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Modified three-term conjugate gradient algorithm and its applications in image restoration

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ABSTRACT

In image restoration, the goal is often to bring back a high-quality version of an image from a lower-quality copy of it. In this article, we will investigate one kind of recovery issue, namely recovering photos that have been blurred by noise in digital photographs (sometimes known as "salt and pepper" noise). When subjected to noise at varying frequencies and intensities (30,50,70,90). In this paper, we used the conjugate gradient algorithm to Restorative images and remove noise from them, we developed the conjugate gradient algorithm with three limits using the conjugate condition of Dai and Liao, where the new algorithm achieved the conditions for descent and global convergence under some assumptions. According to the results of the numerical analysis, the recently created approach is unequivocally superior to both the fletcher and reeves (FR) method and the fletcher and reeves three-term (TTFR) metod. Use the structural similarity index measure (SSIM), which is used to measure image quality and the higher its value, the better the result. The original image was compared with all the noisy images and each according to the percentage of noise as well as the images processed with the four methods.

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1. INTRODUCTION

Conjugate gradient techniques are a powerful family of unconstrained optimization algorithms that have excellent local and global convergence qualities while using little memory. They are also fast and efficient. Researchers have continued to exhibit a strong interest in the performance of the convergence property and the simplicity with which algorithms may be represented in computer programs in a same manner over the previous 50 years and into the current day. Numerous academics have recently been studying conjugate gradient techniques, particularly two types of conjugate gradient methods that have received the most interest recently. Secant conditions are used in the first technique, which includes second-order information from the objective function.

2. LITERATURE REVIEW

According to some researchers, unconstrained optimization may be thought of as the challenge of finding a minimization solution to a real function $F(\chi)$

$$\min_{\chi \in R^n} F(\chi) \quad (1)$$

where $F(\chi)$ is a function that can be differentiable at least once for see more [1]–[3]. Problem (1) may be solved with the use of conjugate gradient (CG) algorithms, which are based on the iterative connection shown in:

$$\chi_{k+1} = \chi_k + \alpha_k\, d_k \; k = 0,1,2 \ldots \quad (2)$$

where α_k, the step length in a exact or inexact linear search is calculated when the equation is nonlinear using the following relationship:

$$F(\chi_k + \alpha_k\, d_k) = \min_{a \geq 0} F(\chi_k + \alpha_k\, d_k) \quad (3)$$

d_k it is the direction of the search and as defined in:

$$d_1 = -\nabla F_1 \; k = 1$$
$$d_{k+1} = -\nabla F_{k+1} + \beta_k\, d_k \; k \geq 1 \quad (4)$$

∇F_{k+1} is a vector matrix that represents the gradient's output. The CG method has a parameter named β_k. Here are the Some conjugate gradient algorithms include the following parameters, these techniques always meet the adequate descent condition, where we calculate $\beta_k$with the direction of the search d_{k+1}I in the following $d_1 = -\nabla F_1$, $y_k = \nabla F_{k+1} - \nabla F_k$

$$\beta_k^{FR} = \frac{\nabla F_{k+1}^T \nabla F_{k+1}}{\nabla F_k^T \nabla F_k}[4];\ \beta_k^{PRP} = \frac{y_k^T \nabla F_{k+1}}{g_k^T g_k}[5];\ \beta_k^{HS} = \frac{y_k^T \nabla F_{k+1}}{\nabla F_k^T d_k}\ [6];\ \beta_k^{DL} = \frac{y_k^T \nabla F_{k+1} - \nabla F_{k+1}^T s_k}{\nabla F_k^T d_k}\ [7],$$

see [8]–[15]. There are trinomial conjugated gradient modalities proposed by Zhang [16], a trinomial conjugate gradient modality was proposed for FR, Polak-Ribière (PR) and Hestenes-Stiefel (HS) parameters. These three approaches consistently accomplish the descent property, and the following is the manner in which the search for some trinomial conjugate gradient methods should be directed:
1- FR three- term is:

$$d_{k+1} = -\nabla F_{k+1} + \beta^{FR} d_k - \theta_k^{(1)} \nabla F_{k+1}$$

where $\theta_k^{(1)} = \frac{d_k^T \nabla F_{k+1}}{\nabla F_k^T \nabla F_k}$

2- PR three – term is:

$$d_{k+1} = -\nabla F_{k+1} + \beta^{PRP} d_k - \theta_k^{(1)} \nabla F_{k+1}$$

where $\theta_k^{(1)} = \frac{d_k^T \nabla F_{k+1}}{\nabla F_k^T \nabla F_k}$

3- HS three - term is:

$$d_{k+1} = -\nabla F_{k+1} + \beta^{HS} d_k - \theta_k^{(1)} \nabla F_{k+1}$$

where $\theta_k^{(1)} = \frac{d_k^T \nabla F_{k+1}}{d_k^T y_k}$ We note that these methods always satisfy $d_k^T \nabla F_k = -\|\nabla F_k\|^2 < 0, \forall k$,
which means that $c = 1$ is a sufficient condition for a descent. In the study of convergence and the application of the CG method, the researcher often needs a exact and inexact line of research such as Wolf's strong terms. Wolf's strong terms are to find an α_k such that:

$$F(\chi_k + \alpha_k d_k) \leq F(\chi) + n1 \alpha_k \nabla F_k^T d_k \quad (5)$$

$$|d_k^T \nabla F_k(\chi_k + \alpha_k d_k)| \leq -n2\ d_k^T \nabla F_k \quad (6)$$

where $0 < n1 < n2 < 1$ is a constant according by Weijun and Li [17].

3. THE AIM AND OBJECTIVES OF THE STUDY

In this paper, we developed a new conjugate gradient algorithm to handle noisy image problems:

- Get the least possible error compared to other methods in the same field.
- Obtaining a structural similarity index measure (SSIM) value close to 1 to show the efficiency of the new developed algorithm.
- Filtering noisy images with high accuracy compared to the rest of the algorithms used in this paper.

4. DEVELOPMENT OF THE THREE-TERM FR ALGORITHM

In this part of the paper we improve the three-term fletcher and reeves (TTFR) method by finding a new value for theta using the conjugate condition of Dai and Liao where we got:

$$d_{k+1} = -\lambda \nabla F_{k+1} + \beta_k^{FR} d_k - \theta_k \nabla F_{k+1}$$

depending on the conjugacy condition of Dai and Liao $y_k^T d_{k+1} = -t s_k^T \nabla F_{k+1}$ the value of $\lambda < 0$, and $t = 1$ we find the value of θ_k^{NEW} [7],

$$y_k^T d_{k+1} = -\lambda y_k^T \nabla F_{k+1} + \beta_k^{FR} y_k^T d_k - \theta_k y_k^T \nabla F_{k+1} = -s_k^T \nabla F_{k+1}$$

$$\theta_k y_k^T \nabla F_{k+1} = -\lambda y_k^T \nabla F_{k+1} + \beta_k^{FR} y_k^T d_k + s_k^T \nabla F_{k+1}$$

$$\theta_k^{NEW} = -\lambda + \beta_k^{FR} \frac{y_k^T d_k}{y_k^T \nabla F_{k+1}} + \frac{s_k^T \nabla F_{k+1}}{y_k^T \nabla F_{k+1}} \tag{7}$$

$$d_{k+1} = -\nabla F_{k+1} + \beta_k^{FR} d_k - \theta_k^{NEW} \nabla F_{k+1} \tag{8}$$

Outline:
In the following, we will write the steps of the new algorithm in detail:
Stage1: let x_0 be a starting point where $\varepsilon > 0$, $k = 0$ and then we find $d_0 = -\nabla F_0$.
Stage2: using Wolfe conditions (5), (6) terms we find the value of the step size α_k.
Stage3: if it was $\|\nabla F_{k+1}\| < \varepsilon$ then stopped else Calculate x_{k+1} from (2).
Stage4: we calculate the direction of the search from the (7), (8), and β_k^{FR}.
Stage5: taking $k = k + 1$ and go to Stage2.

4.1. The new method's descent property

In this section, we will prove the sufficient descent property of the new algorithm by using the FR parameter and by substituting the new value for θ_k^{NEW} in (7), the sufficiency gradient property of the conjugate gradient methods is written in:

$$\nabla F_{k+1}^T d_{k+1} \le -c\|\nabla F_{k+1}\|^2 \text{ for } k \ge 0 \text{ and } c > 0 \tag{9}$$

where the property of sufficient proportions is important to prove the efficiency of the proposed algorithm.

4.1.1. Theorem

To Proof of the sufficient descent property of the proposed new algorithm We take the search direction found in (8) with FR β_k^{FR} parameter and the $\theta_k$value defined in (7) we will get (2) for all values of $k \ge 1$, and using (5), and (6).
Proof: we use mathematical induction to prove descent property:

- where $k = 0$, then $d_0 = -\nabla F_0 \rightarrow \nabla F_0^T d_0 = -\|\nabla F_0\|^2 < 0$
- suppose that the relationship is correct $\nabla F_k d_k < 0$ for all k.
- by multiplying both sides of the (8) by ∇F_{k+1}, we are able to demonstrate that the (9) holds true when k = k+1. we get:

$$\nabla F_{k+1}^T d_{k+1} = -\nabla F_{k+1}^T \nabla F_{k+1} + \beta_k^{FR} \nabla F_{k+1}^T d_k - \theta_k^{NEW} \nabla F_{k+1}^T \nabla F_{k+1}$$

$$\nabla F_{k+1}^T d_{k+1} = -\nabla F_{k+1}^T g_{k+1}(1 + \theta_k^{NEW}) + \beta_k^{FR} \nabla F_{k+1}^T d_k$$

If $\theta_k^{NEW} > 0$ then

$$\nabla F_{k+1}^T d_{k+1} < -\nabla F_{k+1}^T \nabla F_{k+1}(1+\theta_k^{NEW}) + \beta_k^{FR} \nabla F_{k+1}^T d_k$$

$$\nabla F_{k+1}^T d_{k+1} < 0$$

as a result, the new and enhanced method's descent attribute was successfully shown.

4.2. Global convergence of the new proposed method

Now we will prove that the new and known modified algorithm in (7) and (8) with FR β_k^{FR} parameter. This will be done by using the equation to guide our work. In order to investigate whether or not the newly suggested method converges, we need to start with the following assumptions.

ASSUMPTIONS (1)

The following presumptions will be made with regards to the purpose of the common domain (corresponding domain):

- $Q = \{\chi \in R^n : F(\chi) \leq F(\chi_\circ)\}$ It is a set bound in the primary blister and closed.
- The corresponding domain or range function is differentiable and continuous in certain N regions of the set q, and its derivatives are Lipschitz Continuous. In addition, the continuous behavior of the corresponding domain or range function can be seen in its derivatives. This indicates that there is a fixed value known as $D > 0$, and its definition is as follows:

$$\|\nabla F(\chi) - \nabla F(\Upsilon)\| \leq D\|\chi - \Upsilon\| \ \forall \chi, \Upsilon \in N$$

- The domain function F is a convex function that is uniformly convex, where g is a constant integer that accomplishes the variance, for instance.

$$\left(\nabla F(\chi) - \nabla F(\Upsilon)\right)^T (\chi - \Upsilon) \geq \mu\|\chi - \Upsilon\|^2 \text{, for any } \chi, \Upsilon \in Q$$

Using the assumptions (1), on the other hand, we find a positive constant B in the following form:

$$\|\chi\| \leq B, \forall \chi \in Q$$

$$\underline{\gamma} \leq \|\nabla F(\chi)\| \leq \overline{\gamma}, \forall \chi \in Q \quad (10)$$

LEMMA (1)

We use assumptions (1) and (10) is fulfilled by taking (7) and (8) and using Wolfe strong conditions to find the value of the step length α_k, assuming that our assumptions are correct if $\sum_{k>1} \frac{1}{\|d_{k+1}\|^2} = \infty$

we get $\lim_{k\to\infty}(inf\|\nabla F_k\|) = 0$

4.2.1. Theorem

The descent property of verifies the proposition of assumptions (1) and (10). The conjugate gradient technique, together with the β_k^{FR} parameter and θ_k, is provided by the (7), as if α_k is satisfied with Wolf strong conditions (WSC) (5) and (6), respectively. It's uniformly convex in the set Q plane, hence the function, the equation is $\lim_{k\to\infty} inf\|g_k\| = 0$ is satisfy.

Proof:

$$\|d_{k+1}\| = \|-\nabla F_{k+1} + \beta_k^{FR} d_k - \theta_k^{NEW} \nabla F_{k+1}\|$$

$$\|d_{k+1}\| \leq \|\nabla F_{k+1}\| + \beta_k^{FR}\|d_k\| + \theta_k^{NEW}\|\nabla F_{k+1}\|$$

$$\|d_{k+1}\| \leq \|\nabla F_{k+1}\|(1+\theta_k^{NEW}) + \frac{\|\nabla F_{k+1}\|^2}{\|\nabla F_k\|^2}\|d_k\|$$

$$\|d_{k+1}\| \leq \left((1+\theta_k^{NEW}) + \frac{\|\nabla F_{k+1}\|}{\|\nabla F_k\|^2}\|d_k\|\right)\|\nabla F_{k+1}\|$$

$$\sum_{k\geq 1}\frac{1}{\|d_{k+1}\|} \geq \left(\frac{1}{\left((1+\theta_k^{NEW})+\frac{\|\nabla F_{k+1}\|}{\|\nabla F_k\|^2}\|d_k\|\right)^2}\right)\frac{1}{\gamma^2}\sum 1 = \infty$$

using the lemma above $\lim_{k\to\infty}\|\nabla F_k\| = 0$.

5. EXPERIMENTAL RESULTS AND DISCUSSION

The image is typically deformed as a result of noise that occurred either during the process of shooting or transmitting the image. Effective solutions for reducing noise are necessary in order to deliver results that are more dependable across a wide range of image-related applications. In this part of the article, we will cover the method of restoring the original picture from inside an image that has been corrupted by spurious impulse noise. The image in question will have been damaged as a result of spurious impulse noise. Impulse noise, which happens when just a tiny section of a pixel is impacted by the noise, is one of the most common types of noise. It is also one of the noisiest types of noise. As a result of the noise, any information regarding the actual values of the affected pixels is completely obliterated. These difficulties are considered to be among the most challenging in terms of areas for improvement because of their smooth characteristics. The reason for this is that it is tough to find a solution to these problems. It is particularly challenging to find solutions to non-seamless optimization problems due to the fact that many of the existing gradient methods cannot be employed directly. As a result, it is required to build original algorithms or approaches in order to tackle these difficulties. As a result, Yuan *et al*. [18]–[21]. Describe a number of non-linear CG algorithms that can be used for non-smooth optimization problems. These methods get a big number of great results, and more results can be found in the picture problems [22]–[26], and ... Let:

$$Y = \left\{(k,m) \in \Lambda \mid \bar{\xi}_{k,m} \neq \xi_{k,m}, \xi_{k,m} = \delta_{min} \text{ or } \delta_{max}\right\}$$

be the noise candidate's index set, and x be the genuine picture with $X - by - Y$ pixels,$x_{k,m}$ be the grayscale value of x at the specified pixel position (k,m) with $(k,m) \in \Lambda = \{1,2,\cdots,X\} \times \{1,2,3,\cdots,Y\}$, and $\phi_{k,m} = \{(k,m-1),(k,m+1),(k-1,m),(k+1,m)\}$ be in the vicinity of (k,m), where ξ is the observed noisy picture of x distorted by salt-and-pepper noise, $\bar{\xi}$ Is defined as the picture created by applying an adaptive median filter (MED) to the noisy image y, where δ_{min} indicates the noise pixel's minimal value and δ_{max} signifies the noise pixel's highest value. The following issues with picture restoration are examined in detail in this section. They are defined by $\min_v \tau(v)$:
where,

$$\tau(v) = \sum_{(k,m)\in Y}\left\{\sum_{(x,y)\in\phi_{k,m}\setminus N}\chi(v_{i,j} - \xi_{x,y}) + \frac{1}{2}\sum_{(x,y)\in\phi_{k,m}\cap Y}\chi(v_{k,m} - v_{x,y})\right\}$$

it is not hard to comprehend that the regularity of τ is dependent only on χ and that Huber's function is defined as a potential function to keep the edge χ with the same value with:

$$\chi = \begin{cases} e^2/u, & \text{if } |e| \leq u \\ |e| - 2u, & \text{if } |e| > u \end{cases}$$

where $u > 0$. There are numerous good findings concerning τ can be discovered (see [24]–[26]). In this program, all instructions are carried out on a personal computer outfitted with a MATLAB R2021b CPU, an Intel Core i5 processor running at 2.4 GHz, 8.00 gigabytes of random access memory, and the Windows 10 operating system. The parameters are chosen as $\mathrm{a} = 0.5, \mathrm{b} = 0.1, \mathrm{c} = 0.9,$ and $\mathrm{d} = 0.3$. The stop condition is: $\frac{|\tau(v_{k+1}) - \tau(v_k)|}{|\tau(v_k)|} < 10^{-4}$. The proposed filter (HM) is built to process (salt and pepper) noise in digital images. When exposed to noise at different rates (30,50,70,90). The noisy images were entered into filters (FR, TIFR and finally the median filter from the MATLAB system) and processed to get rid of this noise. And to measure improvements and compare the efficiency of the proposed candidate's work. The image (Lena.png) was chosen, known in the digital image processing literature and research. Noises were added to it at different rates (30,50,70,90). The image has been processed with the four noise ratios and for the mentioned filters as well as the suggested filter. The results appeared as shown in Table 1. Use the SSIM, which is used to measure image quality, and the higher its value, the better the result. The original image was compared with all the noisy images and each according to the percentage of noise as well as the images processed with the four filters. It is clear from the table that when the noise ratio is 30, the resulting image after processing from noise with the proposed filter, and comparing it with the image, gave it a match with the original image, the ratio (0.9600). While treatment with FR, TIFR and median filters gave values of 0.8853, 0.9168, and 0.6993, respectively. The same goes for the other noise ratios. As the image exposed to noise 50 was the result of purification and then compared with the original image (0.9392), the image exposed to noise 70 was the result of comparison with the original image (0.8946), the image exposed to noise 90 was the result of (0.7877). The proposed filter with the other filters being compared.

Table 1. Filters processing results by noise ratio

			Lena			
Noise	original Image	noise Image	HM	FR	TTFR	median filtering
30	1.0000	0.0526	0.9600	0.8853	0.9168	0.6993
50	1.0000	0.0262	0.9392	0.5419	0.8326	0.2321
70	1.0000	0.0138	0.8946	0.0885	0.7397	0.0527
90	1.0000	0.0061	0.7877	0.4311	0.6159	0.0115
			Barbara			
Noise	original Image	noise Image	HM	FR	TTFR	median filtering
30	1.0000	0.0959	0.9251	0.8305	0.9118	0.6213
50	1.0000	0.0465	0.8793	0.7180	0.7950	0.2351
70	1.0000	0.0224	0.7997	0.5255	0.6880	0.0587
90	1.0000	0.0081	0.6565	0.4141	0.5678	0.0129

Figure 1 displays the image of the proposed program of work, as the first column shows the original image. The second column shows the image after exposure to noise. The third column shows the images after processing them with the proposed filter, the fourth column shows the filter processing (FR), the fifth column shows the filter processing the TTFR, the sixth column shows the processing by the median filter (MED) of MATLAB. The third column shows the quality of the displayed image after processing it with the proposed filter and its conformity with the original image by discussing the metrics mentioned in the above table. To confirm the results and the accuracy of the proposed filter, another image of the program was introduced, and the results appeared as shown in Figure 2.

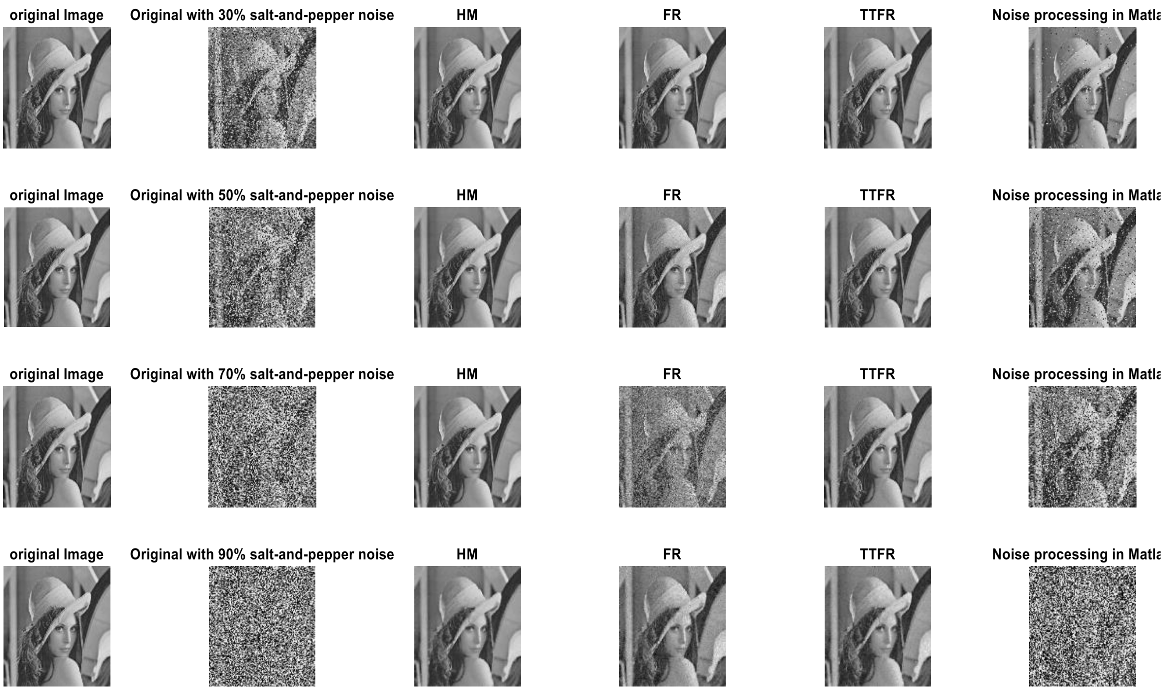

Figure 1. Comparison of image noise removal (Lena.png) using the proposed filter and other filters

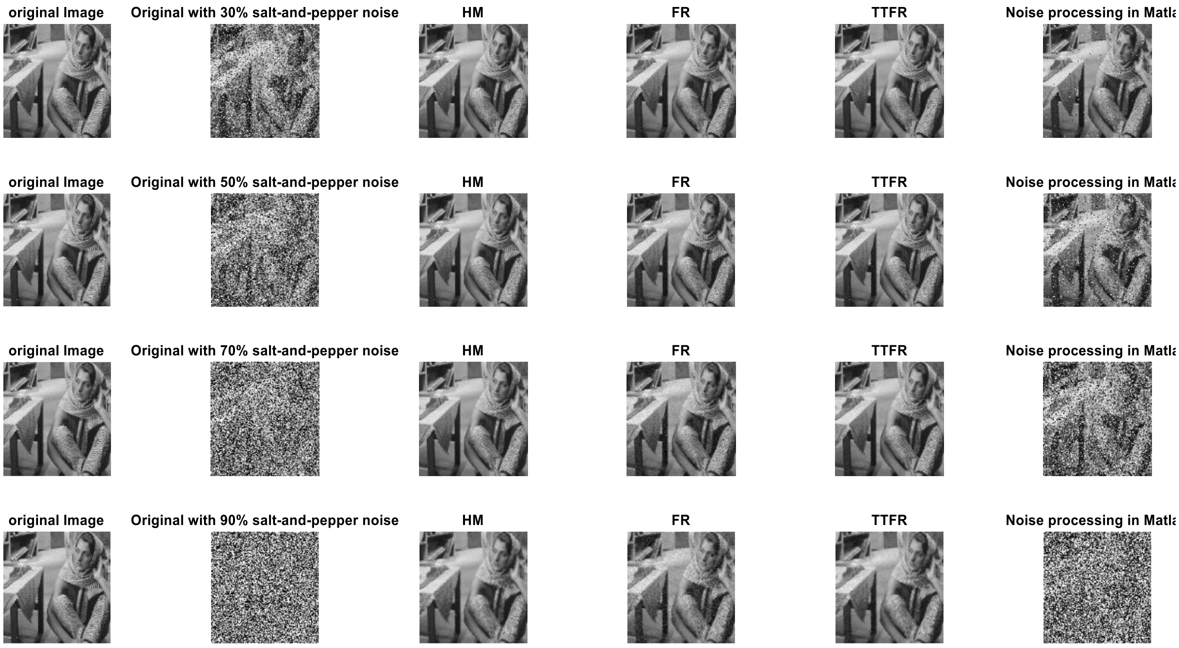

Figure 2. Comparison of image noise removal (Barbara.png) using the proposed filter and other filters

6. CONCLUSION

The proposed paper, we discussed different filtering methods for reducing salt and pepper noise in gray image. Additionally, we presented and compared outcomes for these filtering techniques. The results obtained using median filters, and filters based on methods FR, and TTFR. Filter based on the proposed (HM) guarantees better performance on other filters discussed for (noise free) and image quality as well. The main advantages of this filter are the successfully removing capability of the damaged gray pixels. However, this technique increases the computational complexity. The future scope will be focused on other filters based on mathematical methods to overwhelm other types of noises. As well as to extend the algorithm for color images. A very small error rate was obtained compared to other algorithms in this paper. The value of SSIM was close to one, as detailed in numerical results. The best resolution for filtering noisy images was obtained compared to the FR, and TTFR algorithms.

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