

# Research of Image Compression Based on Quantum BP Network

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## Abstract

Quantum Neural Network (QNN), which integrates the characteristics of Artificial Neural Network (ANN) with quantum theory, is a new study field. It takes advantages of ANN and quantum computing and has a high theoretical value and potential applications. Based on quantum neuron model with a quantum input and output quantum and artificial neural network theory, at the same time, QBP algorithm is proposed on the basis of the complex BP algorithm, the network of a 3-layer quantum BP which implements image compression and image reconstruction is built. The simulation results show that QBP can obtain the reconstructed images with better quantity compared with BP in spite of the less learning iterations.

**Keywords:** Quantum Neural Network; Back-propagation algorithm; Quantum Back-propagation algorithm; image compressing

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## 1. Introduction

With all kinds of digital image communication technology development and wide application of multimedia technology, image data has become one of the major sources of information, therefore, improved image data compression technology of signal and information processing capacity is a key issue that needs to be urgently addressed .

For decades, Research on image compression technology has achieved substantial results, especially with similar artificial neural network for information processing abilities of the human brain, with its unique network of self-organization and structure, good fault tolerance, adaptive, application of image compression has become a highly promising modern compression methods. However, with the increase in the volume and complexity of information processing, neural networks for image compression limitations and shortcomings also came to the fore. These limitations and shortcomings limit the development of the theory of artificial neural networks, also contributed to the theory of artificial neural networks and other combination of interdisciplinary research. As quantum computing emerging out of the powerful calculation ability, many scholars began to consider extension of the neural network to the quantum field, using the unique mechanism and characteristics of quantum computing to improve performance of neural networks, creating an entirely new one by one quantum neural computation model of neural networks. ( Quantum Neural Networks, Referred to as QNN).

Quantum neural network combined with the advantages of quantum computing and neural network, which has a very high theory value and application potential. Group, this paper mainly studies the quantum gate phase shift gate and controlled not gate as the basic unit of quantum neuron model for calculating, depending on BP network theory for image compression, building a three-layer quantum neural network model for image compression. And the image compression and reconstruction process are analyzed, with the aid of plural BP learning rule, and define quantum back propagation algorithm (QBP algorithm) to train the network, to realize image block compression and image reconstruction. Simulation results show that quantum neural network and BP network compression than under the same circumstances, showed faster learning rate, reconstruction of quantum neural networks not only get better image quality, and in optimal learning rate number of iterations less than BP network by nearly half.

## 2. Quantum Computing Basis

### 2.1. Quantum Bit

Quantum bit with a classic bit of difference: a quantum bit is on the superposition of the  $|0\rangle$  and  $|1\rangle$ .

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad (1)$$

$\alpha$  and  $\beta$  is a complex number probability ranges here, met a demand of  $|\alpha|^2 + |\beta|^2 = 1$  ..Using a complex function to describe the state of the quantum state, complex function representation for:

$$f(\theta) = e^{i\theta} = \cos \theta + i \sin \theta \quad (2)$$

Where  $i = \sqrt{-1}$  is the imaginary unit,  $\theta$  phase of a quantum state.  $|0\rangle$  the probability amplitude is expressed in the real part of a complex function,  $|1\rangle$  the probability amplitude is expressed in its imaginary part. Is a quantum state can be described as:

$$|\psi\rangle = \cos \theta |0\rangle + \sin \theta |1\rangle \quad (3)$$

### 2.2. Quantum Gate

Quantum gate is the basis of the physical implementation of quantum computation, quantum gate can be made by a random quantum gate group network is adopted in this paper, which is made by a phase shift gate and two controlled not gate composition of the basic computing unit, as neural network activation function to constitute a new quantum neuron model. To facilitate the application, in the plural said of the quantum state and universal quantum logic gate group. According to the type (2) representation, a phase shift gate and two controlled not gate statement as follows.

$$\text{Phase-shift gate } f(\theta + \theta') = f(\theta) f(\theta') \quad (4)$$

It to quantum state phase shift conversion, make quantum state of a phase rotation Angle.

Controlled not gate(CNOT)

$$f\left(\frac{\pi}{2}\gamma - \theta\right) = \begin{cases} \cos \theta + i \sin \theta & (\gamma = 1) \\ \cos \theta - i \sin \theta & (\gamma = 0) \\ \text{else} & \end{cases} \quad (5)$$

Type,  $\gamma$  is a control parameter. When  $\gamma = 1$ , quantum state happen reverses; When  $\gamma = 0$ , although the  $|1\rangle$  probability amplitude phase happen turnover, but it was observed probability but remain unchanged, so depending on the quantum state for reverses; Different  $\gamma$  correspond to different quantum state, by changing the  $\gamma$  value to realize quantum state of evolution and transformation.

## 3. Quantum Neural Network

### 3.1. Quantum Neuron

Based on a phase shift gate and two controlled not gate a quantum neuron model as shown in Figure 1 shows. Its input using multiple superposition form, through the three quantum branch input quantum state of the amplitude and phase are processing, to get more superposition output.

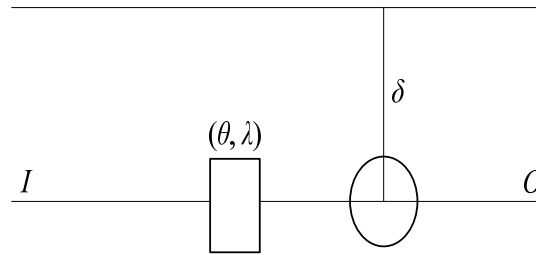


Figure 1. Quantum neuron model

In the quantum neuron model, there are two types of parameter form, one kind is corresponding to the phase shift door phase right value parameter  $\theta$  and threshold parameter  $\lambda$ ; Another flip  $\delta$  control parameters corresponding to the control gate. And the traditional neuron is different, quantum neuron weights in  $f(\theta)$  and input  $x = f(I)$  multiplication result, it is through the phase shift doors neurons state phase shift to achieve.

Here set  $I_i$  to the  $i$ -th input quantum state of the neuron to the  $x_i$  phase, the quantum neuron output formulization as follows:

$$u = \sum_{i=1}^n f(\theta_i) f(I_i) - f(\lambda) \quad (6)$$

$$y = \frac{\pi}{2} g(\delta) - \arg(u) \quad (7)$$

$$O = f(y) \quad (8)$$

$x_i (i=1,2,\dots,n)$  represents  $i$  input to the neural quantum,  $\theta_i (i=1,2,\dots,n)$  for the phase shift of weight coefficients,  $O$  as output state;  $\arg(u)$  is the extraction phase on complex numbers  $u$ ,  $\arg(u) = \text{actag}(\text{Im}(u) / \text{Re}(u))$ , where  $\text{Im}(u)$  for the sake of imaginary part of the complex number  $u$ ,  $\text{Re}(u)$  for the sake of real part of the complex number  $u$ ; The definition of function  $f$  such as formula (2);  $g(x)$  is sigmoid function.

### 3.2. Quantum Gate Neural Network Model of Image Compression

According to the above proposed quantum neuron model, with the aid of artificial BP neural network is used for image compression the principle and network structure, the establishment of used for image compression quantum gate neural network. In order to reduce the operation of the system time and improve the efficiency of the network, the established for image compression quantum BP network only one hidden layer, the network structure as shown in Figure 2 shows. The Figure 2 can see its network structure and the traditional three layer BP network, only one hidden layer neurons, the same between no connection between layer and layer, each neuron is to full connection. Different is its input and output is more of a quantum state is superposition and neurons of the different structure.

Image compression theory of quantum neural network and BP networks, is to make the number of hidden layer neurons of less than the input and output layers, image after you enter data into the input layer, forced through the waist hidden layer of compression purposes, and then by the hidden layer to output layer decoding of image reconstruction. It is shown in Figure 2 of the three layer quantum gate neural network, the network's input layer and output layer take equal neuron number  $N$ ,  $N$  the size of the original image in the compression process depending on the situation to determine, each neuron corresponds to a pixel; Number of hidden neurons for  $K$ , and  $K < N$ ,  $K$  depending on the compression ratio to determine the size.

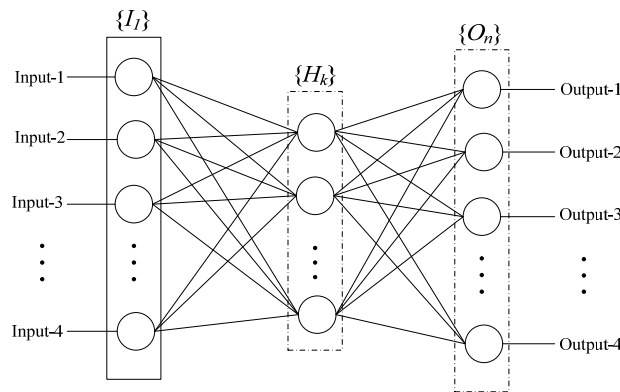


Figure 2. Image compression three layer quantum neural network

#### 4. Quantum Gate Neural Network Image Compression Process Analysis

##### 4.1. Network Parameter Setting

Quantum gate neural network input/output is superposition form, its quantum state using complex function to describe, so network internal parameters of the initial value is its quantum state phase value. The selection of network initial parameter values for quantum BP neural network learning rate and convergence has a great influence. In quantum gate neural network, the phase of the right value parameters and threshold parameters can take any value between the random number  $[-\pi, \pi]$  to take initialized, and phase control parameters  $\delta$  can be through the sigmoid function  $g(\delta)$  make it normalized to the valid range. In the network training process, the network learning coefficient size affects the average iterations and convergence speed.

##### 4.2. Image Compression Process

The original image ( $X \times Y$ ) pixels) divided into each of the overlap of the small pieces, each small piece has  $X_m \times Y_m$  pixel, and for each small piece of pixel value is normalized, its pixels are normalized to  $[0, 1]$  range, will be after the normalization of original image pixels as input mode and target mode, the input to the Figure 2 shows the quantum neural network, the input layer and input layer neuron number  $N = X_m \times Y_m$ . Input mode is input to the input layer, the input layer neurons will make  $(0,1)$  of the input value convert  $[0, \pi/2]$  quantum state between the phase value, its phase modulated by the quantum state of the input layer by each neuron output. Expression is:

$$y_{I_l} = \frac{\pi}{2} I_l \tag{9}$$

$$IO_l = f(y_{I_l}) \tag{10}$$

Among them, the  $I_l (l=0,1,\dots,N)$  said the  $l$ -th input layer neurons input. The  $IO_l$  as the  $l$ -th input layer neurons output, the output of the quantum state was sent to hidden processing, realize image compression coding.

The output of the hidden layer of each neuron, the expression for

$$u_{1k} = \sum_l^N f(\theta_{1k,l}) * IO_l - f(\lambda_{1k}) \tag{11}$$

$$y_{2k} = \frac{\pi}{2} g(\delta_{1k}) - \arg(u_{1k}), \quad k = 0,1,\dots,K \tag{12}$$

$$H_k = f(y_{2k}) \quad (13)$$

Among them,  $\theta_{lk}$  is entering the  $l$ -th neuron hidden layer neurons in the  $k$ -th phase rotation coefficient,  $\lambda_{1k}$  is the hidden layer neurons  $k$ -th threshold factor,  $\delta_{1k}$  is the hidden layer neurons of the  $k$ -th phase control factor,  $H_k$  is the output of the hidden layer neurons in the  $k$ -th.

Output from the output layer neurons, their expression is

$$u_{2n} = \sum_k^k f(\theta_{2n,k}) * H_k - f(\lambda_{2n}) \quad (14)$$

$$y_{3n} = \frac{\pi}{2} g(\delta_{2n}) - \arg(u_{2n}) \quad (15)$$

$$OP_n = f(y_{3n}) \quad (16)$$

Among them,  $\theta_{2n,k}$  is a hidden layer neurons of the  $k$ -th to the output layer neurons in the  $n$ -th phase rotation factor,  $\lambda_{2n}$  is the output layer neurons  $n$ -th threshold factor,  $\delta_{2n}$  is the output layer neurons of the  $n$ -th phase of control factor,  $OP_n$  is the output of the hidden layer neurons in the  $n$ -th.

Quantum in the neurons, quantum state  $|1\rangle$  the equivalent of neurons the activation state, suppression States of quantum state  $|0\rangle$  the equivalent of neurons, so any one quantum state is defined as the activation state of the neurons and inhibit the superposition of States, its final output value of the neuron is activated when the State probabilities.  $n$ -th three-layer BP neural network output layer neurons of the final output:

$$O_n = |\text{Im}(OP_n)|^2 \quad (17)$$

#### 4.3. Training Algorithm

BP algorithm with the plural[4] helps train the neural network of three quantum gates, the back-propagation in the definition of the network as a quantum reverse propagation algorithm (QBP algorithm), using the approximate mean square error of the steepest descent algorithm[5] to adjust network, the phase rotation factor  $\theta$ , the threshold factor  $\lambda$  and the phase control factor  $\delta$ .

The mean square error function  $E$  is defined as:

$$E = \frac{1}{2} \sum_n^N (t_n - O_n)^2 \quad (18)$$

$t_n$  is the desired output of the output layer neurons, and  $O_n$  is the actual output of the output layer neurons. The steepest descent method in accordance with the formula (17), formula (18) and formula (19) to reverse drill adjustment layers parameters until the error value is less than the target error.

$$\delta^{new} = \delta^{old} - \eta \frac{\partial E}{\partial \delta} \quad (19)$$

$$\theta^{new} = \theta^{old} - \eta \frac{\partial E}{\partial \theta} \quad (20)$$

$$\lambda^{new} = \lambda^{old} - \eta \frac{\partial E}{\partial \lambda} \quad (21)$$

Among them,  $\eta$  is the learning coefficient, which reflects the learning rate in the training. It has a great influence on the selection of the neural network training speed and precision of the parameter.

$$\delta_{1k}^{new} = \delta_{1k}^{old} - \eta \frac{\pi}{2} \sum_n d_n \arg'(u_{2n}) m_n g'(\delta_{1k}) \quad (23)$$

$$\theta_{1k,j}^{new} = \theta_{1k,j}^{old} + \eta \sum_n d_n \arg'(u_{2n}) m_n \arg'(u_{1k}) m_{1k} \quad (24)$$

$$\lambda_{1k}^{new} = \lambda_{1k}^{old} - \eta \sum_n d_n \arg'(u_{2n}) m_n \arg'(u_{1k}) s_{2k} \quad (25)$$

among them:

$$m_{1k} = \frac{\cos(\theta_{1k,j} + y_{1l}) \operatorname{Re}(u_{1k}) + \sin(\theta_{1k,j} + y_{1l}) \operatorname{Im}(u_{1k})}{(\operatorname{Re}(u_{1k}))^2} \quad (26)$$

$$s_{1k} = \frac{-\cos(\lambda_{1k}) \operatorname{Re}(u_{1k}) - \sin(\lambda_{1k}) \operatorname{Im}(u_{1k})}{(\operatorname{Re}(u_{1k}))^2} \quad (27)$$

Then, perform input/output of quantum calculation by the new parameters obtained by this three-layer again BP neural network. For there is no parameter in the input layer, the calculation starts from the hidden layer. Get the final output value network error after, if not less error if target is repeated error back propagation, adjusting parameters. Repeat the above course until the error is less than the target.

Trained network in the probability of the quantum state of the implicit layer quantum neuron output value is the result of data compression, whose value of the reconstructed image is the probability of the quantum state of the output value of output layer. Image reconstruction is the inverse process of image block, to restore the output image pixel image  $X \times Y$  reduced matrix elements, respectively, multiplied by 255, so that makes each pixel value recovered from [0,1] to [0,255], which was rebuilt image then.

#### 4.4. Experimental Results and Analysis

Here, we use the image which has a 256 gray-scale value and  $256 \times 256$  pixels as the original image, via the above-mentioned analysis method to make the compression ratio is equal (or similar), separately taking the 64-8-64 structure QNN network and 64-9-64 BP network structure to compare.

BP compression rates were:  $R_{\Omega} = 0.15, R_c = 0.16$ . During the training process, the target error is 0.001, the maximum number of iterations is 5,000, the learning coefficient  $\eta$  values between 0.01 and 5.0. By training each value for 20 times, Figure 3 shows the web average number of iterations of the partial values. As can be seen from Figure 3, the average number of iterations in the same  $\eta$  the quantum door neural network needs are less than BP, and easily achieves convergence. The quantum gates neural networks need 586 iterations at the best coefficient of 3.6 to achieve convergence while BP needs 1084 iterations to converge in best coefficient of 0.09. Figure 4 shows the reconstructed image compression.

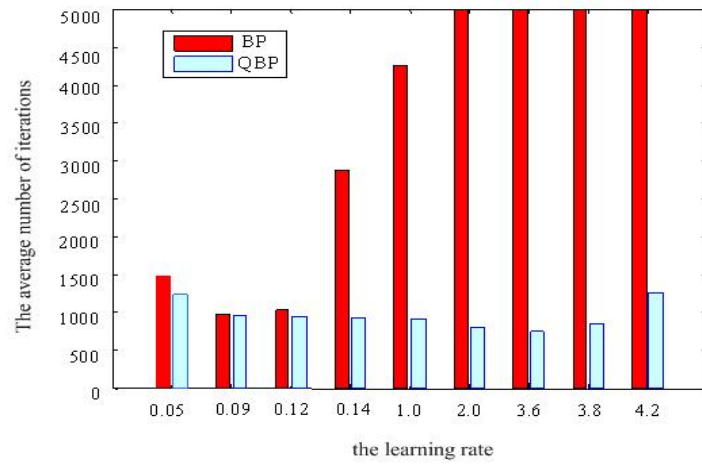


Figure 3. Average number of iterations of the network with different  $\eta$



(A) Original image



(B) QNN rebuild image



(C) CBP rebuild image

Figure 4. The best learning rate network reconstructed image

Table 1 shows its compression performance of the reconstructed image corresponding reconstructed image signal-to-noise ratio and peak signal-to-noise ratio in the comparison table,

quantum gates, the value of the neural network BP, the good quality of the reconstructed image of the neural network of quantum gates. In the course of the experiment, only when  $\eta$  has a value between 0.01 and 1.4, do BP's networks convergence.

Table 1. Compression performance value of the reconstructed image

Beast learning coefficient	Compression rate R	SNR(dB)	PSNR(dB)	Epochs
QNN ( $\eta=3.6$ )	0.15	42.75	45.13	586
BP ( $\eta=0.09$ )	0.16	42.74	45.08	1084

The experimental results show that, in the situation of the compression ratio is almost the same, the BP network neural network of quantum gates not only get better reconstructed image quality, but the number of iterations in the optimal learning rate nearly half less than the BP network, so the quantum neural network used in the field of image compression has the obvious advantages over BP network.

## 5. Conclusion

This paper mainly studies the quantum gate group as the basic calculation unit quantum neuron model, and with the aid of plural BP learning rule and BP neural network used for image compression principle, construct a used for image compression QBP, can be applied to image compression. For the controlled negater of the quantum neuron structure has the function of microcosmic emendation for output, and of for quantum BP neuron can play the role of output, and network calibration parameter, and phase shift parameters exists in the network which varies from 0-1, this will make the quantum neuron output continue to vary and to the minimum. The experimental results show that QBP show faster learning speed, is better than BP network better image compression ability.

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