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# Fuzzy Optimization of Construction Engineering Project Schedule Based on Critical Chain Management

**Jianbing Liu*[1,2], Hong Ren[2], Junshu Du[3]**
[1]School of Economic and Management of Jiangxi University of Science and Technology, No.86 Hongqi Avenue, Zhanggong District, Ganzhou Jiangxi 341000, China, 0086-797-8312413
[2]School of Construction Management and Real Estate of Chongqing University, Shapingba Chongqing 400045, China
[3]School of Business in East China university of Science and Technology, Shanghai 200237, China
*Corresponding author, e-mail: Liujb71@sina.cn

***Abstract***

*Critical chain management was the very important theory innovation in the development of construction project schedule management field in recent years. Compared with the traditional methods of construction project schedule management, the methods of critical chain management considered resource constraints into construction project schedule management, the construction project cycle was shortened and the efficiency was enhanced by using the dynamic thinking and circulation pattern to manage whole project. In view of the progress of critical chain management of the article, fuzzy theory was used to calculate and optimize the buffer zone sizes with the determination of the buffer size of the existing resources in certain critical chain management so as to shorten the cycle.*

***Keywords:*** *optimization, construction project schedule, critical chain management, the buffer zone, fuzzy theory*



## 1. Introduction

Critical chain management was brought out by Israel physicist Goldratt professor in his book which was the critical chain in 1997 in first, which was caused wide echo in academia when was once put forward [1]. The main ideas of critical chain management takes resource constraints into consideration in the project based on the traditional progress management, in this condition, then each working procedure was planned and scheduled in order to achieve efficient allocation of resources and save time [2]. Taking the resource constraints into consideration by professor Goldratt, the most main of the premise is the method could overcome the delay, which in the mental and behavioral factors which lead to the longer of duration and can't complete the task effectively in the traditional project progression, such as Student syndrome, Parkinson’s Law and multitasking influences [3-5].

The traditional methods of project scheduling management such as critical path method and Program Evaluation and Review Technique are not covered those effects, and they don't take into account the resources constraint relation [6, 7]. Critical path method and Program Evaluation and Review Technique view the longest of the path as the project cycle in whole project, but the critical chain method view the longest time line project as the whole construction period which must have the logical relationship between processes and resources restraint into consideration [8, 9]. And in order to complete the mission and guarantee project period, professor Goldratt had three buffers in critical chain management, which is project buffer (PB as briefly), feeding buffer (FB as briefly) and resource buffer (RB as briefly) in respectively. The three buffers were at different positions of the critical chain. The project buffer in critical chain buffer was set in the end which can absorb the critical chain of delay, thus also can guarantee the total duration of project. The feeding buffers was set between the critical chains and non-critical chains which absorb non-critical chain process of delay and ensure the downstream of the critical chain from the influence of the front non-critical chain delays, and it is a kind of time buffer. Resource buffers were set between the critical processes which use different resources and it did not take time by itself, just to provide a warning form [10, 11].

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## 2. Calculation of the Traditional Buffer Size

Calculation of traditional buffer basically has two kinds of methods, which were cut and paste method and root variance method. Cut and paste method referred to each procedure accumulation safety time of the critical chain, and the half of it was cut as project buffer to put at the end of the chain, so that the project period of the project is shortened from the calculation form.

### 2.1. Cut and Paste Method

If a project has $n$ procedures, the calculation formula of project buffer time of these processes is as follows:

$$PB = \frac{1}{2}(\sigma_1 + \sigma_2 + ... + \sigma_n) = \frac{1}{2}\sum_{i=1}^{n} \sigma_i \quad (1)$$

In which $\sigma_i$ was the remaining time of each procedure.

The feeding buffer time is equal to the half of accumulation of the total safety of non-critical chain process, its expression is as follows:

$$FB = \frac{1}{2}(\delta_1 + \delta_2 + ... + \delta_n) = \frac{1}{2}\sum_{i=1}^{n} \delta_i \quad (2)$$

In which, $\delta_i$ was the safety of time of each non-critical process.

The calculation of cut and paste method is easy operation with poor of theory basis. On the one hand, it has each critical chain process time half without considering the influence of each procedure on the critical chain project itself in different sizes, and some influences are small, the approach of one size fits all lacks of scientific. On the other hand, the size of the buffer of critical chain length was influenced directly; the critical chain length has the linear relationship with amount of processes, so there are different lengths of the calculation of the size of the buffer.

### 2.2. Method of Root Variance

The method of root variance put the risk which may be met in the project into account. Through the analysis of variance of risk, the method solves influence of the total duration by uncertainty of the nonlinear time in different processes. It is assumed that the process is independent of each other, and the variance of processes as a safe time, so the total variance is the sum of each procedure of the variance. Suppose there are $n$ procedures and each procedure $i$ will be given two group estimated time: a group of is $t_1$, which is put in the probable estimate of time, the other group of is not put in estimate of time $t_2$, then their deviation time :$\sigma_i = t_1 - t_2$, so the calculation formula of buffer size is as follows:

$$PB = 2 \times \left\{ \sum_{i=1}^{n} \left( \frac{\sigma_i}{2} \right)^2 \right\}^{\frac{1}{2}} = \sqrt{\sum_{i=1}^{n} \sigma_i^2} \quad (3)$$

Compared with cut and paste method, the method of root variance method had a higher rationality. Because it took the possible existence of some accident process and amount of mobile time into consideration, and calculation of the buffer was certain rational. But the premise of calculating of the root variance was that the each working procedure was independent and not related to each other, few of the practical work of each procedure had not correlation between each other, so the results of the calculation formula (3) may be smaller than the practical situation, which lead to the inaccuracy of calculation of the buffer size.

### 3. Calculation of the Buffer Size based on Fuzzy Theory

A lot of times a project progress has uncertainty with two kinds of forms usually, one is uncertainty whether events must have happened to, the other that the thing itself in a state of constant change, in a certain point may be in A state, but not long after into B state, is called fuzzy [11]. A lot of practical projects were difficult to imitate due to the characteristics of itself and can only rely on past experience to estimate, so is needed to use the fuzzy theory. The characteristics of fuzzy theory are able to put fuzziness into mathematical method to quantify, especially for some more complex project period, using the fuzzy theory can be a very good solution.

#### 3.1. Relative Knowledge of Fuzzy Theory

The fuzzy theory was first put forward by the famous American professor L.A. Zadeh, which was used to describe the information method mainly by setting up a fuzzy membership function. Supposes X is on the R fuzzy set, its membership functions $\mu(x)$, $\mu(x) \in [0, 1]$. If X meets that is for each $\alpha \in [0, 1]$, each X has a convex with α interception of the collection, $\mu(x)$ is continual function in the upper part. If $x_0 \in X$, it is $\mu(x_0)=1$, this fuzzy number is X, its shape is shown as follows in Figure 1 which is an upward raised function.

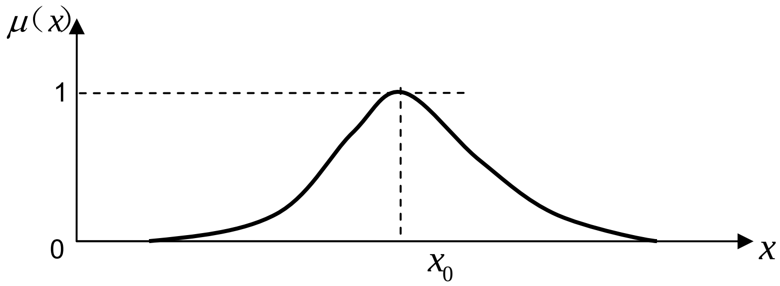


Figure 1. Fuzzy Number Expression General Form

The fuzzy number mainly has two kinds in the fuzzy theory, one is the triangle fuzzy number, the other is the trapezoidal fuzzy number, their corresponding graphs like Figure 2(a) and (b).

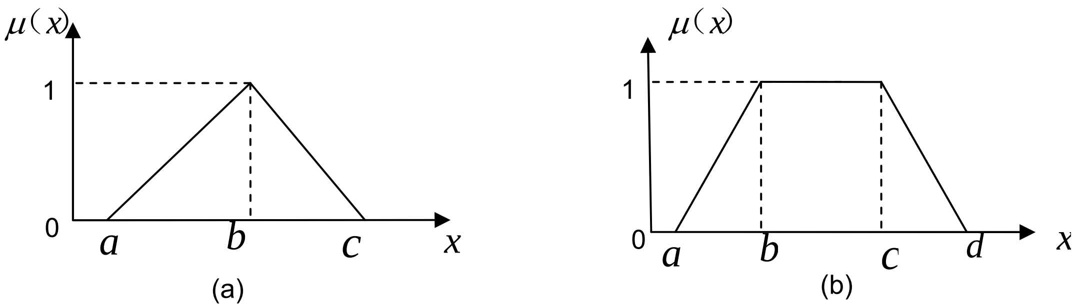


Figure 2. Triangle and Trapezoidal Fuzzy Numbers

Trapezoidal fuzzy number is generally recorded as $N=(a, b, c, d)$, $a$ and $b$ and $c$ and $d$ is respectively the trapezoidal fuzzy number down boundary value, the left value, the right value, the upper boundary value, $a$ and $d$ for the duration of the procedure the minimum value and maximum value, $b$ and $c$ for the duration of the most likely range, and $0 < a \leq b \leq c \leq d$ . Trapezoid fuzzy number point can be formed into triangle fuzzy numbers with a simple Merge. When the upper line $b=c$ in trapezoidal fuzzy number, may be regarded $b$

and *c* points as coincident in the graphic. Then the trapezoidal fuzzy number membership function formula is as follows:

$$\mu_N(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x < b \\ \frac{d-x}{d-c}, & c \le x \le d \text{ ers} \\ 1, & b \le x < c \\ 0, & others \end{cases} \tag{4}$$

The trapezoidal fuzzy number consistent index indicated with the trapezoid interception area with the original area ratio, that is as:

$$Z(t \le X_\alpha) = \begin{cases} 0, & X_\alpha = a \\ 1, & X_\alpha = d \\ \frac{(X_\alpha - a)^2/(b-a)}{c+d-a-b}, & a < X_\alpha \le b \\ \frac{2X_\alpha - a - b}{c+d-a-b}, & b < X_\alpha \le c \\ \frac{(c+d-a-b)-(X_\alpha - a)^2/(c-a)}{c+d-a-b}, & c < X_\alpha \le d \end{cases} \tag{5}$$

**3.2. Calculation of the Buffer Size based on Fuzzy Theory**

Regarding estimation of the working procedure time, it is expressed with fuzzy language. According to the fuzzy theory, qualitative could transform into the quantitative value, we may indicate each working procedure fuzzy time with the trapezoidal fuzzy number for a project value, then calculate this working procedure time. Supposes the working procedure time is $t_e$, then the formula is as follows:

$$t_e = b + \beta \times (c - b), \quad 0 < \beta < 1 \tag{6}$$

In which, $\beta$ expressed the possible coefficient estimated at the working procedure time for a project risk by the project management personnel. If risk circumvented, then $0<\beta<0.5$. If risk preferred, then $0.5<\beta<1$. If risk neutral, then $\beta \to 0.5$.

Then according to the calculated $t_e$ which the most likely time and logic or constraint interaction between the various processes, we could draw chart of the project so as to calculate the buffer time size. Buffer zone calculations are related to security time, and the set of the security time is related to project management personnel personal experience and ability level.

Fuzzy theory can be described, if some process estimated time $t_e$, is not greater than $X_a$, that is less than or equal to $X_a$, then we say that this possibility is $Z(\{t_e \le X_a\})$. The figure $Z(\bullet)$ was decides by the project management personnel who weigh out the risk. For example, the majority people are of the risk circumventions, then we may make $Z(\{t_e \le X_a\})=0.9$. Then according to the fuzzy mathematics operation, calculate $X_a$ back to calculate security time *S*, that is as follows:

$$s = X_\alpha - t_e \tag{7}$$

In which, $X_a$ is working procedure high really degrees the time for a project. $t_e$ is the most possible working procedure for a project.

With calculated safety time *S*, three kinds of the buffer size can be calculated. Regarding project buffer *PB*, its size was equal to some coefficient $\omega$ multiplied by the sum of accumulation security time of various critical working procedure $\sum_{i=1}^{n} s_i$, the formula is as:

$$PB = \omega \times \sum_{i=1}^{n} S_i \quad (8)$$

In which, *n* was the amount of critical working procedure in critical chain. $S_i$ was the safety time of the critical working procedure.

The size of coefficient $\omega$ was determined based primarily on the characteristics of the project itself. According to gather principle, if there are many more procedures in project, the cumulative delay time may be longer when the project is progressing at this point, the size of coefficient $\omega$ can be set large slightly. If the project is big in itself, and relations inner are more complex, in order not to pose inordinate amount of security time, then it can be set small slightly.

Feeding buffer *FB*, its size was a little like the project buffer's, just the object of multiplied by is different. It is equal to use coefficient $\omega$ multiplied by the accumulative total safety time $\sum_{i=1}^{k} s_i$ of non-critical processes which are before the critical processes, the calculation formula is as:

$$FB = \omega \times \sum_{j=1}^{k} s_j \quad (9)$$

In which, $k$ is the accumulative amount of non-critical before this the critical Chain. $S_j$ is the safety time of non-critical procedures before critical procedures in this critical chain.

The arrangement of critical chain above on is not only often used like as the principles of physics circuit current. The discussion before is series form, but it also has a "parallel" form. In this case, security time can be get from each "line", and then take the maxes of them as the security time.

## 4. Case Study

If a small project was finally divided to nine working procedures form *A ~ I* through the project work breakdown structure (WBS), as is shown in Table 1. The table has painted the project resources needed for each working procedure, fuzzy language, corresponding trapezoidal fuzzy number, and logical relationship, using the fuzzy mathematics theory to solve the data.

Table 1. Decomposition of Projection $A$~$I$ each Working Procedure Relation Table

| Process name | Fuzzy language (day) | Trapezoidal fuzzy number (day) | Previous process | Resource needed |
|---|---|---|---|---|
| A | less than 10 | (5,6,8,9) | -- | R1 |
| B | 15 more or less | (9,12,15,17) | A | R2 |
| C | 20 more or less | (15,17,19,23) | A | R3 |
| D | 30 more or less | (22,25,30,35) | A | R4 |
| E | less than 35 | (28,30,32,34) | B | R5 |
| F | more than 20 | (22,25,30,35) | C | R6 |
| G | 25 more or less | (22,25,27,30) | D | R6 |
| H | less than 40 | (25,30,37,39) | F, D | R7 |
| I | 18 more or less | (10,15,18,22) | E, H | R7 |

At first we can draw the project's the corresponding progress network diagram from Table 1, as shown in Figure 1 below.

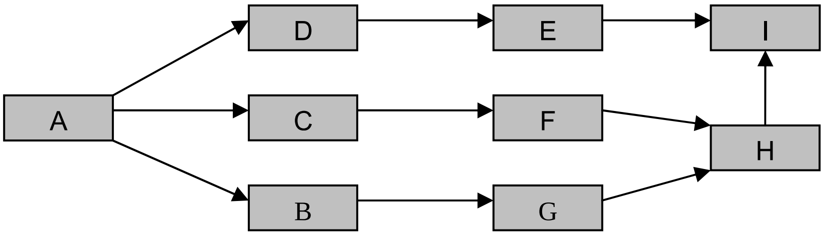


Figure 3. A Project Progress Network Diagram

Suppose that project management is as neutral about risk preferences, namely$\beta \to 0.5$, so each procedure time can be calculated respectively, for example, the first procedure' s most likely time was $t_A$=6+0.5*(8-6)=7(days). By analogy, the others corresponding periods of each working procedure were as $t_B$=13.5(*days*), $t_C$=18(*days*), $t_D$=27.5(*days*), $t_E$=31(*days*), $t_F$=27.5(*days*), $t_G$=26(*days*), $t_H$=33.5(*days*), $t_I$=16.5(*days*).

According to the calculated periods, in the absence of resource constraints, this project critical path was A→B→G→H→I.

We can figure out every process of high real degrees period $X_a$ based on the trapezoidal fuzzy number and consistent index.

$X_A$=8.23(*days*), $X_B$=15.5(*days*), $X_C$=21(*days*), $X_D$=32(*days*), $X_E$=32.8(*days*), $X_F$=32(*days*), $X_G$=28.3(*days*), $X_H$=37(*days*), $X_I$=19.5(*days*).

Then we can calculate each procedure's safety time from above $S_i(S_A \sim S_I)$ is as $S_A$=1.2

3(*days*), $S_B$=2(*days*), $S_C$=3(*days*), $S_D$=4.5(*days*), $S_E$=1.8(*days*), $S_F$=4.5(*days*), $S_G$=2.3(*days*), $S_H$=3.5(*days*), $S_I$=3(*days*) ) .

According to the safety time $S_i$ and critical chain by composed of critical process we can calculate buffer *PB* and the feeding buffer project $FB$ in respectively. In which, the critical processes A and B and G and F and H and I formed the project buffer *PB*. Because of two non-critical chains, so also has two feeding buffers *FB:FB*$_1$ relevant with process C, *FB*$_2$ relevant with processes D and E. Coefficient $\omega$ was supposed to be taken the middle value 0.5,

$$PB = \omega \times \sum_{i=1}^{n} s_i$$

=0.5*(1.23+2+2.3+4.5+3.5+3)=8.25(*days*).

$$FB = \omega \times \sum_{j=1}^{k} s_j$$

,

*FB1*=0.5*3=1.5(*days*), *FB2*=0.5*0.5(4.5+1.8)=3.2(*days*) ..

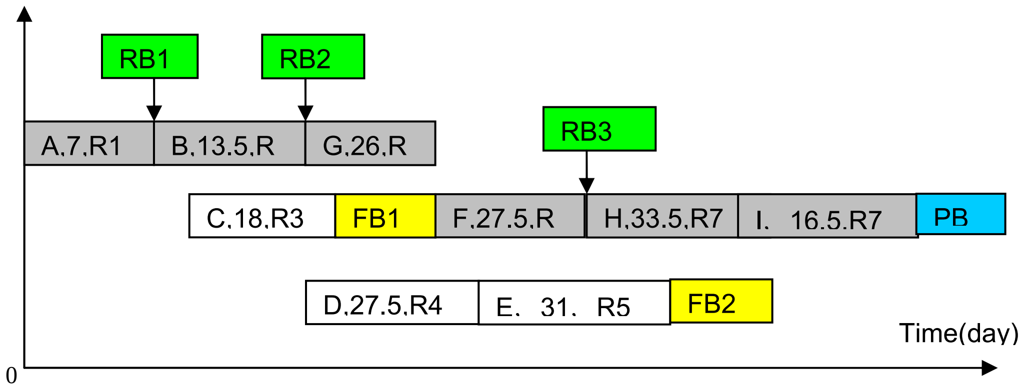


Figure 4. Project Progress Chart under Resource Constrained

The resources buffer *RB* of critical chain was general set in front of the critical process. The front setting method is purposed to convey warning to project management personnel before process has not started and inform resources prepared, let people know the current ready state. In this example, project resources buffer *RB* was set before critical process B and G and H in respectively. The total duration of this project is 145 days. Each buffer's specific setting as shown in Figure 4, the gray boxes are critical processes.

## 4. Conclusion

As can we see from the example analysis, it is a try of the calculation size of buffer based on the fuzzy mathematics theory perspective. The result shows that this method is feasible compared with general calculation method, which save time and shorten the project period, and avoid conflict when sharing the same resource in process at the same time.

Compared with the traditional critical path method and program evaluation and review technology, critical chain theory is a theory progress and innovation in project scheduling management method. The method takes the constraints of time and resources into consideration, which is more practical meaning. But the concrete implementation of critical chain method exists high complexity, especially when the buffer zone was set, the previous critical chain might change, as well as in complex critical chain, when multiple resources are used by multiple procedures at the same time, how to determine the critical chain is also need to study.

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