

## Radiation Characteristics of a Plasma Column Antenna

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### Abstract

By means of the study on electromagnetic (EM) waves propagating along the plasma column, it is found that the radiation of a plasma column is caused by two kinds of operation modes. The first one is a guided mode which leads to a surface current radiation, and its operation condition is obtained by boundary conditions and physical properties of plasma. The second one is a leaky mode which results in a leaky wave radiation, and its operation condition is presented by analyzing normalized complex propagation constant. With further dealing with the leaky mode, its far field is got by applying the Schelkunoff's equivalence principle on the surface of the plasma column. Radiation patterns of two kinds of modes are presented in their respective operation frequency bands, and reveal the radiation characteristics of the plasma antenna. The presented results are very helpful for further investigation in the design and application of plasma column antenna.

**Keywords** plasma column antenna, guided mode, leaky mode, operating condition, radiation characteristics, Schelkunoff's equivalence principle

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### 1. Introduction

Recently, the concept of plasma antenna is well-accepted by more and more people, and has also received considerable interests for various engineering applications, especially in military communications. This is because a self-sustained plasma column can transmit radio frequency (RF) signals as a metal antenna and has a stealth property when plasma is de-energized. So far, there are many works concerning the plasma antenna in many scientific or technical documents. Through typical experimental arrangement, Borg [1] exhibited plasma columns could be used instead of metal antennas in communications systems, and the current distribution along the column could be controlled by the plasma density. Rayner [2] and Kumar [3] further examined the current and conductivity distributions, power patterns, directivity and efficiency of such a plasma column antenna by experimental observations. And Liu [4], Wang [5], Ding [6] and Li [7] studied radiation performances of the plasma column antenna by analytical solutions or numerical simulations. It is worth noting that all plasma antennas analyzed by above researchers radiate from guide modes. However, from these studies it is very difficult to infer the information of a guided mode's working condition required in our theoretical modeling and analysis. Moreover, the radiation of leaky mode did not involve.

In this paper, based on the theory of plasma column waveguides, the operation modes are studied when the plasma antenna is working in radiation condition. The cutoff frequency of guided mode is got through boundary conditions and the radiation pattern is given by developing the surface electric current distribution which is similar with the metal antenna. The operation condition of leaky mode is obtained by analyzing the normalized complex propagation constant, and its radiation field is derived from Schelkunoff's equivalence principle. Finally, it is found that leaky modes could be used for antenna applications at high frequency.

## 2. Radiation of Guided Mode for Plasma Antenna

### 2.1. Operation Condition of Guided Mode

Plasma consists of electrons, ions and neutrons, and it is neutral. In the absence of a magnetic field, the cold plasma is a disperse material[8]. For the special case of negligible collisions, the plasma relative dielectric constant is given by [9]

$$\varepsilon_r = 1 - \omega_{pe}^2 / \omega^2 \quad (1)$$

$$\omega_{pe}^2 = n_e e^2 / m_e \varepsilon_0 \quad (2)$$

where  $\omega_{pe}$  is the plasma angle frequency,  $\omega$  is the angle frequency of incident EM wave,  $n_e$  is the plasma density,  $e$  is the electron charge, and  $m_e$  is the mass of the electron.

When a TM wave is incident upon an interface between dielectric and plasma as shown in Fig.1, where  $\varepsilon_d$  and  $\varepsilon_p$  are relative permittivity of dielectric and plasma, the magnetic field can be written as

$$H_y = e^{-j(k_x x + k_z z)} \quad (3)$$

Corresponding electric field in the z-direction will be obtained from Maxwell's equations as following.

$$E_z = -\frac{1}{j\omega\varepsilon} \frac{\partial H_y}{\partial x} \quad (4)$$

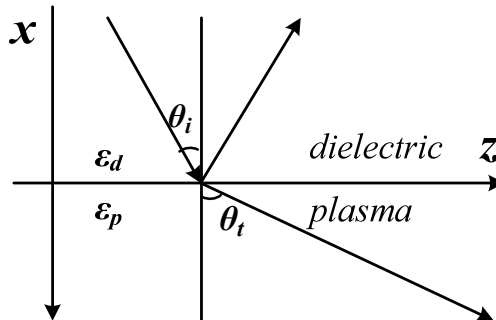


Figure 1. Plane wave incident on a plane boundary

Because the tangential components of the electric field are continuous at the interface, namely,  $E_{dz} = E_{pz}$ , the normal components of the wave-number vector at the interface is got as

$$k_{dx} / \varepsilon_d = k_{px} / \varepsilon_p \quad (5)$$

In terms of Snell's Law, the boundary condition for the tangential component of the wave-number vector is given by

$$k_{dz} = k_{pz} = k_z \quad (6)$$

where

$$k_{dx}^2 + k_{dz}^2 = \varepsilon_d \omega^2 / c^2 \quad (7)$$

$$k_{px}^2 + k_{pz}^2 = \varepsilon_p \omega^2 / c^2 \quad (8)$$

$k_{dx}$  and  $k_{px}$  denote the x-direction components of the wave-number vector for dielectric and plasma,  $k_{dz}$  and  $k_{pz}$  are the z-direction components of the wave-number vector for dielectric and plasma, the quantity  $c$  is the speed of light.

From equations of the boundary conditions, Eq.(5) is established only when  $k_{dx}$  and  $k_{px}$  are imaginary numbers with opposite signs, and has the exact physical meaning. In this case, the EM wave will be decreased exponentially along two directions normal to the interface, and the main energy of that travels on the surface of plasma. Substitute Eqs.(1), (6), (7), and (8) into Eq.(5), we have

$$k_x^2 = \frac{\omega^2}{c^2} \frac{\varepsilon_d(\omega^2 - \omega_p^2)}{(\mathbf{1} + \varepsilon_d)\omega^2 - \omega_p^2} \quad (9)$$

From the above equation, it is found that the surface wave resonates with plasma when  $k_x$  approaches to infinite, that is

$$\omega_p = \omega \sqrt{(\mathbf{1} + \varepsilon_d)} \quad (10)$$

According to the equations (9) and (10), when the dielectric is air, the condition of existence of guided mode is that  $\omega < \omega_p / \sqrt{2}$ , in this case,  $k_x^2 < \mathbf{0}$  and the surface wave will appear in the interface between air and plasma.

Based on the equation of the cutoff frequency for surface wave (guided mode), we will examine the dispersion relation on the propagation constant and signal frequency, and address to the on-going analysis of the radiation for the plasma antenna.

## 2.2. Physical Model of Guided Mode

Consider an isotropic, homogeneous, and cold plasma column. Generally, surface wave is obtained as a superposition of TE and TM modes. However, in a cylindrical symmetric configuration, pure surface wave requires TM mode propagation. So we may get the plasma surface wave dispersion relation for TM mode by Helmholtz's wave equation and boundary conditions, that is [1]

$$\varepsilon_r T_0 I_1(T_p a) K_0(T_0 a) + T_p K_1(T_0 a) I_0(T_p a) = \mathbf{0} \quad (11)$$

where  $T_p^2 = k^2 - \varepsilon_r k_0^2$  and  $T_0^2 = k^2 - k_0^2$ ,  $a$  is the radius of the plasma column,  $k_0 = \omega / c$  is the wave number of free space,  $k$  is the axial propagation constant.  $I_i(\cdot)$  and  $K_i(\cdot)$  denote modified Bessel functions of the first and second kind, respectively.

For a plasma column antenna, supposing the plasma frequency is  $f_p = 2\text{GHz}$ , its size are of length of 1.2m and radius of 0.0125m. Fig.2 shows the propagation constant of the surface wave propagating along the plasma column. It is seen clearly from this figure that there is a high frequency cutoff for signal frequency at 1.41GHz, which can conform to the obtained working condition for a guided mode.

As signal frequency is well below the plasma frequency, the electromagnetic wave propagates along the surface of plasma instead of going deep into it [10], and the plasma exhibits property of conductor. As approximation, the surface current distribution of the plasma column is

$$I(z) = I_0 (e^{jk(l-z)} - e^{-jk(l-z)}) \quad (12)$$

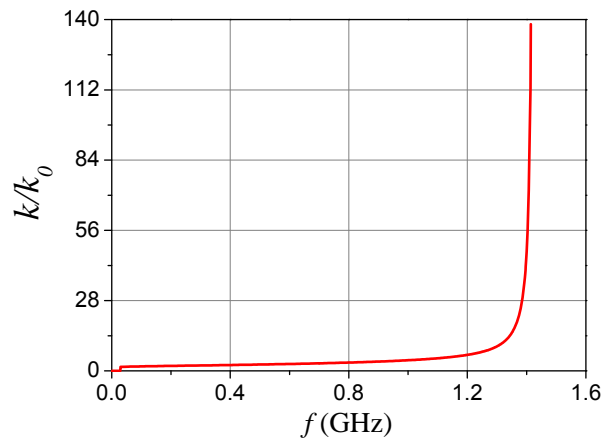


Figure 2. Propagation constant of the surface wave propagating along the plasma column

### 2.3. Result and Analysis

Figure 3 gives the elevation radiation patterns of the monopole plasma antenna at the given signal frequency ( $f=800\text{MHz}$ ) for different plasma frequencies. In Figure 3, When the plasma frequency is reduced, the main lobe of plasma antenna will gradually split into two. The metal antenna can also implement this change of radiation pattern, but it must modify its antenna electrical length, which would be a troublesome thing. So the pattern of the plasma antenna would be reconfigured easily by changing plasma frequency which could be controlled by plasma density due to Eq.(2). The change of the pattern is not very obvious in high plasma frequency with the increase of plasma frequency, the reason is that the propagation constant along axial direction varies slowly in high plasma frequency band for a given signal frequency.

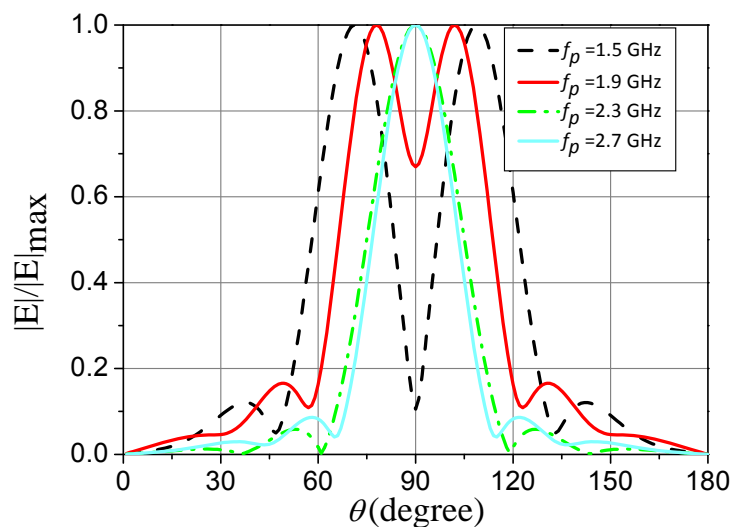


Figure 3. Radiation patterns of the plasma antenna in different plasma frequencies.

## 3. Radiation of Leaky Mode for Plasma Antenna

### 3.1. Operation Condition of leaky Mode

Figure 4 shows the configuration of a plasma antenna ( $l=1.2\text{m}$  and  $a=0.0125\text{m}$ ), which is circular symmetry. Provided that a TM wave is fed into the plasma column, by using Borgnis

function method in the cylindrical coordinate, the equations of the EM fields in region *I* can be written as

$$\begin{aligned} E_{z1} &= \tau_1^2 A_1 J_0(\tau_1 \rho) e^{-j\gamma z} \\ E_{\rho 1} &= j\gamma \tau_1 A_1 J_1(\tau_1 \rho) e^{-j\gamma z} \\ H_{\phi 1} &= j\omega \varepsilon_1 \tau_1 A_1 J_1(\tau_1 \rho) e^{-j\gamma z} \end{aligned} \quad (13)$$

The equations of the EM fields in region *II* can be written as

$$\begin{aligned} E_{z2} &= \tau_2^2 A_2 H_0^{(2)}(\tau_2 \rho) e^{-j\gamma z} \\ E_{\rho 2} &= j\gamma \tau_2 A_2 H_1^{(2)}(\tau_2 \rho) e^{-j\gamma z} \\ H_{\phi 2} &= j\omega \varepsilon_2 \tau_2 A_2 H_1^{(2)}(\tau_2 \rho) e^{-j\gamma z} \end{aligned} \quad (14)$$

According to the continuous condition of tangential components of electric and magnetic fields at the interface, one obtains the dispersion equation for the plasma antenna.

$$\frac{\varepsilon_1 J_1(\tau_1 a)}{\tau_1 J_0(\tau_1 a)} - \frac{\varepsilon_2 H_1^{(2)}(\tau_2 a)}{\tau_2 H_0^{(2)}(\tau_2 a)} = 0 \quad (15)$$

where  $\tau_1^2 = \varepsilon_{r1} k_0^2 - \gamma^2$  and  $\tau_2^2 = k_0^2 \varepsilon_{r2} - \gamma^2$ ,  $\gamma (= \beta - j\alpha)$  is the complex axial propagation constant.  $J_i(\cdot)$  and  $H_i^{(2)}(\cdot)$  denote Bessel function of the first kind and Hankel function of the second kind, respectively.

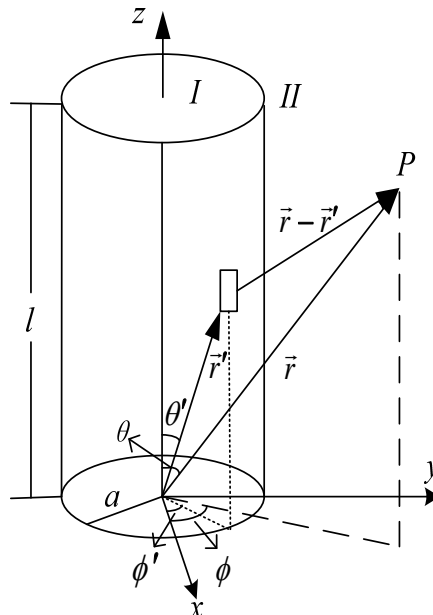


Figure 4. Geometry of plasma column antenna and spherical coordinate system for a differential equivalence element on the surface of the column

As we know, an open dielectric column waveguide can support both slow wave or surface wave for  $\beta/k_0 > 1$  and fast wave or leaky wave for  $\beta/k_0 < 1$ . When a plasma column operates in the leaky mode region, the axial propagation constant  $\gamma (= \beta - j\alpha)$  may be a complex number and be determined by a phase constant  $\beta$  and an attenuation constant  $\alpha$ , and we should note that  $\alpha$  is due to energy leakage rather than cutoff or plasma losses. According to Zeng's theory [11], when  $\beta/k_0 < 1$  and  $\beta/k_0 < \alpha/k_0$ , the wave energy is stored as a form of reactive energy in the structure, and cannot be leaked into the space. Then, to use plasma column as leaky wave antenna, the condition of antenna mode region (physical radiation region,  $\beta/k_0 < 1$  and  $\beta/k_0 > \alpha/k_0$ ) needs to be satisfied, where the wave energy can be radiated into the space. As shown in Fig.5, when normalized plasma frequency  $k_p a (= \omega_p a/c) = 0.526$  ( $f_p = 2\text{GHz}$ ), the normalized phase constant equals to zero at the plasma frequency, and quickly increases and approaches to unity as the normalized frequency  $k_0 a (= \omega a/c)$  becomes higher, the curves of  $\beta$  and  $\alpha$  intersect at the point A ( $k_0 a = 2$ ). Thus, if  $k_0 a$  is greater than 2, the plasma column antenna will work as leaky mode.

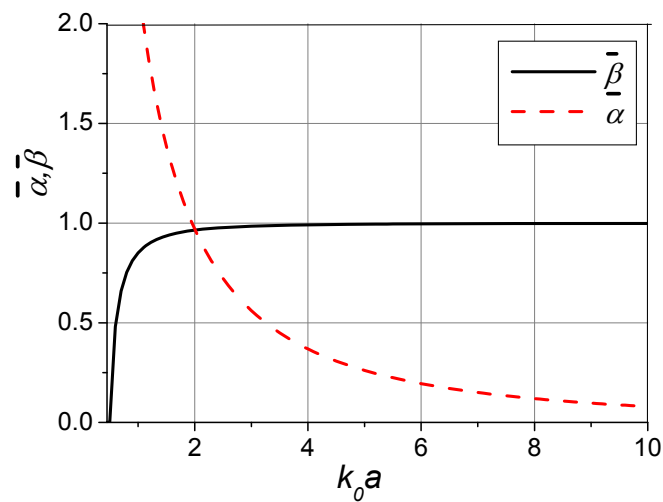


Figure 5. Phase constant and attenuation constant of the surface wave propagating along the plasma column

### 3.2. Physical Model of leaky Mode

Considering  $a \ll l$ , and ignoring the radiation of the ends of the column, the EM field in Fraunhofer zone is to be produced by sheet electric current and sheet magnetic currents on the cylindrical surface. So the far-zone electric field from the leaky mode of plasma antenna can be obtained by Schelkunoff's equivalence principle [12].

$$\vec{E}(\vec{r}) = -j\omega\vec{A}(\vec{r}) - j\frac{\nabla\nabla\cdot\vec{A}(\vec{r})}{\omega\mu\epsilon} - \frac{\nabla\times\vec{A}^m(\vec{r})}{\epsilon} \quad (16)$$

$$\vec{A}(\vec{r}) = \mu\int_S \vec{J}(\vec{r}')G_0(\vec{r},\vec{r}')ds' \quad (17)$$

$$\vec{A}^m(\vec{r}) = \epsilon\int_S \vec{J}^m(\vec{r}')G_0(\vec{r},\vec{r}')ds' \quad (18)$$

where

$$G_0(\vec{r}, \vec{r}') = \frac{e^{-jk|\vec{r}-\vec{r}'|}}{4\pi|\vec{r}-\vec{r}'|}, \vec{J} = \hat{a}_n \times \vec{H}|_{\rho=a}, \vec{J}^m = -\hat{a}_n \times \vec{E}|_{\rho=a}.$$

$\vec{r}$  is the distance vector from the origin to the field point, and  $\vec{r}'$  is the distance vector from the electric (magnetic) current element to the field point in far region. In far zone, the EM fields radiated from a plasma column can be computed in spherical coordinate system. Consider  $k \ll |\vec{r}-\vec{r}'|$ , so that we take  $1/|\vec{r}-\vec{r}'| \approx 1/r$ . Due to  $\vec{r} \parallel \vec{r}'$  in the far field, as the first approximation, we can take  $|\vec{r}-\vec{r}'| = r - \vec{r}' \cdot \hat{a}_r$ , then

$$E_\theta = -j \frac{\omega\mu}{4\pi r} e^{-jkr} \int_S \left( J_\theta - \sqrt{\frac{\varepsilon}{\mu}} J_\phi^m \right) e^{jk\vec{r}' \cdot \hat{a}_r} ds' \quad (19)$$

$$E_\phi = -j \frac{\omega\mu}{4\pi r} e^{-jkr} \int_S \left( J_\phi + \sqrt{\frac{\varepsilon}{\mu}} J_\theta^m \right) e^{jk\vec{r}' \cdot \hat{a}_r} ds' \quad (20)$$

So it is the first step to convert sheet electric and magnetic currents in cylindrical coordinate system into that in spherical coordinate system, then the sheet electric current can be expressed as

$$\vec{J} = (\hat{a}_r \cos \theta - \hat{a}_\theta \sin \theta) H_\phi \quad (21)$$

The sheet magnetic current of the plasma column may be converted by the following equations.

$$J_\theta^m = J_\rho^m \cos \theta \cos(\phi - \phi') + J_\phi^m \cos \theta \sin(\phi - \phi') - J_z^m \sin \theta \quad (22)$$

$$J_\phi^m = -J_\rho^m \sin(\phi - \phi') + J_\phi^m \cos(\phi - \phi') \quad (23)$$

Then we get

$$J_\theta^m = E_{z'} \cos \theta \sin(\phi - \phi') \quad (24)$$

$$J_\phi^m = E_{z'} \cos(\phi - \phi') \quad (25)$$

Through observing Figure 4, we have

$$\vec{r}' \cdot \hat{a}_r = a \sin \theta \cos(\phi - \phi') + z' \cos \theta \quad (26)$$

Then Substitute Eq.(21), Eq.(24), Eq.(25) and Eq.(26) into Eq.(19) and Eq.(20), the far-zone field can be given by

$$E_\theta = -\frac{\omega\mu T_2 A_2}{4\pi r} a e^{-jkr} \left( \omega\varepsilon_2 H_1^{(2)}(T_2 a) \sin \theta f_1(\phi) - jT_2 H_0^{(2)}(T_2 a) f_2(\phi) \sqrt{\frac{\varepsilon}{\mu}} \right) f(\theta) \quad (27)$$

$$E_\phi = -j \frac{\omega\mu T_2 A_2}{4\pi r} a e^{-jkr} T_2 H_0^{(2)}(T_2 a) \cos \theta \sqrt{\frac{\varepsilon}{\mu}} f(\theta) f_3(\phi) \quad (28)$$

where

$$f(\theta) = \frac{e^{j(k \cos \theta - \gamma)l} - 1}{j(k \cos \theta - \gamma)}$$

$$f_1(\phi) = \int_0^{2\pi} e^{jka \sin \theta \cos(\phi - \phi')} d\phi'$$

$$f_2(\phi) = \int_0^{2\pi} \cos(\phi - \phi') e^{jka \sin \theta \cos(\phi - \phi')} d\phi'$$

$$f_3(\phi) = \int_0^{2\pi} \sin(\phi - \phi') e^{jka \sin \theta \cos(\phi - \phi')} d\phi'$$

### 3.3. Result and Analysis

Because  $E_\phi$  is equal to zero from Eq.(28), the far-zone electric field is only  $E_\theta$  which is determined by Eq.(27). Taking the maximum amplitude of the radiation electric field (when  $k_0 a$  is equal to 5.3) as reference, normalized radiation fields are described as shown in Fig. 6. From the figure, the main beam direction angle in E-plane (xz-plane) of the far-zone electric field decreases and the width of main beam becomes wider as the signal frequency increases. It is obvious that the trends of the first side lobe and main lobe are similar at high frequencies region. At the same time, the maximum amplitude of the radiation electric field increases as the signal frequency becomes bigger. So a plasma column can be used as an antenna when it works as a leaky mode, and the pattern is changed as the signal frequency becomes higher, that is to say that the radiation pattern also can be reconfigured by changing plasma frequency for a plasma column leaky wave antenna.

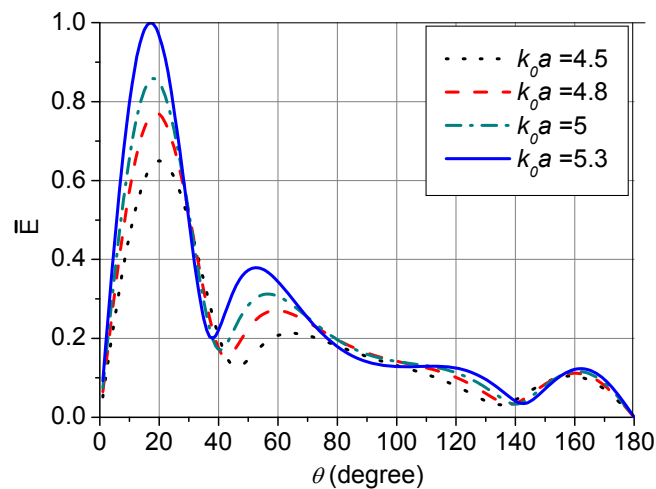


Figure 6. Radiation patterns of the plasma antenna at various normalized signal frequencies.

### 4. Conclusion

In this paper, a theoretical investigation of the radiation of a plasma column antenna has been undertaken. Since the physical property of plasma varies as the signal frequency due to its dispersive nature, it is found that there are two operation modes (used as radiation modes) that exist in different signal frequency bands for a given plasma frequency. The cutoff conditions of the two modes are presented by formula derivation and theoretical analysis for the plasma column. A plasma column works as a guided mode and radiates EM power as a metal antenna in the low frequency, and it may operate as a leaky mode and be used as a plasma column leaky wave antenna in the high frequency. The far field of the leaky mode is derived from Schelkunoff's equivalence principle. Radiation patterns of the two modes are given, and indicate that the pattern could be reconfigured conveniently by controlling plasma density due to the nature of plasma.



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