# Implementing a novel fault prognosis technique based on nonlinear fault observer and online parameters estimation

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# ABSTRACT

This research presents a methodology for predicting errors of parameters, as the algorithm tries to monitor the parameters in order to maintain or replace them when needed to avoid excessive expenses. The presented implementation mechanism is based on monitoring parameters according to a specific number of batches and each batch consists of a number of iterations, which in turn are a number of samples. The proposed algorithm involves designing a new nonlinear observer and writing a secondary algorithm for parameter estimation based on the online nonlinear recursive least squares algorithm associated with the observer states. In addition, the algorithm presents an attempt to find a relationship between the error states and the state of the parameters by creating a new function to determine the weight of the error according to four components; parameter changes, output residuals, output errors and the error diagnosed by the new observer. The algorithm also includes introducing the probability form of the weights using the kernel density function for the average and maximum weights for each batch. Finally, relying on the results, it is possible to take the appropriate decision to maintain or change the parameters as shown a non-linear direct current motor model case study.

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# 1. INTRODUCTION

One of the most important goals of researchers in the automated system is high performance, safety and cost. In order to achieve these problems, error diagnosis has captured the attention of researchers in the past decade. Fault diagnosis algorithms for electrical systems have been studied to monitor the deterioration of their working conditions [1]-[11] while [12] establishes a nonlinear dynamics framework for diagnosis and prognosis in structural dynamic systems by developing an analytically sound means for extracting features.

Focusing on the sequential Monte Carlo method, some papers [11], [13] include a sampling and resampling method for the nonlinear state of a complex dynamical system. When the state of deterioration cannot be directly observed, introduced a model to find the mean residual (residual) and reliability function [14]. Posted fault strategies and rules for diagnosing a fault for a thermo acoustic generator to center and sustain the mover of the electric generator [15]. While Ribot *et al.* [16] presented the problem of maintenance for a system considered as complex and heterogeneous as an airplane. Ekanayake *et al.* [17], reviewed the techniques based on the graphical model to predict errors. Wang and Winters [18], implemented a methodology of prognosis using a neuro-fuzzy techniques for dynamic recurrent system. Liu *et al.* [19] processed a new test device to simulate the multi-component degradation of an aircraft fuel system. Bae *et al.* [20] presented algorithm for the e-prognosis and e-diagnosis purposes. Chuan *et al.* [21] introduced machine-

learning algorithm known as support vector machine (SVM) where Gaussian radial basis function (kernel function) used to accomplish the optimized classifier.

The observers for the fault detection and diagnosis constitute a main part of the fault tolerant control [22]-[32]. The researchers interested in state space-based in presence actuators fault and sensor noise [29], [33], [34]. In addition, some papers were presented to be a solution for fault detection and diagnosis based on aftificial neural networks. In [35]-[42] implemented new algorithms using two types of schemes: recurrent neural networks and fuzzy-neural systems used to predict faults. While a new intelligent nonlinear observer in [43] implied a new fault diagnosing rule based on fuzzy and sequential important sampling filter.

In this paper, a new prognosis algorithm using an optimal nonlinear observer, new online parameter estimation based on estimated observer states and using kernel density function to check the status of the parameters. Therefore, the structure of paper has been organized as: section 2 introduces problem formulation. Section 3 includes the design of the new optimal nonlinear observer. While the implementation of proposed prognosis algorithm described in section 4. Section 5 demonstrates the proposed prognosis algorithm. Finally, section 6 includes the conclusion to discuss the obtained result.

# 2. PROBLEM FORMULATION

The new diagnostic prognosis algorithm (PA) has been implemented to monitor the execution within known number of batches. The execution is divided into batches and each batch includes iterations. For more simplicity, each iteration represents a single implementation with known samples as shown in Figure 1. Therefore, the structure of the new PA diagnostic algorithm is categorized as:

- Designing a nonlinear observer nonfederal organization (NFO) to detect and diagnose the parameters' fault and sensor noise.
- A new online square algorithm was set in order to find the error in parameter estimation due to fault and noise.
- The algorithm generates a new function to calculate the weight for parameter decay. The new function is a cumulative sum of four elements, which are; the expected output error in the parameter estimation algorithm, the fault diagnosis by the new observer, the residual is the difference between the output of the plant and the observer and the error between the plant's output and the desired output.
- In order for the algorithm to be more flexible and linked to information from previous iterations, the unnormalize weight of the previous iteration will be added to the existing weight at the same time sample.
- For each batch, the mean and maximum weights are stored.
- For specified batch numbers, kernel density function is used as a tool for checking the status of monitored parameters based on average and maximum weights.



Figure 1. Prognosis cycles

# **3. DESIGN A NONLINEAR FAULT DETECTION AND DIAGNOSIS OBSERVER (NFO)** The discrete nonlinear dynamic model for the plant has been considered as (1).

$$\begin{cases} x_{k+1} = f(x_k, u_k, \zeta_k) \\ y_k = Cx_k + v_k \end{cases}$$

(1)

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Where  $f(x_k, u_k, \zeta_k)$  is a function of  $x_k \in \mathbb{R}^{n_a}$ ,  $u_k \in \mathbb{R}^{n_{in}}$  that is the non-measurable states  $n_a$  vector of the system and the inputs  $n_{in}$  vector respectively while  $y_k \in \mathbb{R}^p$  express the measurable outputs p vector and its matrix C. The main part of the observer is the diagnosed additive fault (parameter fault)  $\zeta_k \in \mathbb{R}^{n_{in}}$  which occurs due to uncertainty of the parameters in the plant. In addition, the sensors noise  $v_k \in \mathbb{R}^p$  also considered.

The plant has been assumed in (1) a nonlinear Lipschitzian system [44] which will be expressed as:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + \alpha(x_k, u_k) + L_f \zeta_k \\ y_k = Cx_k + v_k \end{cases}$$
(2)

where  $\alpha(x_k, u_k) \in R^{n_a \times n_a}$  represents the nonlinearities of the system as well as  $A \in R^{n_a \times n_a}$ ,  $B \in R^{n_a \times n_{in}}$ ,  $C \in R^{p \times n_a}$  and  $L_f \in R^{n_a \times n_a}$  are constant matrices of appropriate dimensions. Furthermore, the implementation of the observer should satisfy two conditions as:

- The pair (A, B) is controllable and (A, C) is detectable while
- The system is observable and controllable and  $f(x_k, u_k)$  is Lipschitzian with respect to the state  $x_k$  uniformly in the control  $u_k$ , then there exists a constant  $\partial$  such that:

$$\begin{aligned} \|\alpha(x_{k}, u_{k}) - \alpha(x_{k+1}, u_{k+1})\| &\leq \partial \|x_{k} - x_{k+1}\|, \\ x_{k}, x_{k+1} \in R^{n_{a}}, u_{k} \in R^{n_{in}} \\ \|\alpha(x_{k}, u_{k})\| &\leq \partial \|x_{k}\|, \forall u_{k} \in R^{n_{in}} \end{aligned}$$
(3)

the new nonlinear observer to detect and diagnose the fault and estimate the states of the system has been designed as (4):

$$\begin{cases} \hat{x}_{k+1} = A\hat{x}_k + Bu_k + \alpha(\hat{x}_k, u_k) + Hr_k + \hat{L}_f \hat{\zeta}_k \\ \hat{y}_k = C\hat{x}_k \end{cases}$$
(4)

where the gain matrix of the observer  $L \in \mathbb{R}^{n_a \times n_a}$  can be found by the pole placement method so it achieves Lyapunov conditions.  $\hat{L}_f \in \mathbb{R}^{n_a \times n_a}$  is prespecified matrix of a diagnosed fault  $\hat{\zeta}_k$ . In addition,  $\alpha(\hat{x}_k, u_k)$  is Lipschitzian with respect to the state  $\hat{x}_k$ . To find the output residual  $\varphi_k$  the output of the observer needed $\hat{y}_k$  and the output error  $\phi_k$  is the error between the output  $y_k$  and the desired output  $y_{k_{desired}}$  where  $T_f$  is the fault threshold.

$$\varphi_k = y_k - \hat{y}_k \tag{5}$$

$$\phi_k = y_{k_{desired}} - y_k \tag{6}$$

$$\begin{cases} \|\phi_k\| \ge T_f & fault \ has \ been \ occured \\ \|\phi_k\| < T_f & no \ fault \ occurs \end{cases}$$
(7)

To diagnose the fault after the alarm has been generated in (7), the diagnosed fault vector represents the process fault occurred in the system as (8).

$$\zeta_k = [\zeta_1, \dots, \zeta_{r_{in}}]^T \tag{8}$$

When no fault occurs, the fault will be assumed as (9) [45].

$$\zeta_k = 0^{r_{in}}.\tag{9}$$

Therefore, to obtain an accurate estimation, need (10).

$$\lim_{k \to \infty} \hat{\zeta}_k = \zeta_k \tag{10}$$

Furthermore, the faults error can be expressed as (11).

$$\tilde{\zeta}_k = (\zeta_k + v_k) - \hat{\zeta}_k \tag{11}$$

The proposed dynamic fault and dynamic fault error can have rewritten as (12).

$$\hat{\zeta}_{k+1} = -\Gamma_1 \hat{\zeta}_k - \Gamma_2 \varphi_k \tag{12}$$

Where  $\Gamma_1, \Gamma_2$  are the prespecified learning operators to be determined. The proposed diagnostic fault [34] is switch of term  $-\Gamma_1\hat{\zeta}_k$  which is determined by the upper bound of  $\hat{\zeta}_k$  and by the observation output error  $\varphi_k$ . As result, the goal of the fault diagnosis is to find a diagnostic algorithm for  $\hat{\zeta}_k$  and an observer gain vector  $\hat{L}_f$  such that:

$$\begin{cases}
\lim_{k \to \infty} & \varphi_k \to o \\
\lim_{k \to \infty} & \tilde{\zeta}_k \to o
\end{cases}$$
(13)

#### **3.1.** Assumption the theorem

The states error can be written as shown in (14) and (15).

$$e_k = x_k - \hat{x}_k \tag{14}$$

$$e_{k+1} = x_{k+1} - \hat{x}_{k+1} = Ax_k + Bu_k + \alpha(x_k, u_k) + L_f \zeta_k - (A\delta(\hat{x}_k)\hat{x}_k + Bu_k + \alpha(\delta(\hat{x}_k)\hat{x}_k, u_k) + H\varphi_k + \hat{L}_f \hat{\zeta}_k)$$
(15)

Then the equation of states error can be expressed as:

$$e(k+1) = Ax_k + Bu_k + \alpha(x_k, u_k) + L_f \zeta_k - (A\delta(\hat{x}_k)\hat{x}_k + Bu_k + \alpha(\delta(\hat{x}_k)\hat{x}_k, u_k) + H\varphi_k + \hat{L}_f \hat{\zeta}_k)$$
  
$$= Ax_k + Bu_k + \alpha(x_k, u_k) + L_f \zeta_k - (A\delta(\hat{x}_k)\hat{x}_k Bu_k + \alpha(\delta(\hat{x}_k)\hat{x}_k, u_k)) + H(y_k - \hat{y}_k) - \hat{L}_f \hat{\zeta}_k$$
(16)

it can be further expressed as:

$$e_{k+1} = Ax_k + Bu_k + \alpha(x_k, u_k) + L_f \zeta_k - \begin{pmatrix} A\delta(\hat{x}_k)\hat{x}_k + Bu_k + \alpha(\delta(\hat{x}_k)\hat{x}_k, u_k) \\ + H(Cx_k - C\delta(\hat{x}_k)\hat{x}_k) - \hat{L}_f \hat{\zeta}_k \end{pmatrix}$$
(17)

for more simplification, assume:

$$\tilde{L}_f \tilde{\zeta}_k = L_f \zeta_k - \hat{L}_f \hat{\zeta}_k, \tilde{\alpha}(x_k, \delta(\hat{x}_k) \hat{x}_k, u_k) = \alpha(x_k, u_k) - \tilde{\alpha}(\delta(\hat{x}_k) \hat{x}_k, u_k), \bar{A} = A - HC$$

then, the dynamic error rewritten as:

$$e_{k+1} = (A - HC)x_k + \tilde{\alpha}(x_k, \delta(\hat{x}_k)\hat{x}_k, u_k) + \tilde{L}_f \tilde{\zeta}_k) - (A - HC)\delta(\hat{x}_k)\hat{x}_k$$
  
=  $\bar{A}(x_k - \delta(\hat{x}_k)\hat{x}_k) + \tilde{\alpha}(x_k, \delta(\hat{x}_k)\hat{x}_k, u_k) + \tilde{L}_f \tilde{\zeta}_k = \bar{A}e_k + \tilde{\alpha}(x_k, \delta(\hat{x}_k)\hat{x}_k, u_k) + \tilde{L}_f \tilde{\zeta}_k$  (18)

hence, the dynamical error can be rewritten as:

$$e_{k+1} = \bar{A}e_k + \pi_k + w_k \tag{19}$$

assume positive definite matrices  $Q_1$ ,  $Q_2$  to ensure the convergence where exist a symmetric, positive definite matrices  $P_1$ ,  $P_2$  that solves and satisfy.

$$\bar{A}^T P_1 \bar{A} - P_1 = -Q_1, \ \bar{A}^T P_2 \bar{A} - P_2 = -Q_2 \tag{20}$$

#### **3.2.** Proof of the theorem

Define the Lyapunov function  $\Upsilon(e_k, \bar{e}_k, \tilde{\zeta}_k)$  candidate.

$$\Delta \Upsilon(e_k, \tilde{\zeta}_k) = \Upsilon(e_{k+1}) - \Upsilon(e_k) - trace\left(\left(\hat{\zeta}_k - \zeta_k\right)\Gamma_2^{-1}\left(\tilde{\zeta}_k - \zeta_k\right)\right) - trace\left(\zeta_k\Gamma_2^{-1}\left(\hat{\zeta}_k - \zeta_k\right)\right)$$
(21)

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Assume that:

$$\bar{A}^T P_1 \bar{A} - P_1 = -Q_1 \tag{22}$$

$$\bar{A}^T P_2 \bar{A} - P_2 = -Q_2 \tag{23}$$

where  $Q_1, Q_2 > 0$  and matrices of other parameters  $\Gamma_1, \Gamma_2$ . They should all be positively defined to ensure convergence of error diagnoses in (12). ( $\Gamma_1 = \eta I$ ) where  $\eta$  is the scalar and I is the unit matrix. These parameter matrices will affect the convergence speed. So, Lyapunov's function will be as shown in (24):

$$\Delta \Upsilon(e_k, \tilde{\zeta}_k) = \Upsilon \left( (e_{k+1}^T P_1 e_{k+} - e_k^T P_1 e_k) + (e_{k+1}^T P_2 e_{k+1} - e_k^T P_2 e_k) - trace \left( \zeta_k \Gamma_2^{-1} (\hat{\zeta}_k - \zeta_k) \right) \right)$$
(24)

for further simplifying:

$$\Delta Y(e_{k}, \tilde{\zeta}_{k}) = \begin{pmatrix} ([\bar{A}e_{k} + \pi_{k} + w_{k}]^{T}P_{1}[\bar{A}e_{k} + \Pi_{k} + w_{k}] - e_{k}^{T}P_{1}e_{k}) \\ + ([\bar{A}e_{k} + \pi_{k} + w_{k}]^{T}P_{2}[\bar{A}e_{k} + \pi_{k} + w_{k}] - e_{k}^{T}P_{2}e_{k}) \\ - \frac{1}{2}trace\left((\hat{\zeta}_{k} - \zeta_{k})^{T}\Gamma_{2}^{-1}(\hat{\zeta}_{k} - \zeta_{k})\right) - \frac{1}{2}trace(\hat{\zeta}_{k}^{T}\Gamma_{2}^{-1}\hat{\zeta}_{k}) + \frac{1}{2}trace(\zeta_{k}^{T}\Gamma_{2}^{-1}\zeta_{k}) \end{pmatrix}$$
(25)

for stability, the Lyapunov function can be further expressed as an inequality as:

$$\Delta \Upsilon(e_k, \tilde{\zeta}_k) \leq \begin{pmatrix} (-\lambda_{min}(Q_1) \|e_k\|^2 \|P_1\| \|\pi_k\| + \|P_1\| \|w_k\|) \\ + (-\lambda_{min}(Q_2) \|\bar{e}_k\|^2 \|P_2\| \|\pi_k\| + \|P_2\| \|w_k\|) \\ - \frac{1}{2} trace(\hat{\zeta}_k^T \Gamma_2^{-1} \hat{\zeta}_k) + \frac{1}{2} trace(\zeta_k^T \Gamma_2^{-1} \zeta_k) \end{pmatrix}$$
(26)

for further simplification, assumptions are:

$$\lambda_{1} = \lambda_{min}(Q_{1}) \|e_{k}\|, \lambda_{2} = \lambda_{min}(Q_{2}) \|e_{k}\|, \lambda_{3} = \lambda_{max}(P_{1}), \lambda_{4} = \lambda_{max}(P_{2}), \lambda_{5} = \lambda_{min}(\Gamma_{1}^{T}\Gamma_{2}^{-1}), \lambda_{6} = \lambda_{max}(\Gamma_{1}^{T}\Gamma_{2}^{-1}), \lambda_{7} = \lambda_{min}(\Gamma_{2}^{-1}), \lambda_{8} = \lambda_{max}(\Gamma_{2}^{-1}), \lambda_{9} = \lambda_{min}(P_{1}Q_{1}^{-1}P_{1}), \lambda_{10} = \lambda_{min}(P_{2}Q_{2}^{-1}P_{2}), \Gamma_{3} = \sup \|\zeta_{k}\|, \delta_{0} = \min \left\{ \frac{\lambda_{1}}{\lambda_{6}}, \frac{\lambda_{2}}{\lambda_{6}}, \frac{\lambda_{5}}{\lambda_{8}} \right\}$$
(27)

then the inequality will be:

$$\Delta \Upsilon(e_{k}(k), \tilde{\zeta}_{k}) \leq \begin{pmatrix} \frac{-\lambda_{1}}{\lambda_{3}} e_{k}Q_{1}e_{k} + \|P_{1}\|\|\pi_{k}\| + \|P_{1}\|\|w_{k}\|\frac{-\lambda_{2}}{\lambda_{4}}e_{k}Q_{2}e_{k} + \|P_{2}\|\|\pi_{k}\| + \|P_{2}\|\|w_{k}\| \\ -\frac{\lambda_{5}}{2\lambda_{8}}trace(\hat{\zeta}_{k}^{T}\Gamma_{2}^{-1}\hat{\zeta}_{k}) + \lambda_{6}\|\zeta_{k}\| \end{pmatrix}$$
(28)

the inequality will be further expressed as (29).

$$\Delta \Upsilon \left( e_{k}(k), \tilde{\zeta}_{k} \right) \leq \begin{pmatrix} \frac{-\lambda_{1}}{\lambda_{3}} \| e_{k} \| + \frac{\lambda_{3}}{\lambda_{3}} (\| \pi_{k} \| + \| w_{k} \|) \frac{-\lambda_{2}}{\lambda_{4}} \| e_{k} \| \\ + \frac{\lambda_{4}}{\lambda_{3}} (\| \pi_{k} \| + \| w_{k} \|) \\ - \frac{\lambda_{5}}{2\lambda_{8}} trace \left( \left( \hat{\zeta}_{k} - \zeta_{k} \right)^{T} \Gamma_{2}^{-1} \left( \hat{\zeta}_{k} - \zeta_{k} \right) \right) + \lambda_{6} \Gamma_{3}^{2} \end{pmatrix}$$

$$(29)$$

For the first term, the Rayleigh-Ritz inequality, the Cauchy-Schwarz inequality, and the index matrix rule were used for the second. The linearity can be satisfied by approximating the nonlinear dynamic equation to the linear dynamic equation which leads to:

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$$\left(\lim_{\|e_k\|\to 0}\frac{\|\pi_k\|}{\|e_k\|} + \lim_{\|e_k\|\to 0}\frac{w_k}{\|e_k\|}\right) = 0$$

this mean that for any  $\sigma_1, \sigma_2 > 0$ , no matter how small, there exists  $\varepsilon_1, \varepsilon_2 > 0$ , yields that:

$$\|e_k\| < \sigma_1 \Rightarrow \frac{\|\pi_k\|}{\|e_k\|} < \varepsilon_1, \frac{\|w_k\|}{\|e_k\|} < \varepsilon_2$$

$$\tag{30}$$

to satisfy the condition in (30),  $||e_k|| < \sigma_1$  and  $u_k$  can be chosen to obtain:

$$\Delta \Upsilon \left( e_k(k), \tilde{\zeta}_k \right) \leq \begin{pmatrix} \|e_k\| \left( \frac{-\lambda_1}{\lambda_3} + \frac{\lambda_1}{\lambda_3 \|e_k\|} \left( \|\pi_k\| + \|w_k\| \right) \right) \\ \|e_k\| \left( \frac{-\lambda_2}{\lambda_4} + \frac{\lambda_2}{\lambda_4 \|e_k\|} \left( \|\pi_k\| + \|w_k\| \right) \right) \\ - \frac{\lambda_5}{2\lambda_8} trace \left( \left( \hat{\zeta}_k - \zeta_k \right)^T \Gamma_2^{-1} \left( \hat{\zeta}_k - \zeta_k \right) \right) + \lambda_6 \Gamma_3^2 \end{pmatrix}$$
(31)

it can be more represented as:

$$\Delta \Upsilon \left( e_k(k), \tilde{\zeta}_k \right) \leq \left( \|e_k\| \left( \frac{-\lambda_1}{\lambda_3} + \frac{\lambda_1}{\lambda_3} (\sigma_1 + \sigma_2) \right) \|e_k\| \left( \frac{-\lambda_2}{\lambda_4} + \frac{\lambda_2}{\lambda_4} (\sigma_1 + \sigma_2) \right) \|\hat{\zeta}_k - \zeta_k\|^2 \left( \frac{-\lambda_7}{\lambda_8} + \lambda_6 \Gamma_3^2 \right) \right)$$
(32)

to satisfy stability; the equilibrium of a nonlinear system must be asymptotically stable by Lyapunov stability theory, and the condition  $\Delta \Upsilon(e_k, \tilde{\zeta}_k) < 0$  should be realized.

$$(\varepsilon_1 + \varepsilon_2) < \frac{1}{2}, \Gamma_3 < \sqrt{\frac{\lambda_7 \lambda_6}{\lambda_8}}$$
(33)

# 4. PROGNOSIS ALGORITHM (PA)

A new prognosis algorithm PA was implemented to show the relationship between the occurrence of the observer-diagnosed fault and the parameter estimation error simultaneously by online iterative least square. The new idea also creates a new function to calculate the weight of the error. In addition, the implementation of the algorithm is a known number of batches, and each batch is a set of iterations with specific time samples for each iteration. To evaluate the algorithm, the average and maximum weights for each batch are kept in memory. Subsequently, the EKD kernel density function is used to find the probabilities of the mean and maximum weights, as shown in the following steps of the proposed algorithm.

## 4.1. At first batch, initializing the matrices

Due to parameters fault, the model of the system in (2) will be introduced as a stochastic plant to express the parameter matrices as a nonlinear ARMAX model:

$$\hat{A} = \begin{bmatrix} \hat{a}_{1,1} & \cdot & \cdot & \hat{a}_{1,n_a} \\ \cdot & \cdot & \cdot & \cdot \\ \hat{a}_{1,n_a} & \cdot & \cdot & \hat{a}_{n_a,n_a} \end{bmatrix}, \hat{B} = \begin{bmatrix} \hat{b}_{1,1} & \cdot & \cdot & \hat{b}_{1,n_a} \\ \cdot & \cdot & \cdot & \cdot \\ \hat{b}_{n_a,1} & \cdot & \cdot & \hat{b}_{n_a,n_b} \end{bmatrix}$$
(34)

$$\bar{\theta}_{k,i} = \begin{bmatrix} \hat{A} & \hat{B} \end{bmatrix}$$
(35)

$$\bar{\phi}_{k,i} = f_p \big( \hat{x}_{1,k,i}, \hat{x}_{2,k,i}, \dots, \hat{x}_{n_a,k,i}, u_{1,k,i}, u_{2,k,i}, \dots, u_{n_b,k,i}, \breve{w}_{k,i} \big)$$
(36)

where  $f_p$  is the unknown nonlinear plant equation, bounded and first order differentiable with respect to all of its variables.  $\hat{x} \in R^{n_a \times 1}$  and  $u \in R^{n_b \times 1}$  represent the observed state of the observer and the input as in (4) and (2) respectively.

Based on the above, the nonlinearity of the estimated system causes an error between the estimated and true values as in most practical engineering systems. Hence, bounded noise  $\breve{w}_{k,i}$  can be assumed using least-squared method [46] by choosing a number of sampling points as (37).

$$f_{k,i} = \bar{\theta}_{k-1,i}^T \bar{\phi}_{k,i} \tag{37}$$

# 4.2. NEW: starting for a new prognosis batch

From the observed output of the observer in (4), predicated least square parameters in (35) and predicted input vector in (36), the predicated error  $\bar{\varepsilon}_{k,i}$  will be expressed as (38).

$$\bar{\varepsilon}_{k,i} = \hat{y}_{k,i} - \bar{\theta}_{k-1,i} \,\bar{\phi}_{k,i} \tag{38}$$

Form the assumed probability  $\overline{P}_{k,i}$  using (39).

$$\bar{P}_{k,i} = \bar{P}_{k-1,i} \left( I - \frac{\bar{\phi}_{k,i} \bar{\phi}_{k,i}^{T} \bar{P}_{k-1,i}}{\left( 1 + \bar{\phi}_{k,i}^{T} \bar{P}_{k-1,i} \bar{\phi}_{k,i} \right)} \right)$$
(39)

Update the least squares estimate  $\bar{\theta}_{k,i}$ 

$$\bar{\theta}_{k,i} = \bar{\theta}_{k-1,i} + \bar{\varepsilon}_{k,i} \bar{P}_{k,i} \bar{\phi}_{k,i} \tag{40}$$

furthermore, the parameters changes will be defined as (41):

$$\tilde{\theta}_{k,i} = \theta_{k,i}^{setpoin\bar{t}_{k,i}} \tag{41}$$

where  $\theta_{k,i}^{setpoint}$  represents the desired values of the parameters. Calculating the weight function at the sample *k*.

The suggested weight in the sample k for  $i^{th}$  iteration is a function of four items; The parameter amount changes in (42), the output residual in (5), the output error in (6), and the dynamic diagnosed fault in (12) according as (42).

$$f_{w_{k,i}} = \tilde{\theta}_{k,i} + \varphi_{k,i} + \hat{\zeta}_{k+1,i}$$
(42)

In addition, the unnormalize weight  $\hat{f}_{w_{k,i-1}}$  of the previous weight will be added to the existence weight to show whether the fault is persistent in the same sample as (43).

$$\hat{f}_{w_{k,i}} = \hat{f}_{w_{k,i}} + \hat{f}_{w_{k,i-1}} \tag{43}$$

Calculate mean and maximum weight values for each batch. After  $N_{iteration}$ , the mean  $E_b$  and maximum values  $M_b$  for  $N_B$  batches are (45) and (46).

$$E_b = \frac{1}{N_{iteration}} \sum_{N=1}^{N_{iteration}} \hat{f}_{w_{k,N_{iteration}}}$$
(44)

$$M_b = \sup\left\{\hat{f}_{w_{k,N_{iteration}}}\right\} \tag{45}$$

#### 4.3. Finally, at the end of the $N_B$ batches

The estimated density kernel (EDK) is used to find the probabilities for the mean and maximum values as shown in (46) and (47):

$$\hat{f}_{E_b} = \frac{1}{N_B H} \sum_{i=1}^{N_B} K_r \left( \frac{r - E_b}{H} \right) \tag{46}$$

$$\hat{f}_{M_b} = \frac{1}{N_B H} \sum_{i=1}^{N_B} K_r \left(\frac{r - M_b}{H}\right)$$
(47)

where  $N_{B}$ , H are the batches number and the bandwidth for the kernel smoothing function  $K_r$ .

#### 5. CASE STUDY AND RESULTS

To validate the results of the new proposed algorithm, the continues time nonlinear model of the DC Motor system is considered as (48) [31].

$$\begin{bmatrix} \dot{I}_{A}(t) \\ \dot{\omega}(t) \end{bmatrix} = \begin{bmatrix} -R_{a}/L_{a} & -\Psi/L_{a} \\ (\Psi/J) & -M_{F1}/J \end{bmatrix} \begin{bmatrix} I_{A}(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} 1/L_{a} & 0 \\ 0 & -1/J \end{bmatrix} \begin{bmatrix} U_{A}(t) \\ M_{L}(t) \end{bmatrix} + \begin{bmatrix} -K_{B}|\omega(t)|I_{A}(t) \\ -M_{F0}\,sign(\omega(t)) \end{bmatrix}$$

$$\begin{bmatrix} I_{A}(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_{A}(t) \\ \omega(t) \end{bmatrix}$$

$$(48)$$

The parameter matrix in (49) is assumed to be affected by the assumption of the parameter fault where the nonlinear model of the motor can be as:

$$\begin{bmatrix} \zeta_1(t) \\ \zeta_2(t) \end{bmatrix} = \begin{bmatrix} -R_a/L_a & -\Psi/L_a \\ (\Psi/J) & -M_{F1}/J \end{bmatrix} \begin{bmatrix} I_A(t) \\ \omega(t) \end{bmatrix} - \begin{bmatrix} -R_a/L_a & -\Psi/L_a \\ (\Psi/J) + fault(t) & -M_{F1}/J \end{bmatrix} \begin{bmatrix} I_A(t) \\ \omega(t) \end{bmatrix}$$

then the system with the additive fault  $\zeta(t)$  and sensor noise v(t) can be represent as:

$$\begin{bmatrix} I_{A}(t) \\ \dot{\omega}(t) \end{bmatrix} = \begin{bmatrix} -R_{a}/L_{a} & -\Psi/L_{a} \\ (\Psi/J) & -M_{F1}/J \end{bmatrix} \begin{bmatrix} I_{A}(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} 1/L_{a} & 0 \\ 0 & -1/J \end{bmatrix} \begin{bmatrix} U_{A}(t) \\ M_{L}(t) \end{bmatrix} + \begin{bmatrix} -K_{B}|\omega(t)|I_{A}(t) \\ -M_{F0} sign(\omega(t)) \end{bmatrix} + \begin{bmatrix} \zeta_{1}(t) \\ \zeta_{2}(t) \end{bmatrix}$$
$$\begin{bmatrix} I_{A}(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_{A}(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ v(t) \end{bmatrix}$$

to verify the PA algorithm, a discrete nonlinear has been used where the following matrices values have been chosen to satisfy the conditions of observer design in (20) and (12).

$$P_{1} = \begin{bmatrix} 1.1897 & 0.0947 \\ 0.0947 & 0.0792 \end{bmatrix}$$

$$Q_{1} = \begin{bmatrix} 0.92 & 0 \\ 0 & 0.04 \end{bmatrix}$$

$$P_{2} = \begin{bmatrix} -1.1556 & 0.0259 \\ 0.6751 & -1.2956 \end{bmatrix}$$

$$Q_{2} = \begin{bmatrix} -0.3583 & 0.117 \\ 0.6774 & -1.2615 \end{bmatrix}$$

$$H = \begin{bmatrix} -0.512 & -0.325350 \\ 0.56 & 0.5949 \end{bmatrix}$$

$$\hat{L}_{f} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$$

$$\Gamma_{1} = \begin{bmatrix} -0.8 & 0 \\ 0 & -0.9 \end{bmatrix}$$

$$\Gamma_{2} = \begin{bmatrix} -2 & 0 \\ 0 & -15 \end{bmatrix}$$

The following lines show the detection of various faults under the open loop strategy of eight batches, each batch containing ten iterations of sixty seconds in length. To study the algorithm, Gaussian sensor noise has been considered, the mean has zero value and the variance is 0.3. In addition, the PA algorithm is realized by introducing two types of faults in the second and eighth iteration of each batch where the mean Gaussian fault is 1.2, the variance=0.5 but the non-Gaussian fault is proposed to be fault(t) = 2.5 + 3 sin(t).

To investigate the performance of the paper, two types of faults have been assumed two obtain the changes in the second and eighth batches according to the maximum values; i) in existence Gaussian fault with mean, Figure 2 and Figure 3 include the means and maximums values of the weights for the eight batches respectively while Figure 4 and Figure 5 demonstrate the symmetrical shape of the estimated density kernel for the means and maximums values respectively, and ii) in presence non Gaussian fault, the Figure 6 and Figure 7 show the means and maximums values of the weights for the batches whereas Figure 8 and Figure 9 demonstrate the symmetrical shape of the estimated density kernel for the means and maximums values of the estimated density kernel for the means and maximums values of the estimated density kernel for the means and maximums respectively. Hence, it is evident from the following figures that the designed prognosis algorithm successfully detects all parameter changes since the efficiency of the proposed PA algorithm is achieved under the evaluation of conditions values.



Figure 2. The means for the weights in eight batches (Gaussian fault)



Figure 3. The maximums for the weights of the weights in eight batches (Gaussian fault)



Figure 4. The estimated density kernel for the means of weights in eight batches (Gaussian fault)



Figure 5. The estimated density kernel for the maximum weights in eight batches (Gaussian fault)



Figure 6. The means for the weights in eight batches (non-Gaussian fault)



Figure 8. The estimated density kernel for the means of the weights in eight batches (non-Gaussian fault)



Figure 7. The maximums of the weights for the weights in eight batches (non-Gaussian fault)



Figure 9. The estimated density kernel for the maximum weights in eight batches (non-Gaussian fault)

# 6. CONCLUSION

To overcome two critical problems of non-linear automated systems; uncertainty and unnecessary plant maintenance, a new prognosis (alarm) algorithm was implemented and studied. The uncertainty is due to uncertainties in the nonlinear model, process perturbations, parameter changes and measurement noise. The main concern of this paper is the development of a new diagnostic algorithm based on the nonlinear fault diagnostic observer and online parameter estimation. The objectives were investigated by looking at the relationship between fault occurrence and parameter uncertainty in the time sample. The implementation of the algorithm depends on dividing the time into a number of batches, each batch consisting of a number of iterations and each iteration, a number of time samples.

To realize the algorithm; a new non-linear fault observer is designed that detects and diagnoses the fault simultaneously, introducing a new optimal online parameters estimation and generating a function to calculate the weight at each time sample. Furthermore, to achieve the prediction with precision and craftsmanship to avoid sudden stops in the parameters, the new function was generated based on four elements; parameter changes, output residuals, output errors and the error diagnosed by the new observer. However, to monitor the parameter condition which is the main goal; the mean and maximum values of the weights in iterations for one batch are saved in the memory. Subsequently, the measured values for one batch are used to find mean and maximum values for a known number of batches where the estimated density kernel is used as an additional tool to make correct decision about a maintenance. To summarize, the nonlinear model of the proposed algorithm has been successfully demonstrated with the help of a realistic example of a nonlinear DC motor assuming that it has been exposed to Gaussian and non-Gaussian faults and disturbances.

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# **BIOGRAPHIES OF AUTHORS**

