TELKOMNIKA, Vol. 11, No. 12, December 2013, pp. 7580~7588
e-ISSN: 2087-278X

Multiple-symbol Differential Sphere Decoding for Network Coding

Xinqiang Han[1], Xiaoping Jin*[1], Zhengquan Li[1,2], Ruixin Zhu[1]
[1]Department of Information Engineering, China Jiliang University, Hangzhou, China
[2]National Mobile Communications Research Laboratory, Southeast University, Nanjing, China
*Corresponding author, e-mail: jxp1023@cjlu.edu.cn

Abstract

In order to shorten 3dB performance gap between the conventional differential detection and correlation detection in network coding, we consider multiple-symbol differential detection (MSDD) for two-way relay channel (TWRC) model. MSDD, which makes use of continuously N symbols to jointly detect N-1 symbols. However, the complexity of the maximum likelihood differential detection increases exponentially with the detection group length and the modulation constellation points. In this paper, we propose multiple-symbol differential sphere decoding (MSDSD) to circumvent this excessive computational complexity. Simulation results show that the combination of MSDSD and differential network coding can not only reduce the computational complexity, but also overcome error platform caused by High-Doppler frequency offset at high signal-to-noise ratio, and obtain the optimum detection performance simultaneously. Hence, MSDSD can be regarded as a low complexity detection algorithm in differential network coding scheme.

Keywords: *network coding, multiple-symbol differential detection, multiple-symbol sphere decoding*

1. Introduction

Network coding (NC) [1, 2] can solve the problem of low frequency spectrum efficiency in the wireless channel environment. Meanwhile, NC can also save time slot resources effectively. Compared with relay store and forward scheme in the traditional cooperative system, NC scheme takes relevant operation to merge several received signal before broadcast it to the destination at the relay node. This type of transmission scheme can achieve higher data transmission. Hence, NC can be a effective transmission method to get higher capacity and improve transmission efficiency in relay network.

Generally, maximum likelihood detection (MLD) has been used in signals detection [3]. In the study of network coding, relay and destination nodes often employ the traditional maximum likelihood coherent detection [4, 5] to detect signals, but coherent detection always needs to estimate various channel information. However, sometimes it is hard to obtain accurate channel information when wireless equipments are located in a poor environment. Therefore, differential modulation [6] was employed to avoid channel estimation. Some researchers committed to the research on differential network coding [6, 7]. For example, a differential modulation analog network-coding scheme proposed in [6], in which a differential detection scheme was developed and system performance analysis was given. In [8, 9], the author adopted maximum likelihood detection in physical layer differential network coding scheme, and proposed several improvement schemes to reduce the high detection complexity.

Although differential detection schemes above mainly reduce the detection complexity of NC, they cannot improve the performance gap between differential detection and correlation detection effectively. In order to shorten the performance gap, the differential detection scheme was extended to the multiple symbols differential detection (MSDD) [8]. In this work, we proposed a differential network coding (D-NC) scheme based on detection and forward protocol at relay node. At relay node and source nodes, this work firstly uses maximum likelihood differential decoding to detect signals. For high computational complexity of this algorithm, then we employ the depth-first multi-symbol differential sphere decoding (MSDSD) [10, 11] to reduce computational complexity.

Received June 28, 2013; Revised August 16, 2013; Accepted September 3, 2013

Simulation results verify the performance of proposed scheme. We compare the performance influence on different Doppler frequency offset [12]. We also compare with bit error rate (BER) of ML detection and MSDSD detection. Results show that MSDSD works fine in the slow fading channel. MSDSD apply to differential network coding that can reduce computational complexity effectively caused by ML decoding and keep the optimum detection performance.

In this paper, main work of the research on differential network coding is as following:

1) We proposed a differential network-coding scheme based on detection and forward mode at relay node. We also compare this scheme with the store and forward scheme in the TWRC model. For improving the probable error of the relay decoding, a kind of error correction method called magic genie [13] used to decrease the detection bit error rate.

2) For the high computational complexity of MSDD-ML detection, we proposed low complexity MSDSD.

Notations: In this paper, I_k represents the $k \times k$ unit matrix. $[\cdot]^T$, $[\cdot]^H$, $[\cdot]^*$, and $[\cdot]^{-1}$ represent transpose, conjugate transpose, conjugate and inverse of matrix, respectively. $diag\{.\}$ denotes a diagonal matrix. $\varepsilon\{\cdot\}$ denotes the statistical expectation. $|.|$ denotes taking absolute value. $\|\cdot\|$ denotes Frobenius norm of matrix.

2. The Information Exchange in TWRC Model

2.1. System Model

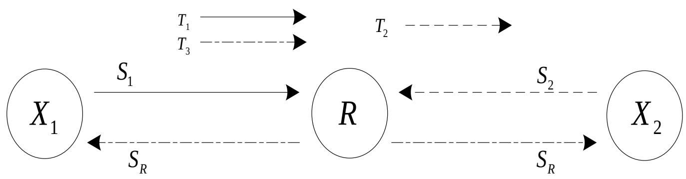

Figure 1. System model of Network Coding Scheme

Figure 1 shows the information exchange model in which X_1, X_2 denote source nodes and R denotes relay node. Each source node complete information exchange with the help of relay node. At first, NC scheme can be expressed as the source node X_k (k=1, 2) sends signal S_k, X_1 send S_1 to R at the first time slot. X_2 send S_2 to R at the next time slot. Then the relay merge two signals S_1 and S_2, and broadcast it. The source nodes can get information of other source node through decoding and eliminating their self-interference. Therefore, it will spend 3 time slots when each time information exchange completed. However, traditional store and forward scheme need spend 4 time slots. In this scheme, source X_1 send signal to R, and then R forward signal to X_2. It is same for X_2. This scheme does not need relay re-encode signals. Hence, NC scheme can save more time slots resources.

At time slot n, X_k transmits M-ary information that mapped by *M-PSK* symbols $a_k[n]$, which comes from the constellation set Q, where $Q = \{a_i = e^{j\frac{2\pi i}{M}} \mid i \in \{0,1,2,\cdots,M-1\}\}$. To avoid the channel estimation, S_k is processed by $S_k[n] = a_k[n] S_k[n-1]$. Suppose channel fading coefficient h_k is an independent Additive Gaussian random variable with zero mean and variances σ_k^2. For the temporal correlation of the fading coefficients, we adopt a Clarke's model with $\varphi_{h,k}(i) = \varepsilon\{h_k[n+i] h_k^*[n]\} = \sigma_k^2 J_0(2\pi B_k T i)$, where $J_0(.)$ denotes zero order Bessel function, B_k denotes normalized fading bandwidth and T denotes time interval between two observations.

2.2. Network Coding

This scheme can be divided into 3 phases. At the first time n, X_1 sends $S_1[n]$ to R and the received signal is given by:

$$y_{r1}[n]=h_1[n]S_1[n]+n_{r1}[n]=y_{r1}[n-1]a_1[n]+z_{r1}[n]\ , \quad (1)$$

Where $n_{r1}[n]$ denotes Additive White Gaussian Noise (AWGN) with zero mean and variance σ_{r1}^2, and $z_{r1}[n]=n_{r1}[n]-n_{r1}[n-1]a_1[n]$ denotes AWGN with zero mean and variance $2\sigma_s^2$. Based on detection and forward protocol, the relay firstly needs to detect the receiving signals. We can obtain $\hat{a}_1[n]$ by using the differential maximum likelihood decoding, namely:

$$\hat{a}_1[n]=\underset{a_1(n)\in\Omega}{\arg\min}\,\|\,y_{r1}[n]-a_1[n]y_{r1}[n-1]\,\|^2\cdot \quad (2)$$

Similarly, at the second time $n+1$, R receives signals from the source node X_2. After decoded, the signal is denoted by $\hat{a}_2[n+1]$. Then R completes XOR network coding and broadcasts the merged signals. In order to improve system performance and reduce performance degradation, (2) can be extended to multiple symbols detection. We will discuss it in the next section.

The inverse mapping of $\hat{a}_1[n]$ is reverted to bit information b_1, which corresponds to $e^{j\frac{2\pi}{M}l_1}$ in the constellation set Q. In addition, the inverse mapping of $\hat{a}_2[n+1]$ is reverted to bit information b_2, which corresponding to $e^{j\frac{2\pi}{M}l_2}$ in set Q. Then the relay completes XOR operation on b_1 and b_2, i.e. $b_1\oplus b_2$. The combined information corresponds to $e^{j\frac{2\pi}{M}l_r}=e^{j\frac{2\pi}{M}(l_1+l_2)}$ in set Q. If signal $e^{j\frac{2\pi}{M}l_r}$ is denoted by $a_r[n+1]$. The transmitted signals at relay are modulated as $S_r[n+1]=a_r[n+1]S_r[n]$, where $S_r[n+1]\in Q$.

From (2), the probable decoding error of the relay node may cause system performance degradation. In order to enhance the performance during simulation, a kind of error correction method called magic genie [13] is used. This method can decrease BER of relay detection effectively. Moreover, it can be expressed as: only when the relay obtains the correct detection, the signals are re-encoded and forwarded. This method can greatly improve the system performance.

At the last time slot, the relay R broadcasts the merged signals. In the downlink, the X_1 and X_2 can be discussed similarly. Therefore, we mainly discuss source X_1. At time n, the signal received by X_1 is given by:

$$y_1[n]=h_1[n]S_r[n]+n_1[n]=y_1[n-1]a_r[n]+z_1[n]\ , \quad (3)$$

Where $h_1[n]$ denotes channel fading coefficient from R to X_1, and $z_1[n]=n_1[n]-n_1[n-1]a_r[n]$ denotes AWGN with zero mean and variance $2\sigma_s^2$. We can obtain $\hat{a}_r[n]$ by using of ML decoding. It can be expressed as:

$$\hat{a}_r[n]=\underset{a_r(n)\in\Omega}{\arg\min}\,\|\,y_1[n]-a_r[n]y_1[n-1]\,\|^2\cdot \quad (4)$$

In order to overcome performance degradation, (4) also can be extended to multiple symbols differential detection.

The source X_1 achieve $\hat{a}_r[n]$ by differential detection. After detection, the source X_1 eliminates its self-interference to obtain the signal transmitted by X_2. Because of signal $\hat{a}_r[n]$

comes from the constellation set Q. It can be denoted by $e^{j\frac{2\pi}{M}\hat{l}_r}$, which corresponds to bit information $\hat{b}_r$. If $\hat{l}_r = \hat{l}_1 + \hat{l}_2$, source X_1 eliminates self-interference by $\hat{l}_2 = \hat{l}_r - l_1$. Hence, bit information $\hat{b}_2$ can be achieved by $\hat{b}_r$ XOR operation with b_1. Namely,

$$\hat{b}_2 = \hat{b}_r \oplus b_1 = (\hat{b}_1 \oplus \hat{b}_2) \oplus b_1 . \tag{5}$$

From (5), source X_2 can achieve bit information $\hat{b}_2$, which sent by source X_2. Similarly, we can also obtain the bit information $\hat{b}_1$ that come from X_1 through the above analysis.

3. Multiple-symbol Differential Sphere Decoding

The conventional differential detection may be prone to decoding error and lead to performance degradation. Therefore, we can use MSDD to improve detection performance of the receiver and shorten performance gap between traditional differential detection and correlation detection. If group length is N, (3) can be expressed as:

$$y_1 = S_r h_1 + n_1 , \tag{6}$$

Where $S_r = diag\{S_r[n+1], \cdots, S_r[n+N]\}$, $y_1 = [y_1[n+1], \cdots, y_1[n+N]]^T$, $h_1 = [h_1[n+1], \cdots, h_1[n+N]]^T$, $n_1 = [n_1[n+1], \cdots, n_1[n+N]]^T$.

Similarly, (1) also can be expressed as:

$$y_{r1} = S_1 h_1 + n_{r1} . \tag{7}$$

MSDD can be described as the receiver received continuously N symbols to jointly detect N-1 symbols. Generally, we can achieve the optimal signals by ML detection. For (6), ML detection decision formula is given by:

$$\begin{aligned} \hat{a}_r &= [\hat{a}_r[n+1], \cdots, \hat{a}_r[n+N-1]] = \arg\min_{a_r \in Q} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \| y_1[n+j-1] - a_r[n+j-1] y_1[n+i-1] \|^2 \\ &= \arg\min_{a_r \in Q} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \| y_1[n+j-1] - (\prod_{m=n+i}^{n+j-1} a_r[m]) \times y_1[n+i-1] \|^2 \end{aligned} . \tag{8}$$

In (8), all signal lattice points are visited. The detection complexity of ML detection grows exponentially with increasing of modulation constellation points and group length. Therefore, the ML detection is unsuited as the receiver detection algorithm when modulation constellation points and group length are large. In order to reduce computational complexity, we use more effective detection algorithm, namely, MSDSD.

MSDSD algorithm sets an initial sphere radius C^2. At first, we can set radius as a great value. Through the formula (8), MSDSD decision condition is given by:

$$\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \| y_1[n+j-1] - (\prod_{m=n+i}^{n+j-1} a_r[m]) \times y_1[n+i-1] \|^2 \le C^2 \cdot \tag{9}$$

There are $N(N-1)/2$ nonnegative items in (9). Each item is a norm square which needs to meet the constraints. If we regard signals detection as a detection tree, MSDSD starts from the root node of detection tree, namely i=N. Then detection tree reduce i continuously until i=1.Thus we get a set of results. Comparing the current metric with the instant radius, if the metric is smaller, the radius update and the detection goes back to the above layer, and then repeats the searching algorithm until the radius does not update. If the searching ended in the constraints range, we can obtain the optimal detection result, the searching of MSDSD finished. Next, we describe MSDSD algorithm in detail. Flow diagram of MSDSD algorithm is given by Figure 2.

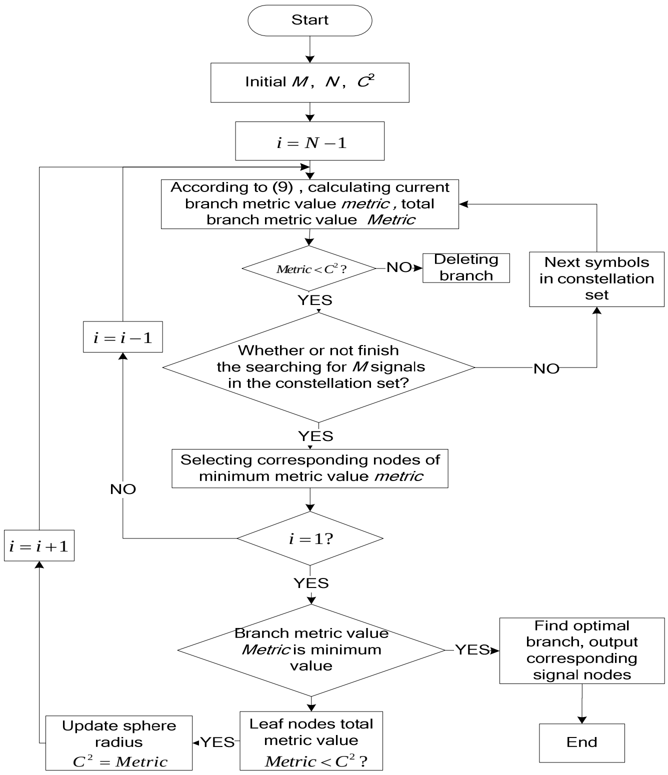

Figure 2. Flow Diagram of MSDSD Algorithm (i denotes detection layer, M denotes modulation constellation points. C^2 denotes sphere radius, *metric* denotes each layer branch metrics, *Metric* denotes total metric of current nodes, and N denotes group length)

From (9), MSDSD is inverted tree search process. At first, MSDSD set an initial sphere radius, for example, $C^2 \to \infty$. The algorithm starts from the root node of detection tree, namely, i=N. Downwardly expand M branches, and store M signal node which correspond to M branch norm square. In each layer, total metric value consist of the sum of current norm square and the above layer metric value. When detection tree reduce i to i=N-1, according to (9), $\prod_{m=n+i}^{n+j-1} a_r[m] = a_r[n+N-1]$, the algorithm meets:

$$\| y_1[n+N-1[-a_r[n+N-1]y_1[n+N-2]\|^2 \le C^2. \quad (10)$$

Through different values of $a_r[n+N-1]$ which comes from set Q , we can achieve the smallest branch norm square $metric_1$ and the corresponding signal. Then detection tree reduces i to i=N-2. From (9), the algorithm meets:

$$\| y_1[n+N-2]-a_r[n+N-2]y_1[n+N-3]\|^2 + \| y_1[n+N-1]-a_r[n+N-2]a_r[n+N-1]y_1[n+N-3]\|^2 \le C^2. \quad (11)$$

Here, $\prod_{m=n+i}^{n+j-1} a_r[m] = a_r[n+N-2]a_r[n+N-1]$.

The left part of (11) is total metric value of $a_r[n+N-2]$. Where the smallest current norm square $metric_2$ also can be achieved by the different value of $a_r[n+N-2]$. Then the algorithm continues to reduce i to i=N-3, total metric value of this layer node needs to meet:

$$\begin{aligned}&\| y_1[n+N-3]-a_r[n+N-3]y_1[n+N-4]\|^2+\| y_1[n+N-2]-a_r[n+N-3]a_r[n+N-2]y_1[n+N-4]\|^2+\\&\| y_1[n+N-1]-a_r[n+N-3]a_r[n+N-2]a_r[n+N-1]y_1[n+N-4]\|^2\le C^2\end{aligned}. \quad (12)$$

Minimizing the left part of (12), the current norm square $metric_3$ can be achieved. Similarly, detection tree reduce i continuously until i=1. From (9), the total metric value of this layer node meets:

$$\begin{aligned}&\| y_1[n+1]-a_r[n+1]y_1[n]\|^2+\| y_1[n+2]-a_r[n+1]a_r[n+2]y_1[n]\|^2+\cdots+\\&\| y_1[n+N-1]-a_r[n+1]a_r[n+2]\cdots a_r[n+N-1]y_1[n]\|^2\le C^2\end{aligned}. \quad (13)$$

From (13), the algorithm achieves current norm square $metric_{N\text{-}1}$. The total metric is: $Metric$= $metric_1$+$metric_2$+⋯+$metric_{N\text{-}1}$, it corresponds to a set of signals $\left[a_r[n+1],a_r[n+2],\cdots,a_r[n+N-1]\right]$.

Then, the radius C^2 is updated to $Metric$. The detection goes back to the above layer, Comparing the total metric value with the instant radius in this layer, If the value is less than C^2, downwardly expand M branches until i=1, and make sure whether update the radius by comparing with the metric square at last layer. The searching process above finished until the radius does not update. We can obtain the smallest metric value which corresponding to a set of optimal signals $\left[\hat{a}_r[n+1],\hat{a}_r[n+2],\cdots,\hat{a}_r[n+N-1]\right]$.

4. Simulation Results and Analysis

This section mainly verifies the performance of MSDD which is used in differential network coding (D-NC) scheme. During the simulation, we suppose two sources X_1 and X_2 transmit signals independently. The noise and fading of different nodes are independent. Signals modulated by DQPSK. We assume that the channel is standard static Rayleigh fading channel when the Doppler-frequency offset [12] fd is less than 0.03.

4.1. System Performance Analysis

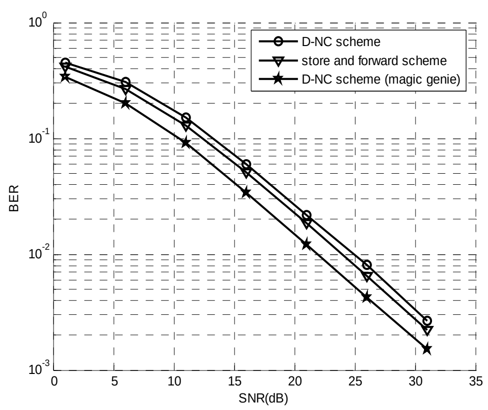

Figure 3. Performance Comparison of Differential Network Coding Scheme

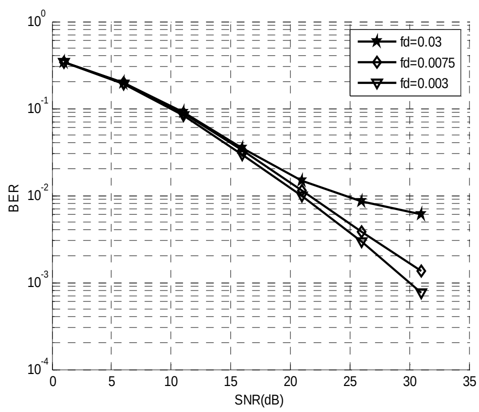

Figure 4. Performance Comparison of MSDSD under Different Doppler Frequency Offset

Figure 3 shows BER performance of D-NC and store and forward scheme. MSDSD are used for signal detection. The Doppler frequency offset of channel is 0.0075. In the downlink, source node X_k use MSDSD with group length N=2. The BER of D-NC scheme is larger than store and forward scheme. It means more performance loss. Aimed at this performance loss, we use a kind of error correction method called magic genie to achieve error correction during our simulation. From Figure 3, we can see the performance of D-NC improved. When BER is 10^{-2}, the SNR is about 3dB better than that of D-NC without magic genie.

Figure 4 shows BER performance of MSDSD under the different Doppler frequency offset when group length N=2. As shown in Figure 4, when fd=0.03, error platform may occur at high SNR. When fd=0.0075, the BER decreases with the SNR increases and system performance becomes better. In fd=0.003, the BER is even smaller than fd=0.0075 and fd=0.03. Hence, the fading speed of channel has a great influence on system performance. When channel changes faster, it causes performance degradation and an error platform occurs with the increase of SNR. In addition, when channel changes slower, the performance will be better with the increase of SNR.

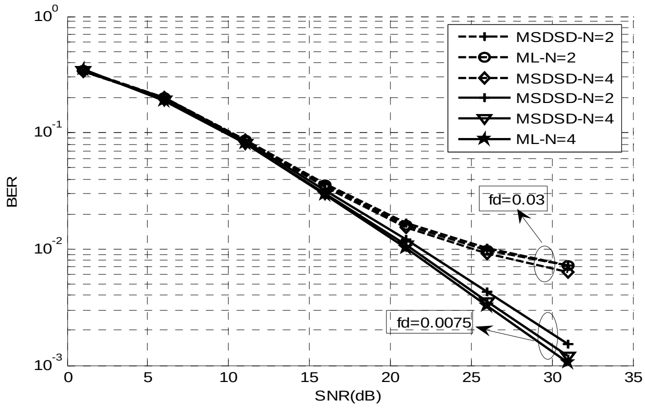

Figure 5. Performance Comparison of MSDSD and ML Decoding under Different Group Length

Figure 5 shows BER performance comparison of D-NC scheme when source nodes use ML decoding algorithm and MSDSD algorithm with different group length. As shown in Figure 5, the two algorithms have same performance when N=2. And the decoding performance of MSDSD closes to ML decoding when N=4. Figure 5 also give the comparison of decoding performance under the conditions of Doppler frequency offset fd=0.03 and fd=0.0075. We can learn that the performance of slower fading channel is better than faster fading channel. When channel fading is same, the BER of group length N=4 is smaller than N=2. Therefore, under the slower fading channel, the system performance will be better with the increasing of SNR and group length.

4.2. Calculational Complexity Analysis

We can use total visited nodes as a benchmark for complexity analysis [14]. At first, we analyze maximum likelihood algorithm, if the modulation constellation points number is M, the visited nodes of the ML algorithm can be expressed as:

$$x = 1 + \sum_{i=1}^{N-1} M^i \ . \tag{14}$$

The MSDSD algorithm limits visited nodes in a certain spherical range. For MSDSD algorithm, under each SNR, the algorithm of each layer average retention branches are $t_0, t_1, ..., t_{N-1}$. When $N = 2$, The MSDSD algorithm has same visited nodes with the ML algorithm, therefore two kinds of algorithm has same computational complexity. When $N > 2$ for MSDSD, the visited nodes number is as follows:

$$x = 1 + \sum_{i=1}^{N-1} t_{i-1} \ . \tag{15}$$

For example, when modulation method is D8PSK, the visited nodes number of ML algorithm under the different SNR is gven by Table 1.

Table 1. The Visited Nodes Number with ML Algorithm

SNR(dB)	1	4	7	10	13	16	19
N=2	9	9	9	9	9	9	9
N=4	585	585	585	585	585	585	585

For MSDSD, under each corresponding SNR, we calculate average visited nodes bumber when group length is different. MATLAB verifies that the computational complexity of MSDSD is shown by Table 2.

Table 2. The Visited Nodes Number with MSDSD Algorithm

SNR(dB)	1	4	7	10	13	16	19
N=2	9	9	9	9	9	9	9
N=4	136	118.2	104.4	93.05	71.05	51.95	27.75

From Table 1 and Table 2, when group length is N=2, MSDSD algorithm has same computational complexity with ML algorithm. When group length is N=4, the computational complexity of MSDSD algorithm reduced by the increasing of SNR obviously, but ML algorithm has not change.

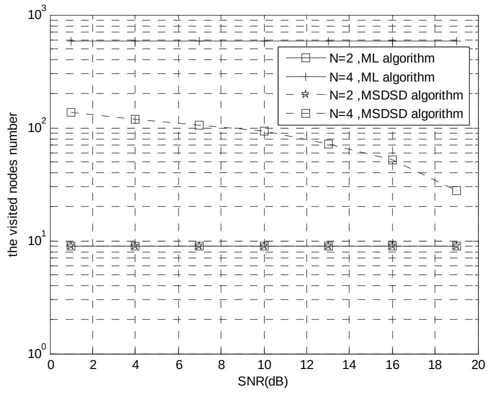

Figure 6. The Visited Nodes Number of MSDSD and ML Decoding under Different Group Length

We use Figure.6 to show the visited nodes number under different group length N=2 and N=4. The computational complexity of ML algorithm is not changing with the increasing of SNR. But MSDSD can significantly reduce computational complexity when N=4 and high SNR.

5. Conclusion

In this paper, we mainly consider the D-NC scheme in TWRC model. The depth-first MSDSD was applied to D-NC scheme to reduce ML detection's high computational complexity. We describe MSDSD alghorithm and analyse computational complexity in detail. Simulation results show the performance of D-NC scheme which has a better performance under the slower fadng channel. Meanwhile, the using of MSDSD in proposed scheme can reduce computational complexity effectively and overcome deep channel fading.

At present, many researchers devote to study of network coding in TWRC model. More and more new thoughts are put forward for further improve performance. Hower, it is still a huge challenge to propose more complex models and more advanced technology.

Acknowledgement

This paper is supported by the Zhejiang Provincial Natural Science Foundation of China (No. LQ12F01010), the National Natural Science Foundation of China (No. 60902011) and the open research fund of National Mobile Communications Research Laboratory, Southeast University (No. 2011D18).

References

[1] Wei Li, Jie Li. Network Coding for Two-Way Relaying Networks Over Rayleigh Fading Channels. *IEEE Transactions on Vhicular Technology.* 2010; 59(9): 4476-4488.
[2] Moonseo Park, Seong-LyunKim. *Minimum Distortion Network Code Design for Source Coding Over Noisy Channels.* IEEE 19th International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC). Cannes. 2008: 1-5.
[3] Wenli Shen. Encoding performance of orthogonal space-time coded continuous phase modulation system. *TELKOMNIKA Indonesian Journal of Electrical Engineering.* 2013; 11(5): 2415-2423.
[4] Roghayeh Joda, Farshad Lahouti. Network Code Design for Orthogonal Two-Hop Network with Broadcasting Relay: A Joint Source-Channel-Network Coding Approach. *IEEE Transactions on Communications.* 2011; 60(1): 132-142.
[5] Xiang-Gen Xia, Qinye Yin. A Linear Analog Network Coding for Asynchronous Two-Way Relay Networks. *IEEE Transactions on Wireless Communications.* 2010; 9(12): 3630-3637
[6] Lingyang Song, Guo Hong, Merouane Debbah. Joint Relay Selection and Analog Network Coding Using Differential Modulation in Two-Way Relay Channels. *IEEE Transactions on Vehicular Technology.* 2010; 59(6): 2932-2939.
[7] Tao Cui, Feifei Gao, Chintha Tellambura. *Physical Layer Differential Network Coding for Two-Way Relay Channel.* IEEE Global Telecommunications Conference. New Orleans, LO. 2008: 1-5.
[8] Tao Cui, Feifei Gao, Chintha Tellambura. Differential Modulation for Two-Way Wireless Communications: A Perspective of Differential Network Coding at the Physical Layer. *IEEE Transactions on Communications.* 2009; 57(10): 2977-2987.
[9] Feifei Gao, Tao Cui, Nallanathan A, Tellambura C. Maximum likelihood detection for differential unitary space-time modulation with carrier frequency offset. *IEEE Transactions on Communications.* 2008; 56(11): 1181-1891.
[10] Lampe L, chober R, Paul Vi, Windpassinger C. Multiple-symbol differential sphere decoding. *IEEE Transactions on Communications.* 2005; 53(12): 1981-1985.
[11] Tao Cui, Chinthananda Tellambura. Bound Intersection Detection for Multiple Symbol Differential Unitary Space Time Modulation. *IEEE Transactions on communications.* 2005; 53(12): 2114-2123.
[12] Hua Li, Jing Bai, Shujian Liao, Juanping Wu. Simulation models for MIMO wireless channels. *TELKOMNIKA Indonesian Journal of Electrical Engineering.* 2013; 11(1): 158-166
[13] Andreas Meier Dr John Thompson. *Cooperative Diversity in Wireless Networks.* 6th IEEE International Conference on 3G and Beyond. Washington, DC. 2005: 1-5.
[14] Volker Pauli, Lutz Lampe. On the Complexity of Sphere Decoding for Differential Detection. *IEEE Transactions on information theory.* 2007; 53(4): 1595-1603.