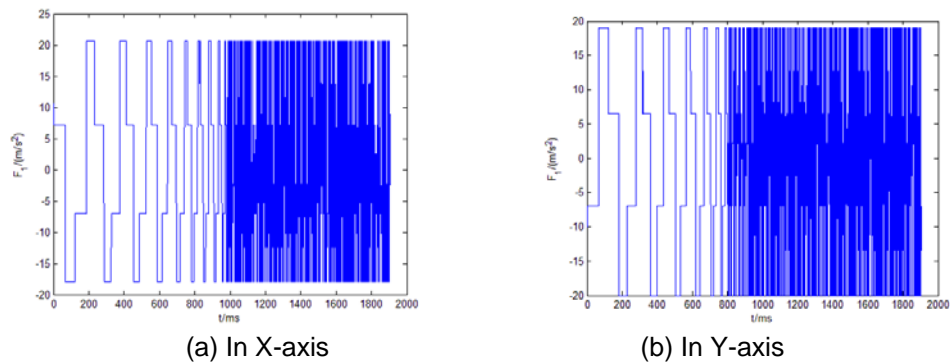


Figure 10. The pendulum nine-point controller output of long rod

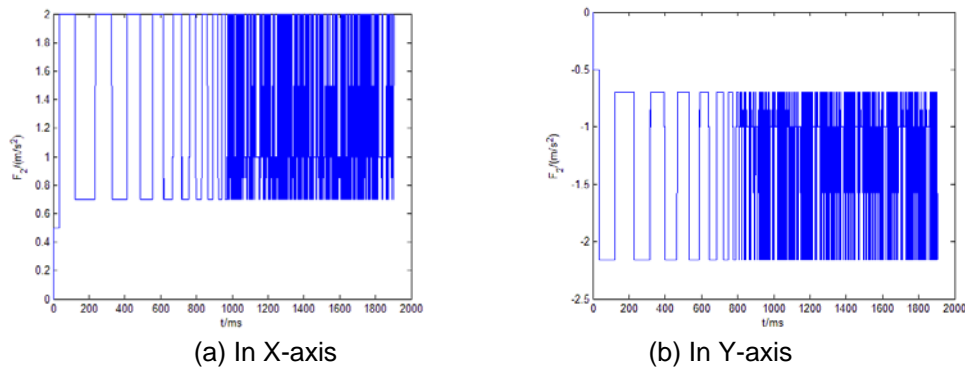


Figure 11. The cart nine-point controller output of long rod

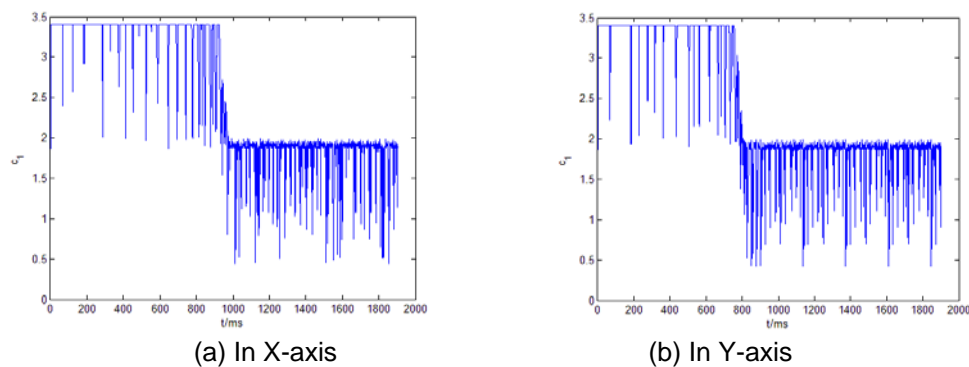


Figure 12. The change tendency of c_1 with long rod

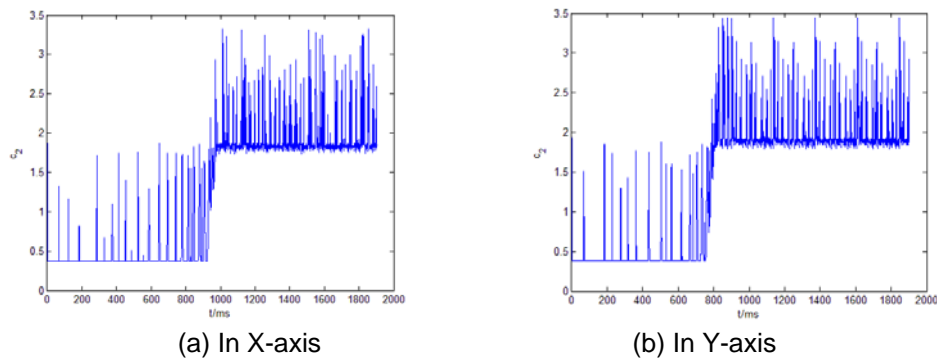


Figure 13. The change tendency of c_2 with long rod

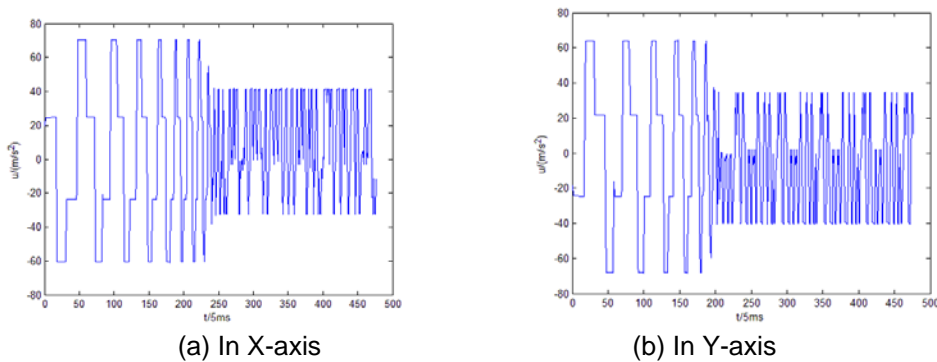


Figure 14. The control force u of long rod

In order to confirm the effectiveness of the designed control strategy, also to show its robust feature, a controlled trial is designed. This time the pendulum is replaced by a short rod with a length of 40 centimeter and the weight is 90 gram. Besides, all the other parameters of the controller is the same with the first experiment with a long rod. This time, the outcome of the experiment are shown as Figure 15 to Figure 21.

The outcome in Figure 15 is quite similar with Figure 8. In Figure 16 (a) and (b), the largest displacement in X-axis and Y-axis is 12.5 centimeter and 12 centimeter respectively. Both the pendulum and the cart can go back to its equilibrium position (For the displacement, it's very close to its original point) and keep fine adjustment.

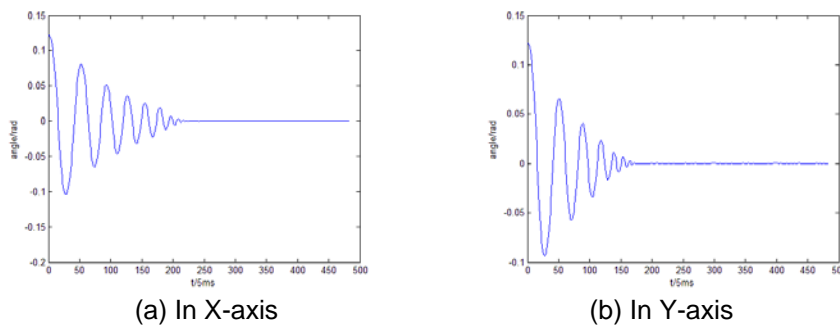


Figure 15. The rod error of short rod

Figure 17 and Figure 18 are similar to Figure 10 and Figure 11 respectively, the pendulum/cart controller act as the pendulum/cart nine-point controller designed in table 1 and table 2. Figure 19 and Figure 20 are similar to Figure 12 and Figure 13 respectively; the curves confirm that the priority of the pendulum controller is higher than that of the cart. In Figure 21 (a) and (b), the range of the control force in X-axis is $[-60, 70]$, and in Y-axis, it is $[-70, 65]$.

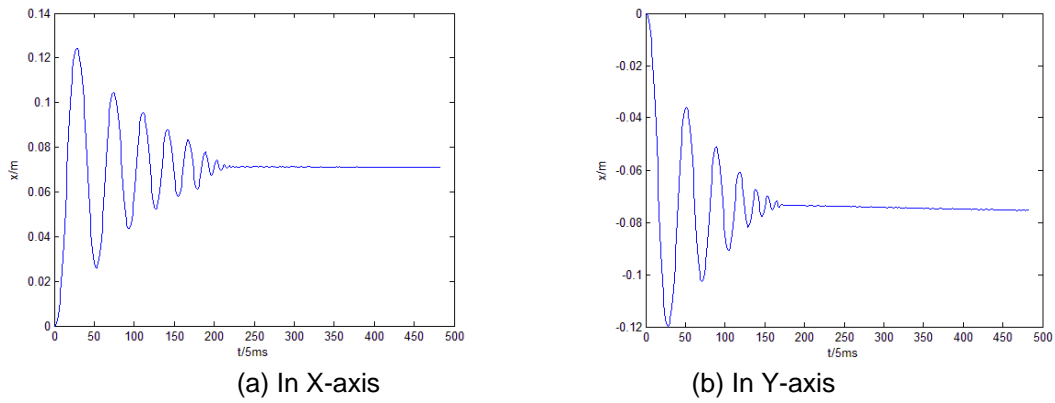


Figure 16. The cart displacement of short rod

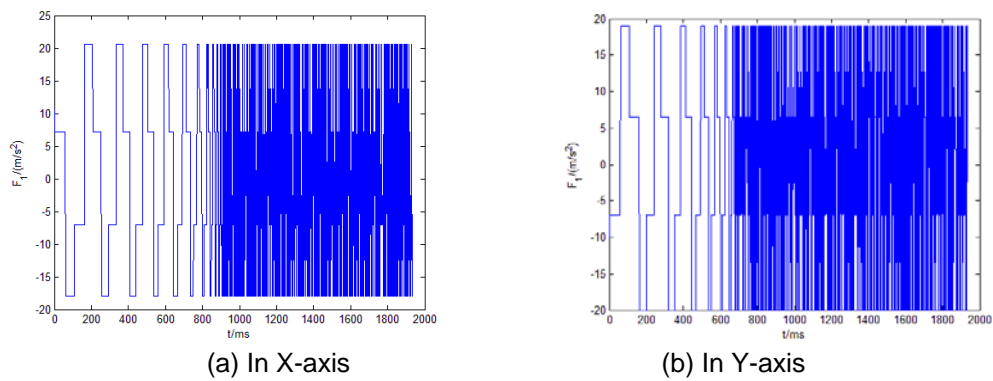


Figure 17. The pendulum nine-point controller output of short rod

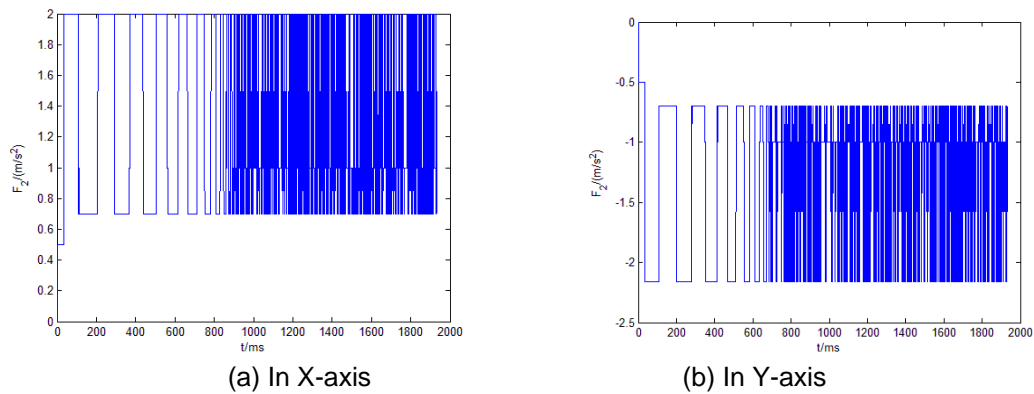


Figure 18. The cart nine-point controller output of short rod

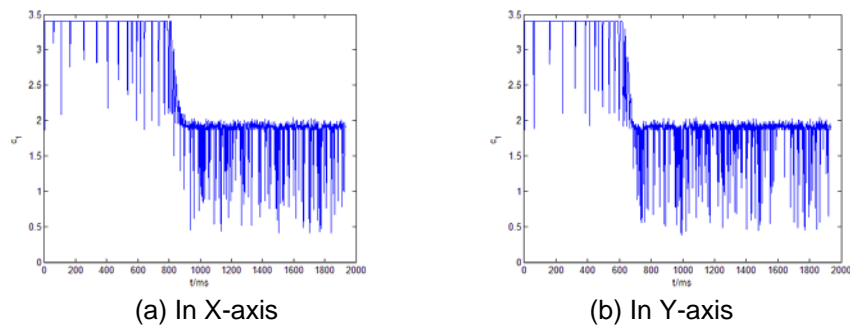


Figure 19. The change tendency of c_1 with short rod

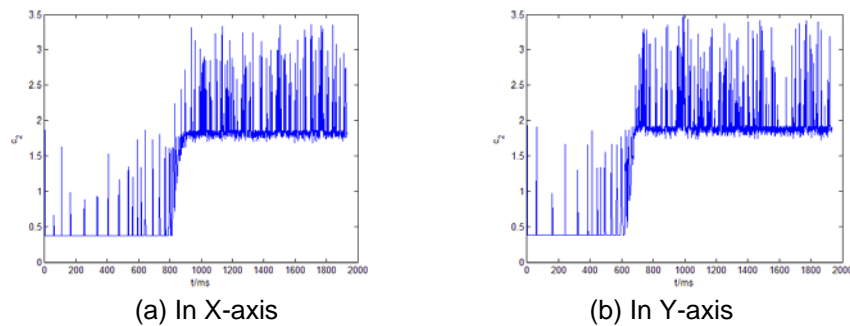


Figure 20. The change tendency of c_2 with short rod

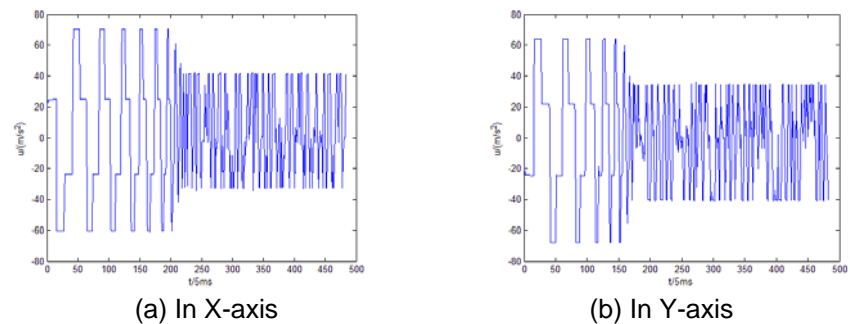


Figure 21. The control force u of short rod

The robust control experiments show the effectiveness of the designed control strategy.

5. Conclusion

This paper analyze the physical features of planar inverted pendulum and its control logic, then design a pendulum nine-point controller and a cart nine-point controller for X-axis and Y-axis based on the nine-point controller theory. What's more, this paper designed a coordinator to harmonize these two nine-point controllers in each axis based on the priority of each controller. Robust control experiments show that the control strategy is effective, as well as has a good robust feature. It is a new control strategy for expressing human control experiments, and human beings or control specialists are always the master in control system.

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