Improvement of conjugate gradient methods for removing impulse noise images

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ABSTRACT

Optimization problems occur in most disciplines like engineering, physics, mathematics, economics, administration, commerce, social sciences, and even politics. The conjugate coefficient is the cornerstone of conjugate gradient algorithms with the desired conjugate property. In this study, we discovered fresh second order information for the Hessian from the target function, which might lead to a new search direction. Based on a unique search direction, we proposed the update formula and nonlinear conjugate gradient technique. Under Wolfe line search and moderate objective function assumptions, the strategy has acceptable descent property and is always globally convergent. According to numerical results, the technique is successful and competitive in recovering the original picture from an image corrupted by impulsive noise.

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1. INTRODUCTION

This paper presents an iterative technique for solving optimization problems using an edge-preserving regularization (EPR) objective function. In general, two equations explain impulse noise. The first equation uses an adaptive median filter (AMF) [1] to detect pixels that may be contaminated. Let X be the true picture and $A = \{1,2,3,\ldots.. M\} \times \{1,2,3,\ldots.. N\}$ be its index set. Let $N \subset A$ be the indices of the first phase noise pixels. Let $P_{i,j}$ be the set of four closest neighbors of the pixel at $(i,j) \in A$, $y_{i,j}$ be the observed pixel value at (i,j), and $u_{i,j} = [u_{i,j}]_{(i,j)\in N}$ be a lexicographically ordered column vector of length c. N has c components. This is done by reducing the following functional:

$$f_\alpha(u) = \sum_{(i,j)\in N}\left[|u_{i,j} - y_{i,j}| + \frac{\beta}{2}(2 \times S^1_{i,j} + S^2_{i,j}\right]. \tag{1}$$

where β is the regularization parameter, and $S^1_{i,j} = 2\sum_{(m,n)\in P_{i,j}\cap N^c}\phi_\alpha(u_{i,j} - y_{m,n}), S^2_{i,j} = \sum_{(m,n)\in P_{i,j}\cap N}\phi_\alpha(u_{i,j} - y_{m,n})$. The $\phi_\alpha = \sqrt{\alpha + x^2}, \alpha > 0$ is an example of an edge-preserving potential. In reality, the non-smooth data-fitting term is unnecessary in the second phase, when only noisy pixels are restored, Yu *et al*. [2]. Thus, several optimization techniques may be extended to minimize the smooth edge-preserving regularization (EPR) functional:

$$f_\alpha(u) = \sum_{(i,j)\in N}[2 \times S^1_{i,j} + S^2_{i,j}] \tag{2}$$

Image restoration uses conjugate gradients, image restoration problems are expressed as:

$$Minf(u)\ ,\ \mathrm{u} \in R^n \tag{3}$$

where $f: R^n \rightarrow R$ is smooth function, Gilbert and Nocedal [3]. A general conjugate gradient algorithm generates a sequence of iterates by the rule:

$$x_{k+1} = x_k + \alpha_k d_k \tag{4}$$

where the step size α_k is positive and the directions d_k are computed using the updating formula:

$$d_{k+1} = -g_{k+1} + \beta_k d_k \tag{5}$$

which β_k is a scalar known as the conjugate gradient parameter. Different choices of β_k lead to various conjugate gradient methods, Andrei [4]. In this paper, we focus our attention on well-known method such as Fletcher and Reeves, [5] given by:

$$\beta_k^{FR} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k} \tag{6}$$

it is well known that the FR possess nice convergence properties. In the past decades, a variety of conjugate gradient methods are developed. There are some well known conjugate gradient methods, such as [6]-[10]. Other nonlinear conjugate gradient methods and their global convergence can be found in [11]. We can get the step-size α_k using the exact line research:

$$\alpha_k = \frac{-g_k^T d_k}{d_k^T Q d_k}, \tag{7}$$

see, [12]. Usually, in (2), the steplength α_k is computed using the Wolfe line search conditions:

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \delta \alpha_k g_k^T d_k \tag{8}$$

$$d_k^T g(x_k + \alpha_k d_k) \geq \sigma\, d_k^T g_k \tag{9}$$

where $0 < \delta < \sigma < 1$, Wolfe [13], [14]. During the last decade, much effort has been devoted to develop new conjugate gradient methods which not only possess strong convergence properties but are also computationally superior to the classical methods. The most typical feature of conjugate gradient methods is conjugacy, namely, the search directions generated by (3) should possess the following conjugacy condition:

$$d_{k+1}^T Q d_k = 0 \tag{10}$$

researchers focus their attention on conjugacy condition. One of the remarkable results is obtained in conjugate gradient methods. For a good reference for studies describing the latest CG coefficients with important result and various modifications from β_k, Xue *et al*. [1]. For further references on the optimization methods, please refer to [15]-[18].

Using the conjugacy condition , we are now ready to give some new formulas of nonlinear conjugate gradient methods. Global convergence of these formulas have been established. Finally, some of the numerical results have been reported, which show the effectiveness of the new formula.

2. NEW CONJUGATE GRADIENT COEFFICIENT

In our paper, By adopting some idea that in the literature [19]. We first consider the second order Taylor series of f (x) as (11).

$$f(u) = f(u_{k+1}) + g_{k+1}^T (u - u_{k+1}) + \frac{1}{2}(u - u_{k+1})^T Q(u_k)(u - u_{k+1}) \tag{11}$$

Finding the derivative yields:

$$g_{k+1} = g_k + Q(u_k) s_k \tag{12}$$

The parameter, β_k, in the linear conjugate gradient method is given by:

$$\beta_k = \frac{g_{k+1}^T Q s_k}{d_k^T Q s_k} \tag{13}$$

where Q is Hessian matrix and where β_k is satisfies the conjugacy condition, Hassan and Sulaiman [19]. Now, we shall consider another expression of the denominator $d_k^T Q s_k$. Using (11) and (7) in (11), we obtain:

$$s_k^T Q(u_k) s_k = 2/3 s_k^T y_k + 2/3(f_k - f_{k+1}) \tag{14}$$

which implies that:

$$d_k^T Q(u_k) s_k = 2/3 d_k^T y_k + 2/3(f_k - f_{k+1})/\alpha_k \tag{15}$$

from this we advance formula,

$$\beta_k = \frac{g_{k+1}^T y_k}{2/3 d_k^T y_k + 2/3(f_k - f_{k+1})/\alpha_k} \tag{16}$$

since f is quadratic model by using exact line search, then (16) reduces to:

$$\beta_k = \frac{\|g_{k+1}\|^2}{2/3 d_k^T y_k + 2/3(f_k - f_{k+1})/\alpha_k} \tag{17}$$

and

$$\beta_k = \frac{\|g_{k+1}\|^2}{-2/3 d_k^T g_k + 2/3(f_k - f_{k+1})/\alpha_k} \tag{18}$$

and

$$\beta_k = \frac{\|g_{k+1}\|^2}{2/3 g_k^T g_k + 2/3(f_k - f_{k+1})/\alpha_k} \tag{19}$$

our formula, so-called BA1, BA2 and BA3. They show that the new method globally convergent for general functions under some proper conditions. Below we present the BA algorithms:

- Stage 1 : Set $k = 1$, then$d_1 = -g_1$ and select u_1.
- Stage 2 : Test for Continuation of Iterations.
- Stage 3 : Calculate the step length by using Wolfe line search (5) and (6).
- Stage 4 : Compute d_{k+1}using (5).
- Stage 5 : Compute u_{k+1}using (4).
- Stage 6 : Set k=k+1, and go to Step 1.

Theorem (2.2)

The search direction defined by (3) with (17)-(19) satisfy:

$$d_{k+1}^T g_{k+1} < 0 \text{ and } d_{k+1}^T g_{k+1} = \beta_k d_k^T g_k \tag{20}$$

Proof :

Clearly by (3), if $k = 0$ then $g_0^T d_0 = -\|g_0\|^2$ holds, let $d_k^T g_k < 0$ for all k. Multiplying both sides of (3) with g_{k+1}^T, we get:

$$\begin{aligned} d_{k+1}^T g_{k+1} &= -g_{k+1}^T g_{k+1} + \beta_k d_k^T g_{k+1} \\ &= -\beta_k(2/3 d_k^T y_k + 2/3(f_k - f_{k+1})/\alpha_k) + \beta_k d_k^T g_{k+1} \end{aligned} \tag{21}$$

From (21), we obtain:

$$d_{k+1}^T g_{k+1} = \beta_k[d_k^T g_{k+1} - (2/3 d_k^T y_k + 2/3(f_k - f_{k+1})/\alpha_k)] \tag{22}$$

also, by using (17) and (21) we get:

$$d_{k+1}^T g_{k+1} = \beta_k d_k^T g_k \tag{23}$$

by the $d_k^T g_k < 0$and (23), we can write:

$$d_{k+1}^T g_{k+1} < 0 \tag{24}$$

hence $d_{k+1}^T g_{k+1} < 0$ and $d_{k+1}^T g_{k+1} = \beta_k d_k^T g_k$ holds, which completes the proof. Similarly, other methods can be proved.

3. CONVERGENCE ANALYSIS

In the above section, we have proved that the new methods has decsent property property that is independent of the line search and the function. convexity. In this section, we will make use of this property to establish the global convergence for the new methods using a variety of line searches. Suppose that the objective function satisfies the following assumption.

Assumptions:

I) The level set $\Omega = \{u \in R^n / f(u) \leq f(u_1)\}$, is bounded.

II) Gradient is Lipschitz continuous, that is, for $L > 0$:

$$\|g(P) - g(O)\| \leq L \quad \|P - O\|, \forall P, O \in \Lambda \tag{25}$$

for more details see [20]-[22].

By having these assumptions, Zoutendjik [23] has proven the following Lemma.

Lemma (3.1):

Suppose Assumption are satisfied. In any iteration method if α_k is satisfied Wolfe line search, then:

$$\sum_{k=1}^{\infty} \frac{(g_k^T d_k)^2}{\|d_k\|^2} < \infty \tag{26}$$

Proof : For see Sulaiman and Hassan [24] and Zhang and Xu [25].

Theorem (3.2):

Suppose that Assumption and Lemma 1 holds. Then,

$$\lim_{k \to \infty} inf \|g_k\| = 0 \tag{27}$$

Proof :

By induction, let that (27) is not true. Suppose that there exists $c_1 > 0$ such that $\|g_k\| \geq c_1$ for all $k \in n$. Squaring both sides of (8), we get:

$$\|d_{k+1}\|^2 + \|g_{k+1}\|^2 + 2d_{k+1}^T g_{k+1} = (\beta_k)^2 \|d_k\|^2 \tag{28}$$

using (23), yields:

$$\|d_{k+1}\|^2 = \frac{(d_{k+1}^T g_{k+1})^2}{(d_k^T g_k)^2} \|d_k\|^2 - 2d_{k+1}^T g_{k+1} - \|g_{k+1}\|^2 \tag{29}$$

divide by (29) by $(d_{k+1}^T g_{k+1})^2$, we get:

$$\frac{\|d_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} = \frac{\|d_k\|^2}{(d_k^T g_k)^2} - \frac{\|g_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} - \frac{2}{d_{k+1}^T g_{k+1}} \tag{30}$$

using (3), (17), and (30), we obtain:

$$\frac{\|d_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} \leq \frac{\|d_k\|^2}{(d_k^T g_k)^2} - \left(\frac{\|g_{k+1}\|}{(d_{k+1}^T g_{k+1})} + \frac{1}{\|g_{k+1}\|^2} \right) + \frac{1}{\|g_{k+1}\|^2} \leq \frac{\|d_k\|^2}{(d_k^T g_k)^2} + \frac{1}{\|g_{k+1}\|^2} \tag{31}$$

hence,

$$\frac{\|d_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} \leq \sum_{i=1}^{k+1} \frac{1}{\|g_i\|^2} \tag{32}$$

therefore,

$$\frac{\|d_{k+1}\|^2}{(d_{k+1}^T g_{k+1})^2} < \frac{k+1}{c_1^2} \tag{33}$$

from (33), we obtain:

$$\sum_{k=1}^{\infty} \frac{(g_k^T d_k)^2}{\|d_k\|^2} = \infty \tag{34}$$

based on Lemma 1, we get $\lim_{k\to\infty} \inf \|g_k\| = 0$ holds.

4. NUMERICAL RESULTS

The numerical findings in this section indicate the effectiveness of New in the reduction of salt-and-pepper impulse noise. New and FR methods are tested in our trials. MATLAB r2017a is used to write and execute all of the programs. The following are the stopping criteria for both methods (35).

$$\frac{|f(u_k)-f(u_{k-1})|}{|f(u_k)|} \leq 10^{-4} \text{ and } \|f(u_k)\| \leq 10^{-4}(1+|f(u_k)|) \tag{35}$$

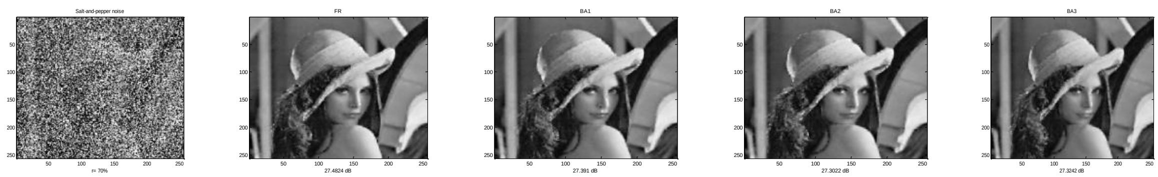
Lena, House, Cameraman, and Elaine make up the test photos. PSNR (peak signal-to-noise ratio) is a quantitative metric that may be used to evaluate the quality of the restoration process:

$$PSNR = 10\, log_{10} \frac{255^2}{\frac{1}{MN}\sum_{i,j}(u_{i,j}^r - u_{i,j}^*)^2} \tag{36}$$

in this case, $u_{i,j}^r$ and $u_{i,j}^*$ represent the restored and original image's pixel values, respectively.

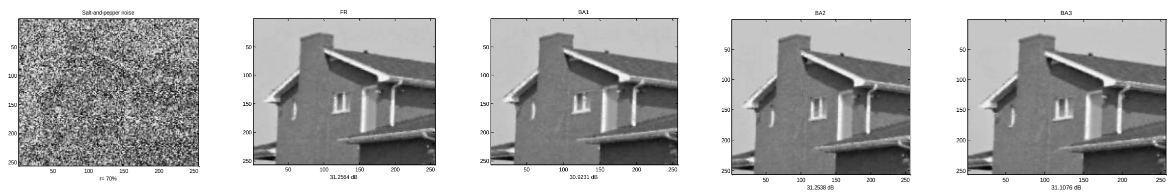
The number of iterations (NI) and the number of function evaluations (NF) needed for the whole denoising process, as well as the PSNR of the recovered picture, are reported in this paper. We can observe from Table 1 that the New technique is much quicker than the FR method for the vast majority of the test photographs. Furthermore, we see that the PSNR values obtained by the New and FR methods are fairly close. Conclusion. Table 1 shows that the suggested methods outperform the FR approach in terms of number of iterations, function evaluations, and peak signal to noise ratio when it comes to eliminating impulse noise from photos.

Figures 1-4 show the obtained results by denoised images. Figures (a1), (a2), (a3) and (a4) are the images corrupted with 70% salt-and-pepper noise; Figures (b1), (b2), (b3) and (b4) are results of FR method; Figures (c1), (c2), (c3) and (c4) are results of the BA1 method; Figures (d1), (d2), (d3) and (d4) are results of the BA2 method; and Figures (e1), (e2), (e3) and (e4) are results of the BA3 method.



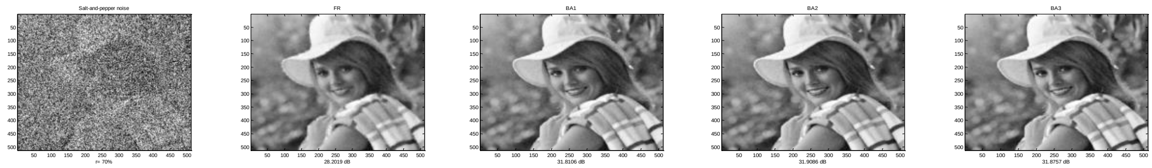
a1 b2 c1 d1 e1

Figure 1. Demonstrates the results of algorithms: (a1) denoised images with 70% salt-and-pepper noise, (b1) FR method, (c1) BA1 method, (d1) BA2 method and (e1) BA3 method of 256 * 256 Lena images



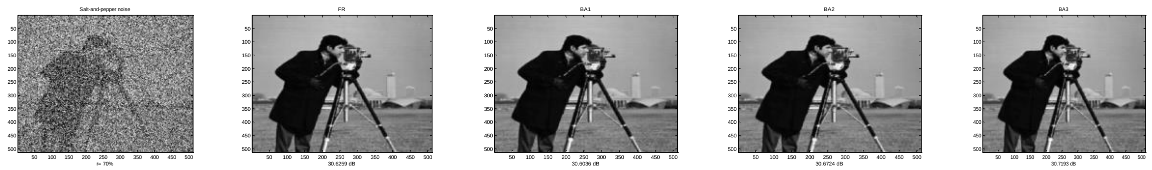
a2 b2 c2 d2 e2

Figure 2. Demonstrates the results of algorithms: (a2) denoised images with 70% salt-and-pepper noise, (b2) FR method, (c2) BA1 method, (d2) BA2 method and (e2) BA3 method of 256 * 256 House image



a3 b3 c3 d3 e3

Figure 3. Demonstrates the results of algorithms: (a3) denoised images with 70% salt-and-pepper noise, (b3) FR method, (c3) BA1 method, (d3) BA2 method and (e3) BA3 method of 256 * 256 Elaine image



a4 b4 c4 d4 e4

Figure 4. Demonstrates the results of algorithms: (a4) denoised images with 70% salt-and-pepper noise, (b4) FR method, (c4) BA1 method, (d4) BA2 method and (e4) BA3 method of 256 * 256 Cameraman image

Table1. Numerical results of FR, BA1, BA2 and BA3 algorithms

Image	Noise level r	FR-Method			BA1-Method			BA2-Method			BA3-Method		
	(%)	NI	NF	PSNR (dB)	NI	NF	PSNR (dB)	NI	NF	PSNR (dB)	NI	NF	PSNR (dB)
Le	50	82	153	30.5529	42	89	30.7873	42	91	30.4787	44	93	30.6782
	70	81	155	27.4824	45	94	27.391	40	80	27.3022	41	81	27.3242
	90	108	211	22.8583	54	109	23.0297	84	167	22.6043	49	94	23.0253
Ho	50	52	53	30.6845	28	57	34.8651	32	67	34.7529	33	68	34.7004
	70	63	116	31.2564	37	71	30.9231	37	73	31.2538	32	63	31.1076
	90	111	214	25.287	48	99	25.1515	69	137	24.9917	47	95	25.1155
El	50	35	36	33.9129	23	43	33.895	21	40	33.875	25	47	33.8553
	70	38	39	31.864	300	56	31.8106	32	61	31.9086	29	53	31.8757
	90	65	114	28.2019	38	72	28.1511	53	103	28.2248	39	72	28.1069
c512	50	59	87	35.5359	26	56	35.3508	23	49	35.7124	30	61	35.4487
	70	78	142	30.6259	37	76	30.6036	49	100	30.6724	36	72	30.7193
	90	121	236	24.3962	52	109	24.9562	70	141	24.7639	52	109	24.9182

5. CONCLUSIONS

In this research, we offer novel CG approaches for minimizing the smooth regularization functional for impulse noise reduction, which we call smooth regularization functions. The parameters of the new technique are generated from a quadratic model, which is described in this paper. The numerical findings demonstrate that the novel technique has a low computing cost and is effective at solving signal processing difficulties.

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