Zero Dynamics Analysis for Inverse Decoupling Control of Asynchronous Traction Motor

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Abstract

Considering the problem for inverse system method in EMU AC induction traction motor linear decoupling, the zero dynamics subsystem will be separated from the original dynamic system through coordinate transformation. Firstly, a getting method for zero dynamics of the multiple input multiple output nonlinear system is discussed when γ ﹤n. Second, the zero dynamics analysis for five order nonlinear model of asynchronous traction motor which base on the stationary coordinate system is given by using inverse decoupling method. The analysis results show that if the stability of the zero dynamics can be ensured, then the entire linearization of original nonlinear system is not necessary, need only partial linearization which effect on the external dynamic portion. The inverse decoupling process of asynchronous traction motor can be simplified by this conclusion.

Keywords: asynchronous traction motor, inverse decoupling control, nonlinear , zero dynamics

1. Introduction

AC induction traction motor has been widely used in high-speed EMU of CRH series. Asynchronous traction motor is a very complex nonlinear control object [1]. Due to cross-coupling between variables,the speed of the induction motor and the rotor flux must be dynamic decoupling to improve the control performance of the AC traction motor [2]. Inverse decoupling control method [3] is a nonlinear feedback linearization method which has intuitive simple and easy-to-understand features.The inverse system control method has be introduced into the field of AC variable speed by some scholars to achieve the stator flux and electromagnetic torque dynamic decoupling control [4-6].

When the system relative γ is less than the number of system order n,the zero dynamics will be separated from the original dynamic system through the coordinate transformation in Linearized decoupling of asynchronous traction motor which use the inverse system method. The zero dynamic is a internal dynamic behavior of the system, which with close links to the stability of the system.If the zero dynamics equation is unstable,then the linearized system is also unstable. It is necessary to analyze its zero dynamics of $\gamma < n$ system when using inverse system method to linearization.

The zero dynamics is discussed for the asynchronous motor direct feedback linearization in the literature [7,8].The first-order zero-dynamic stability is analyzed for the nonlinear system in the literature [9]. The zero dynamic characteristics is studied for the non-linezero control of DC motorin the literature [10]. This paper deals with the problem of the zero dynamic analysis for asynchronous traction motor nonlinear model which established in the stationary reference frame.We particularly focus our study on getting method of the zero dynamic for the multi-input multi-output affine nonlinear systems when $\gamma < n$.

2. Zero Dynamic of Multi-input Multi-output Nonlinear Systems

Consider the following nonlinear system

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i \qquad (1)$$

$$\begin{cases} y_1 = h_1(\boldsymbol{x}) \\ y_2 = h_2(\boldsymbol{x}) \\ \quad\vdots \\ y_m = h_m(\boldsymbol{x}) \end{cases} \tag{2}$$

Where, $\boldsymbol{x}$ is a n dimensional state vector, $\boldsymbol{f}(x)$ and $\boldsymbol{g}(x)$ are n dimensional smooth vector field, $\boldsymbol{u}_i$ for the i-th control amount, y_i for i-th output, $h_i(x)$ for $\boldsymbol{x}$ scalar function.

The system relative is $\gamma = \gamma_1 + \gamma_2 + \cdots + \gamma_m$, where each output $y_i = h_i(x)$ has a corresponding relative γ_i. We assume system relative is $\gamma = \gamma_1 + \gamma_2 + \cdots + \gamma_m < n$ and using the coordinates transformation $z = \varphi(x)$ as follows

$$\boldsymbol{z} = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{r_1} \\ z_{r_1+1} \\ z_{r_1+2} \\ \vdots \\ z_r \\ z_{r+1} \\ \vdots \\ z_n \end{bmatrix} = \begin{bmatrix} \varphi_{11}(x) \\ \varphi_{12}(x) \\ \vdots \\ \varphi_{1r_1}(x) \\ \varphi_{21}(x) \\ \varphi_{22}(x) \\ \vdots \\ \varphi_{mr_m}(x) \\ \varphi_{r+1}(x) \\ \vdots \\ \varphi_n(x) \end{bmatrix} = \begin{bmatrix} h_1(x) \\ L_f^0 h_1(x) \\ \vdots \\ L_f^{r_1-1} h_1(x) \\ h_2(x) \\ L_f^0 h_2(x) \\ \vdots \\ L_f^{r_m-1} h_m(x) \\ \varphi_{r+1}(x) \\ \vdots \\ \varphi_n(x) \end{bmatrix} \tag{3}$$

The system (1), (2) can be transformed into the following form by using the coordinates transformation $z = \varphi(x)$.

$$\dot{z}_1 = \dot\varphi_{11}(x) = \varphi_{12}(x)$$
$$\dot{z}_2 = \dot\varphi_{12}(x) = \varphi_{13}(x)$$
$$\vdots$$
$$\dot{z}_{r_1-1} = \dot\varphi_{1r_1-1}(x) = \varphi_{1r_1}(x)$$
$$\dot{z}_{r_1} = \dot\varphi_{1r_1}(x) = L_f^{r_1} h_1(x) + L_g L_f^{r_1-1} h_1(x) u$$
$$\dot{z}_{r_1+1} = \dot\varphi_{21}(x) = \varphi_{22}(x)$$
$$\dot{z}_{r_1+2} = \dot\varphi_{22}(x) = \varphi_{23}(x)$$
$$\vdots$$
$$\dot{z}_{r-1} = \dot\varphi_{mr_m-1}(x) = \varphi_{mr_m}(x)$$
$$\dot{z}_r = \dot\varphi_{mr_m}(x) = L_f^{r_m} h_m(x) + L_g L_f^{r_m-1} h_m(x) u$$
$$\dot{z}_{r+1} = \dot\varphi_{r+1}(x) = L_f \varphi_{r+1}(\varPhi^{-1}(z))$$
$$\vdots$$
$$\dot{z}_n = \dot\varphi_n(x) = L_f \varphi_n(\varPhi^{-1}(z))$$

Due to the system relative γ is less than the number of system order n, the remaining mapping relationship can be obtained by the coordinates transformation $z=\varphi(x)$. The remaining mapping relationship is described by:

$$\boldsymbol{\eta} = \left[z_{r+1}, z_{r+2}, \ldots, z_n\right]^{\mathrm{T}} = \left[\eta_1, \eta_2, \ldots, \eta_{n-r}\right]^{\mathrm{T}} \tag{4}$$

where the Jacobian matrix of Vector function $\varphi(x)$ is non-singular at $x = x^0$.

Generally, it is possible to appropriately select the system output function $h_i(x)$ so that the equilibrium point x^0 at the value zero. Considering the output $y_i = h_i(x)$ is essentially the dynamic deviations for the actual output of the system dynamic response with respect to the output function of the equilibrium point. If the dynamic deviations of system output is forced to be zero at any time by the control method, this means that system output remains unchanged in any interference. The system is highly stable from the external dynamic view. Let the output of system (1) and (2) as

$$y_1 = y_2 = \ldots = y_m = 0$$

That

$$h_1(\boldsymbol{x}) = h_2(\boldsymbol{x}) = \ldots = h_m(\boldsymbol{x}) = 0$$

And then

$$\begin{cases} \varphi_{11}(x) = \varphi_{12}(x) = \ldots = \varphi_{1r_1}(x) = 0 \\ \varphi_{21}(x) = \varphi_{22}(x) = \ldots = \varphi_{2r_2}(x) = 0 \\ \vdots \\ \varphi_{m1}(x) = \varphi_{m2}(x) = \ldots = \varphi_{mr_m}(x) = 0 \end{cases} \tag{5}$$

After the conversion, the former equations will disappear. The remaining dynamic equation are given by

$$\begin{cases} \dot{z}_{r+1} = \dot{\eta}_1 = \dot{\varphi}_{r+1}(x) = L_f \eta_1(x) \\ \dot{z}_{r+2} = \dot{\eta}_2 = \dot{\varphi}_{r+2}(x) = L_f \eta_2(x) \\ \vdots \\ \dot{z}_n = \dot{\eta}_{n-r} = \dot{\varphi}_n(x) = L_f \eta_{n-r}(x) \end{cases} \tag{6}$$

The system's internal dynamic behavior can be described by the set of differential equations as formula (6). So the equations which decided within the system internal dynamic behavior are called the zero dynamics equations for the original system (1), (2), referred to as the zero dynamic.

Finally, it is necessary to verify the stability of the zero dynamics equations by numerical analysis. If the zero dynamics equation is stable at x^0, then the entire system is stable in the field of x^0.

3. Zero Dynamics Analysis

In this section, we consider a five order nonlinear model of asynchronous traction motor which established in the stationary coordinate system (alpha-beta) .The speed and rotor flux of traction motor can be dynamic decoupled by using Inverse system method.

3.1. Asynchronous Traction Motor Inverse Decoupling

In order to achieve high-performance control for asynchronous traction motor speed and rotor flux, we have defined a state variable $x=[\ i_a, i_b, \psi_a, \psi_b, \omega]^T = [x_1, x_2, x_3, x_4, x_5]^T$ by using the following variables: stator current vector, rotor flux vector and rotational speed. $u=[u_a\ , u_b]^T$ is defined with control variable by using the stator voltage vector. $y=[h_1(x), h_2(x)]^T = [\omega, \psi^2_a + \psi^2_b]^T$ is defined with output variable by using the following variables: rotor flux and rotational speed. We obtain a 5-order nonlinear model of asynchronous traction motor [16], such as:

$$\dot{x} = f(x,u) = \begin{cases} -k_1 x_1 + k_2 k_3 x_3 + k_3 k_4 x_4 x_5 + k_5 u_a \\ -k_1 x_2 - k_3 k_4 x_3 x_5 + k_2 k_3 x_4 + k_5 u_b \\ k_2 k_7 x_1 - k_2 x_3 - k_4 x_4 x_5 \\ k_2 k_7 x_2 + k_4 x_3 x_5 - k_2 x_4 \\ k_6 (x_2 x_3 - x_1 x_4) - k_8 T_L \end{cases} \tag{7}$$

$$y = h(x) = \left[\omega, \psi^2_a + \psi^2_b \right]^T = \left[x_5, x_3^2 + x_4^2 \right]^T \tag{8}$$

where $k_1 = (L^2_m R_r / \sigma L_s L^2_r) + (R_s / \sigma L_s), k_2 = R_r / L_r, k_3 = L_m / \sigma L_s L_r, k_4 = n_p, k_5 = 1/\sigma L_s, k_6 = n_p L_m / JL_r, k_7 = L_m, k_8 = 1/J$. The rotor motor speed is given by ω; the rotor flux are ψ_a and ψ_b; i_a and i_b are the two-phase stator current; n_p denotes the number of pole pairs; J is the moment of inertia and T_L is the load torque; R_s, R_r are the stator and rotor resistances; L_s, L_r are the stator and rotor self-inductances; and L_m is the s mutual inductance between the stator and rotor. Let σ denote an angle such that $d\sigma/dt = n_p\omega$.
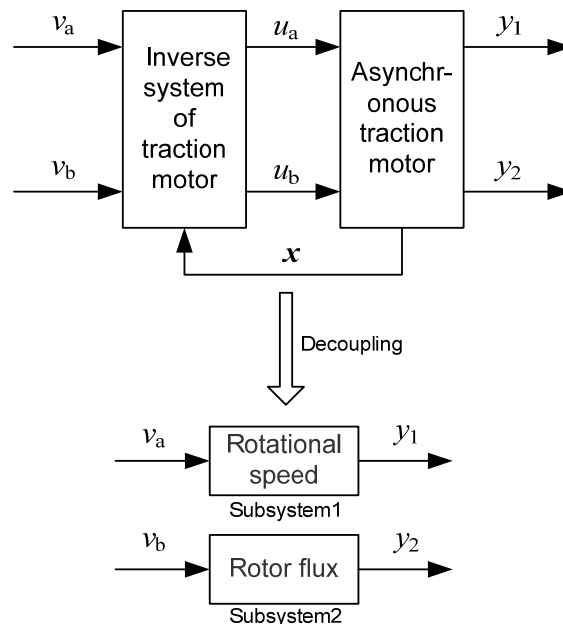

Figure 1. Dynamic decoupling structure of asynchronous traction motors based on inverse system method

Our method uses the inverse system method to decouple the traction motor system. Dynamic decoupling structure of asynchronous traction motors based on inverse system method is shown in Figure 1. For the inverse system method, the algorithm need to determine the relative degree of the system to determine whether the system reversible. The formula (8) is calculated as follows

$$L^0_{f(x,u)}h_1(x) = h_1(x) = x_5$$

$$L^1_{f(x,u)}h_1(x) = \frac{\partial h_1(x)}{\partial(x)}f(x,u) = k_6(x_2 x_3 - x_1 x_4) - k_8 T_L$$

$$L^2_{f(x,u)}h_1(x) = \frac{\partial(L^1_{f(x,u)}h_1(x))}{\partial(x)}f(x,u) = \begin{bmatrix} -k_6 x_4 & k_6 x_3 & k_6 x_2 & -k_6 x_1 & 0 \end{bmatrix}f(x,u)$$

$$L^0_{f(x,u)}h_2(x) = h_2(x) = x_3^2 + x_4^2$$

$$L^1_{f(x,u)}h_2(x) = \frac{\partial h_2(x)}{\partial(x)}f(x,u) = \begin{bmatrix} 0 & 0 & 2x_3 & 2x_4 & 0 \end{bmatrix}f(x,u)$$

$$L^2_{f(x,u)}h_2(x) = \frac{\partial(L^1_{f(x,u)}h_2(x))}{\partial(x)}f(x,u) = \begin{bmatrix} 2k_2 k_7 x_3 & 2k_2 k_7 x_4 & -4k_2 x_3 + 2k_2 k_7 x_1 & 2k_2 k_7 x_2 - 4k_2 x_4 & 0 \end{bmatrix}f(x,u)$$

The necessary and sufficient conditions of the system reversible are given in the literature [11] by

$$\frac{\partial(L^k_{f(x,u)}h_i(x))}{\partial u_j} = 0 \qquad j=1,2; \quad i=1,2; \quad k=0,1 \tag{9}$$

$$A(x,u) = \begin{bmatrix} \dfrac{\partial L^2_{f(x,u)}h_1(x)}{\partial u_a} & \dfrac{\partial L^2_{f(x,u)}h_1(x)}{\partial u_b} \\[2mm] \dfrac{\partial L^2_{f(x,u)}h_2(x)}{\partial u_a} & \dfrac{\partial L^2_{f(x,u)}h_2(x)}{\partial u_b} \end{bmatrix} = \begin{bmatrix} -k_5 k_6 x_4 & k_5 k_6 x_3 \\ 2k_2 k_5 k_7 x_3 & 2k_2 k_5 k_7 x_4 \end{bmatrix} \tag{10}$$

Due to $\det A(x,u) = -2 k_2 k_5^2 k_6 k_7(x_4^2 + x_3^2)$, $A(x,u)$ is non-singular when $x \in \Omega = \{ x \in R^5 : x_4 \neq 0, x_3 \neq 0 \}$. rank $A(x,u)=2$, so the system relative are $\gamma = \{2, 2\}$. Let γ-order integral inverse system input is $v=[v_a, v_b]^T$, the equations of decoupled pseudo-linear system can be described as

$$y = \begin{bmatrix} y_1^{(2)} \\ y_2^{(2)} \end{bmatrix} = \begin{bmatrix} v_a \\ v_b \end{bmatrix} \tag{11}$$

where $v_a = L^2_{f(x,u)}h_1(x)$, $v_b = L^2_{f(x,u)}h_2(x)$.

3.2. The Solving of Zero Dynamics Equation
The relative summation of asynchronous traction motor system is less than the number of system order.

$$\gamma = \sum_{i=1}^{2} \alpha_i = 4 < n = 5 \tag{12}$$

There will be appear $n-\gamma$ zero dynamics equation by using the coordinate transformation $z=\varphi(x)= [z_1, z_2, z_3, z_4, z_5]^T$

$$\begin{cases} z_1 = y_1 = h_1(x) = x_5 \\ z_2 = y_1^{(1)} = \dfrac{\partial h_1(x)}{\partial(x)}f(x,u) \\ z_3 = y_2 = h_2(x) = x_3^2 + x_4^2 \\ z_4 = y_2^{(1)} = \dfrac{\partial h_2(x)}{\partial(x)}f(x,u) \\ z_5 = \eta_1 = \arctan(x_4/x_3) \end{cases} \tag{13}$$

where the five state variables is selected as the angle of the rotor flux vector.

The zero dynamic equation can be obtained by equation (6)

$$z_5 = \dot{\eta}_1 = \dot{\varphi}_5(x) = L_f \eta_1(x) \tag{14}$$

The above formula can be rewritten as

$$L_f \eta_1(x) = \frac{\partial \eta_1(x)}{\partial x} f(x,u) = \left[\frac{\partial \eta_1(x)}{\partial x_1}, \frac{\partial \eta_1(x)}{\partial x_2}, \frac{\partial \eta_1(x)}{\partial x_3}, \frac{\partial \eta_1(x)}{\partial x_4}, \frac{\partial \eta_1(x)}{\partial x_5} \right] f(x,u)$$

$$= \left[0, 0, \frac{-x_4}{x_3^2 + x_4^2}, \frac{x_3}{x_3^2 + x_4^2}, 0 \right] f(x,u) = \frac{-x_4}{x_3^2 + x_4^2} \left(k_2 k_7 x_1 - k_2 x_3 - k_4 x_4 x_5 \right) + \frac{x_3}{x_3^2 + x_4^2} \left(k_2 k_7 x_2 + k_4 x_3 x_5 - k_2 x_4 \right)$$

$$= \frac{1}{x_3^2 + x_4^2} \left[k_4 \left(x_3^2 + x_4^2 \right) x_5 + k_2 k_7 \left(x_2 x_3 - x_1 x_4 \right) \right] = k_4 x_5 + \frac{k_2 k_7 \left(x_2 x_3 - x_1 x_4 \right)}{x_3^2 + x_4^2}$$

$$= n_P x_5 + \frac{R_r J k_6 \left(x_2 x_3 - x_1 x_4 \right)}{(x_3^2 + x_4^2) n_P}$$

The state variable x in the above formula can be converted to z by coordinate transformation $z=\varphi(x)$

$$L_f \eta_1(x) \Big|_{x=\varphi^{-1}(z)} = n_P z_1 + \frac{R_r \left(J z_2 + T_L \right)}{z_3 n_P} \tag{15}$$

3.3. Analysis for Stability of The Zero Dynamics Equation

It is necessary to analysis stability of the zero dynamics equation, in order to verify the inverse decoupling system stability. The zero dynamics equations must be stable to ensure that asynchronous traction motor Inverse decoupling is valid.Generally, a balance state of the system should be obtained first, and then combining with equation (15)γvariable of converted zero dynamics equation value is taken as zero based on the definition of zero dynamics. Finally, according to the characteristic roots which are obtained from the dynamic equation whether have negative real part to determine the system at equilibrium is asymptotically stable.

Taking into account the control target of asynchronous traction motor is to meet the expectations of the motor rotor speed and rotor flux amplitude, , so take the rotor speed setpoint ω_{ref} and rotor flux amplitude setpoint ψ_{ref} as a balance point to force the system to reach work-balance point. Zero dynamics equation becomes as

$$\dot{z}_5 = n_P \omega_{ref} + \frac{R_r T_L}{\psi_{ref}^2 n_P} \tag{16}$$

Although z_5 is increasing with time and is divergent in the Lyapunov stability from the formula (16), but its actual physical meaning of state is amplitude angle of the rotor flux vector, which growing does not affect the actual system state stability. The state variables which is not directly related to energy storage aspects In the actual control system, such as the growing of the angle and displacement over time will not damage the stability of the system. The literature [7] shows that zero dynamics of the system is asymptotically stable, if the system external dynamic is asymptotically stable, then the entire system is asymptotically stable.

4. Conclusion

This paper analyzes the zero dynamics for inverse decoupling of asynchronous traction motor when $\gamma < n$. The zero dynamics subsystem will be separated from the original dynamic system through coordinate transformation in asynchronous traction motor linear decoupling by inverse system method. Thus, a getting method for zero dynamics of multi-input multi-output

nonlinear system is presented, and then the zero dynamics for linearization of a 5 order nonlinear model of asynchronous traction motor which established in the stationary coordinate system (alpha-beta) by using inverse system method were analyzed.

For the practical point of view, we main interest in the external dynamics of the system which is need to not only stable but also have good quality, and the internal dynamics need to stable only. Therefore this approach will simplify the process based on inverse system method asynchronous traction motor linear decoupling, only necessary to design the control law to guarantee the stability of the zero dynamics.In this case,the original nonlinear system is not necessary entirety linearization , while only need to linearized a part whose effect on the external dynamic.

References
[1] Long Z, Shiming G. Electrical Machinery and Apparatus of EMU. Cheng Du: Southwest Jiaotong University Press. 2009: 12-16.
[2] Leiming S, Zhongping Y. Transmission and Control of EMU. Bei Jing: China Railway Press. 2009: 24-27.
[3] Qing L, Liyong Y, Zhengxi L, Huade L. *Stator Flux and Torque Decoupling Control of Induction Motor Using Inverse System Method*. Proceedings of the CSEE. 2006; 26(6): 146-150.
[4] Qinghui W. The Inverse Decoupling Control for the Stator Flux and Torque in Induction Motors and Its Existence. *Control Theory & Applications*. 2009; 26(9): 983-987.
[5] Qinghui W, Shuxian L. Research on Adaptive Inverse Decoupling Control of Induction Motor Based on Stator and Rotor Resistance Error Compensation. *Acta Automatica Sinica*. 2010; 36(2): 297-303.
[6] Qinghui W, Shuxian L, Xiaoheng C, Zuoyou Y. Inverse Decoupling Control of Induction Motor Based on PCA-NN Compensation. *Transactions of China Electrotechnical Society*. 2011; 26(1): 40-45.
[7] Chunpeng Z, Fei L, Wenchao S, Lianwei J. *Nonlinear Control of Induction Motors Based on Direct Feedback Linearizationg*. Proceedings of the CSEE. 2003; 23(2): 99-102.
[8] Chunpeng G, Ben W, Yueheng Z, Tai L. Decoupling Control of Asynchronous Motor Based on Feedback Linearization. *Da Dianji Jishu*. 2010; (4): 31-37.
[9] Xiaorong Z, Yonglong P, Heming L, Xinchun S. *Nonlinear Control of Current-Source PWM Rectifier*. Proceedings of the CSEE. 2007; 27(28): 96-101.
[10] Fang L, Meimei S, Lei Z. Research on Nonlinear Control of DC Motors. *Modern Electronics Technique*. 2011; 34(24): 202-205.
[11] Xinghua Z, Xianzhong D. Speed Control System of Induction Motor Based on Inverse System Method. *Control and Decision*. 2000; 15(6): 708-711.