TELKOMNIKA, Vol. 11, No. 12, December 2013, pp. 7213~7221
e-ISSN: 2087-278X

Adaptive Iterated Square-Root Cubature Kalman Filter and Its Application to SLAM of a Mobile Robot

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Abstract

For the mobile robot Simultaneous Localization and Mapping (SLAM), a new algorithm is proposed, and named Adaptive Iterated Square-Root Cubature Kalman Filter based SLAM algorithm (AISRCKF-SLAM). The main contribution of the algorithm is that the numerical integration method based on cubature rule is directly used to calculate the SLAM posterior probability density. To improve innovation covariance and cross-covariance, the latest measurements are iteratively used in the measurement updating. The algorithm can reduce linearization error and improve the accuracy of the SLAM algorithm. The algorithm also used adaptive iterating estimation restricted by the iterative sentencing guideline to adjust the proportion of the observation and dynamic model, to make the estimated square root of the error covariance more accurate and reasonable. In experiments, the proposed algorithm is compared with Extended Kalman Filter based SLAM algorithm (EKF-SLAM), Unscented Kalman Filter based SLAM algorithm (UKF-SLAM) and Square-Root Cubature Kalman Filter based SLAM algorithm (SRCKF-SLAM. The results indicate that the proposed algorithm having with the higher accuracy of the state estimation is obtained to compare with the EKF-SLAM algorithm, the UKF-SLAM algorithm and the SRCKF-SLAM algorithm.

Keywords: *mobile robot, adaptive iterate, square-root cubature kalman filter, SLAM*

1. Introduction

The SLAM is that the mobile robot builds map in the unknown environment and localization by using built map under the condition of itself position uncertainty [1]. During the process of the SLAM, it has a lot of uncertain factors. Firstly, the robot is of uncertainty because of the noise data of the senor. Secondly, the detective environment of the robot is also unpredicted. These uncertain factors make the SLAM more difficult. So, recently, the probability theory is used in the SLAM algorithm by the increasing researchers. For this problem, the EKF algorithm is used to estimate it in the early days [2]. Then the method is extensively used, and the EKF-SLAM algorithm is proposed in different environments. But the EKF algorithm has two main shortcomings. Foremost, the bottleneck of the EKF algorithm is that it is of bigger computational complexity. So the mapping is difficult to satisfy the requirement of real time in the big environment [3]. Then, the EKF algorithm will go against local linear hypothesis when being the strongly nonlinear [4]. The EKF algorithm may be divergented when neglecting the bigger error of the higher order term [5].

To avoid these limitations of the mentioned factors above, in recent years, many researchers propose some new algorithms for the SLAM. Juan Andrade-Cetto etc propose the SLAM algorithm based on Unscented Kalman Filter [6]. The nonlinear system model is directly used in the UKF algorithm. It can avoid errors which are created becsuse of higher order term truncation. However, it can improve the systerm accuracy. The square-root Unscented Kalman Filter(SR-UKF) based on cholesky decompose is proposed to apply to the SLAM algorithm in the paper [7]. The algorithm is iteratively calculated by using the square root of the covariance to substitute the covariance. It can ensure the nonnegative definiteness of the covariance matrix and the numerical stability of the filter. But, with the observation and the update proceed, the sigma point which used to the predicted value for center and used to the predicted variance for covariance generate will gradually deviate the estimated value of the real state when the SR-UKF algorithm is applied to the SLAM. Montermerlo etc proposed the

Received May 24, 2013; Revised July 28, 2013; Accepted August 20, 2013

FastSLAM algorithm based on particle filter has the same disadvantages as the EKF algorithm [8]. The FastSLAM algorithm estimates the trajectory of the robot by using particle filter and the landmark position by using the EKF algorithm. It can reduce computational complexity. But the FastSLAM algorithm may create the particle subset degradation, and particle subset diversification is decreased. So, it reduces the accuracy of the SLAM algorithm [9].

Recently, a new SLAM algorithm based on square-root cubature kalman filter (SRCKF) is proposed in the paper [10]. The SRCKF-SLAM algorithm solves the integral problem of the Recursive Bayesian Filters by using the cubature point subset of the equal weighting. It can obtain the better linear approximation property, the numerical value precision and the stability of the filter. But the pose estimation of the mobile robot is a problem of the highly nonlinear state estimation. It has bigger initial error, and the measurement equation of the SRCKF-SLAM algorithm is of highly nonlinear properties. So, those reduce precision of the SRCKF-SLAM algorithm state estimation. To solve the problems above, a new AISRCKF-SLAM algorithm will be proposed in this paper. The algorithm absorbs advantages of the SRCKF -SLAM algorithm and it makes full use of the latest measurement information. It can effectively reduce error of state estimation. The algorithm also used adaptive iterating estimation restricted by the iterative sentencing guideline to adjust the proportion of the observation and dynamic model, to make the estimated square root of the error covariance more accurate and reasonable.

2. SLAM Algorithm based on Adaptive Iterated Square-root Cubature Kalman Filter

2.1. Selection of the Iteration Strategy

In the paper [12], the proposed Gauss-Newtow iteration strategy has global convergence properties. But it can not ensure the increase of the quasi-likelihood and the judgment threshold of the proposed Gauss-Newtow iteration impact seriously also the performance of the algorithm. In the paper [12], the decision threshold as following is adopted.

$$x_k^{i+1} - x_k^{i} \leq \lambda \tag{1}$$

Here, λ is the predetermined threshold. For improving the performance of the algorithm, the decision threshold as following is adopted in the paper.

$$[h(\hat{x}_k^{(i+1)})-h(\hat{x}_k^{(i)})]^T Q_k^{-1} [h(\hat{x}_k^{(i+1)})-h(\hat{x}_k^{(i)})]+[\hat{x}_k^{(i+1)}-\hat{x}_k^{(i)}]^T[S^{T(i)}_{zz,k} S^{(i)}_{zz,k}]^{-1}[\hat{x}_k^{(i+1)}-\hat{x}_k^{(i)}]>0 \tag{2}$$

2.2. Selection of the Adaptive Factor

By adaptive factors, robust adaptive filter adjusts the state of information for the filtering estimation function, making the state covariance parameters predicted value more reasonable, improving the filtering accuracy obviously [13]. So, the idea of robust adaptive filter is applied to SRCKF in this paper. The square root of covariance matrix of state parameter predicted value is adjusted by using adaptive factor, and then improving the filtering stability and estimated accuracy. Predicted residual is used as discriminant statistics in this paper. ζ_k is defined as:

$$\zeta_k = (z_k - \overline{z_k}) \tag{3}$$

$$\Delta \zeta = \mathrm{sqrt}\left(\frac{\zeta_k^T \zeta_k}{tr[(S_{zz,k})^T S_{zz,k}]}\right) \tag{4}$$

The selected rules of adaptive factor are as the following [14].

$$\Im_k = \begin{cases} 1 & \Delta\zeta_k \leq q \\ \dfrac{q}{\Delta\zeta_k} & \Delta\zeta_k > q \end{cases} \tag{5}$$

Where q is experience value constant. It is usually 1.5—2.5.

2.3. Adaptive Iterated Square-Root Cubature Kalman Filter

(a) The standard SRCKF algorithm is used to initialize, computing cubature point and Measurement updating. Mu Jin and Cai Yuanli propose the square root cubature Kalman filter [11].

2.2.1. Time Adapting

a. The cubature point can be obtained by:

$$X_{j,\,k-1/k-1}=S_{k-1/k-1}\,\xi_j+\overline{x_{k-1/k-1}} \tag{6}$$

b. The transmission cubature point can be obtained by:

$$X^{*}_{j,\,k-1/k-1}=f(X_{j,k-1/k-1}) \tag{7}$$

c. The state prediction and the variance prediction can be given by Equation (8) and (9), respectively.

$$\overline{x_{k/k-1}}=\frac{1}{m}\sum_{i=1}^{m} w_i X^{*}_{j,k/k-1} \tag{8}$$

$$P_{k-1}=\overline{S_{k/k-1}}\;\overline{S^{T}_{k/k-1}} \tag{9}$$

$$\overline{S_k}=Tf\left([X^{*}_{k/k-1}\;\sqrt{Q_{k-1}}\,]\right) \tag{10}$$

Where $Q_{k-1}=S_{Q,\,k-1}S^{T}_{Q,\,k-1}$, and Q_{k-1} is system noise the k-1th time. Tf( ) is that the square matrices of the matrices is obtained by using diagonalization of the matrices. $\overline{S_k}$ is the Cholesky factor of the variance prediction. $X_{k/k-1}^{*}$ is defined as:

$$X_k^{*}=\frac{1}{\sqrt{n}}[X^{*}_{1,k/k-1}-\overline{x_{k/k-1}}\,,X^{*}_{2,k/k-1}-\overline{x_{k/k-1}}\,,\ldots\ldots X^{*}_{m,\,k/k-1}-\overline{x_{k/k-1}}\,] \tag{11}$$

2.2.2. Measurement Updating

a. The cubature point can be obtained by:

$$X_{j,\,k、}=\overline{S_{k/k-1}}\;\xi_j+\overline{x_{k/k-1}} \tag{12}$$

b. The transmission cubature point can be obtained by:

$$Z_{j,\,k}=h(X_{j,k/k-1}) \tag{13}$$

c. The factor and the covariance of the Cholesky division of the measurement prediction and the new information variance can are given by Equation (14), (15) and (17), respectively.

$$\overline{z_k}=\frac{1}{m}\sum_{i=1}^{m} w_i Z_{j,k} \tag{14}$$

$$\overline{S_{zz,k}}=Tf\left([\aleph_k\;\;\sqrt{R_k}\,]\right) \tag{15}$$

$$R_k=S_{R,k}S_{R,k}^{T} \quad (16)$$

Here, R_k is the observation noise.

$$P_{xz/x}=\Im_{k/k-1}\aleph_{k/k-1}^{T} \quad (17)$$

Here, the matrix $\aleph_k$ and $\Im_k$ are given by Equation (18) and (19), respectively.

$$\aleph_k=\frac{1}{\sqrt{n}}[Z_{1,k/k-1}-\overline{z_{k/k-1}},Z_{2,k/k-1}-\overline{z_{k/k-1}},\ldots\ldots Z_{m,k/k-1}-\overline{z_{k/k-1}}] \quad (18)$$

$$\Im_k=\frac{1}{\sqrt{n}}[X_{1,k/k-1}-\overline{x_{k/k-1}},X_{2,k/k-1}-\overline{x_{k/k-1}},\ldots\ldots X_{m,k/k-1}-\overline{x_{k/k-1}}] \quad (19)$$

(b) The Equation (3)-(5) are use to select the adaptive factor.

(c) The estimation value of the Cholesky division factor of the gain, the kth time state and covariance can be obtained by the following equations.

$$S_{zz,k}=[S_{zz,k}-\mathrm{chol}(R_k)]/\sqrt{\Im_k}+\mathrm{chol}(R_k) \quad (20)$$

$$W_k=W_k/\Im_k \quad (21)$$

$$\hat{x}=\overline{x_k}+W_k(z_k-\overline{z_k}) \quad (22)$$

$$S_k=\mathrm{Tria}([\chi_k-W_k\gamma_k W_kS_{R,k}]) \quad (23)$$

(d) According to the state estimation vector $\hat{x}$ and covariance S_k of the kth iteration, produce the cubature point and complete the calculation of the new cubature point and the measurement update. The concrete method is the same standard SRCKF algorithm.

(e) The procedure (b), (c) are used again to select the adaptive factors and to renew the Corresponding equation.

(f) The iteration is terminated when satisfy the inequality (2), or it will return the procedure (c).

2.4. Design and Realization of AISRCKF based SLAM Algorithm for Mobile Mobots

(a) State prediction

According to Equation (6)~(10), estimate the square root factor of the state prediction and the prediction error covariance of the robot at kth time.

(b) Observation

The observation data is obtained by the observation model of the robot. That is, it obtains the landmark feature of the robot detected in the detecting range of the sensors, and computes the range r_i of each detected landmark feature to the robot and angle θ_i of each detected landmark feature relative to the forward direction of the robot.

(c) Data association

According to the observation data of the sensor, take the new feature observation value and the existing feature in the map to the data association. The correct data association is a sufficient condition of the obtaining consistent map. The traditional ICNN algorithm is relatively simple. But it can’t solve the problem which robot exist unknown motion, because the prediction equation will produce errors. The nearest-neighbor algorithm becomes invalid. The Feature Matching Algorithm (FMA) [15] is used to achieve the data association in this paper.

(d) Update

During the data association, if new measure value corresponds to existing feature in the map, then use observation formation to update existed state based on the AISRCKF algorithm. That is to say, according to (20)~(23),update the state vector and the square root factor of the prediction error covariance. If new measure value does not corresponds to existing feature in the map, and then it obtains a new feature. So, it needs to initialize the feature in the map, and augment state.

(e) Mapping

The observations S are decomposed into the association observation S_k and the observation of the new feature S_{nk}.. It is defined as the following [16]:

$$S = [S_k\ S_{nk}]^T \tag{24}$$

It achieves map by using the augment state, and the method as the following [16]:

$$x_k^{new} = f\,(S_{nk},\ x_k) \tag{25}$$

$$X_k = [X_{k-1},\ x_k^{new}]^T \tag{26}$$

Where S_{nk} and x_k^{new} are observation value of the new feature and feature point of the new observation, respectively.

3. Experiment Modeling and Analysis

3.1. Experiment Modeling

Before the SLAM simulation experiment, we need build a system model for the mobile robot. The established model mainly includes system model, robot location model, control command model, environment map model, robot motion model, sensor measurement model and system noise model. In this paper, the Bailey SLAM model is used [17].

(1) The motion model can be obtained by:

$$X_{V,k+1} = \begin{bmatrix} x_{Vx,k+1} \\ x_{Vy,k+1} \\ x_{V\theta,k+1} \end{bmatrix} = \begin{bmatrix} x_{Vx,k} + \Delta TV_{k+1}\cos(x_{V\theta,k} + \phi_{k+1}) \\ x_{Vy,k} + \Delta TV_{k+1}\cos(x_{V\theta,k} + \phi_{k+1}) \\ x_{V\theta,k} + \dfrac{\Delta TV_{k+1}\sin\phi_{k+1}}{B} \end{bmatrix} \tag{27}$$

Input: $X_{V,k}$ specifies the pose of the robot at time k. ΔT specifies the sampling time of the dead recking sensors. Vk specifies the velocity of the robot. ϕ_k is rudder angle. B is two interaxial wheelbases. Output: $X_{V,k+1}$ specifies the pose of the robot at time k+1.

(2) Observation model can be obtained by:

$$Z_k = \begin{bmatrix} r_i \\ \theta i \end{bmatrix} = \begin{bmatrix} \sqrt{(x_i - x_{Vx,k})^2 + (y_i - x_{Vy,k})^2} \\ \arctan\dfrac{y_i - x_{Vy,k}}{x_i - x_{Vx,k}} - x_{V\theta,k} \end{bmatrix} \tag{28}$$

Iuput: (x_i, y_i) specifies the position coordinates of detected the ith landmark features. Output: r_i and θ_i respectively specify the range of the ith landmark feature relative to the robot and angle of the ith landmark feature relative to the robot direction.

3.2. Experimental Environments

The 300m*250m outdoor environment area is used in the experiment. The 10 waypoint and the 34 landmark are preestablished in experiment environment area. Robot start moving at (0,0) counterclockwise with waypoint. From the coordinates (0,0), the robot starts moving counterclockwise along the trace ensured by the waypoint The simulation parameters are shown in Table 1. In this experiment environment, we make simulation experiment through MATLAB/Simulink simulations.

Table 1. The Simulation Parameters

The simulation parameters	Value
velocity	1.5m/s
Maximum steering angle	$\pm 40^0$
Maximum steering angular velocity	$\pm 20^0$/s
L wheel distance	1.5m
The sensor scanned Maximum distance	45m
laser scanner range	0^0-120^0
control frequency	100Hz
observing frequency	10Hz
Feature points	34

3.3. Experimental Results and Analysis

In this paper, under the same experiment condition, we make simulation experiment by using the EKF, UKF, SRCKF and AISRCKF algorithm, respectively. Randomly taken the one time experiment results and taken the average result of the 50 time repeated the experiment as final result were comparing analyzed.

Figure 1 and Figure 2 are corresponding to the simulation experimental results of the SRCKF-SLAM algorithm and the AISRCKF-SLAM algorithm, respectively. From Figure 1 and Figure 2 shown, in terms of the robot navigation and positioning estimation, the obtained Integrating degree of the estimating path with the actual path of the robot moving by using the AISRCKF-SLAM algorithm is higher than that by using the SRCKF-SLAM algorithm. This means that the AISRCKF-SLAM algorithm estimation precision is higher. In terms of map accuracy, the AISRCKF-SLAM algorithm building map accuracy is higher.

For Figure 3 and Figure 4 experimental results, they have the following analysis results. From the algorithm estimation stability analysis, the stability of the AISRCKF algorithm is the best among the four algorithms on the X-axis. The error value variation of the AISRCKF algorithm is within 3m. The stability of the UKF algorithm is next to the stability of the AISRCKF algorithm. The error value variation of the UKF algorithm is within 4m. We can see from Figure 3(a), the UKF algorithm appeared two time larger deviation. The maximum deviation of the UKF algorithm is more than 40m. The stability of the SRCKF algorithm is poorer. The stability of the SRCKF algorithm is only better than that of the EKF algorithm. The error value variation of the SRCKF algorithm is within 6m. We can see from Figure 3(a), the SRCKF algorithm appeared three time larger deviation. The maximum deviation of the SRCKF algorithm is more than 40m. And the stability of the EKF algorithm is the worst. The error value variation of the EKF algorithm is within 8m. And we can see from Figure 3(a), the EKF algorithm appeared three time larger deviation. The maximum deviation of the EKF algorithm is more than 80m. The stability of the AISRCKF algorithm is the best among the four algorithms on the Y-axis. The error value variation of the AISRCKF algorithm is within 4m.

The stability of the SRCKF algorithm is next to the stability of the AISRCKF algorithm. The error value variation of the SRCKF algorithm is within 5m. We can see from Figure 4(a), the SRCKF algorithm appeared three time larger deviation. The maximum deviation of the SRCKF algorithm is more than 30m. The stability of the UKF algorithm is poorer. The stability of the UKF algorithm is only better than that of the EKF algorithm. The error value variation of the UKF algorithm is within 6m. We can see from Figure 4(a), the UKF algorithm appeared three time larger deviation. The maximum deviation of the UKF algorithm is more than 50m. And the stability of the EKF algorithm is the worst. The error value variation of the EKF algorithm is within 8m. And we can see from Figure 4(a), the EKF algorithm appeared three time larger deviation. The maximum deviation of the EKF algorithm is more than 90m.

From the algorithm estimation precision analysis, the estimation precision of the AISRCKF algorithm, the SRCKF algorithm and the UKF algorithm has increased significantly on the second half X-axis. The estimation precision of the EKF algorithm is the worst and Shows a trend of decreasing. The estimation precision of the UKF algorithm has increased on the second half X-axis. The estimation precision of the UKF algorithm is slightly better than that of the SRCKF algorithm. But the estimation precision of the UKF algorithm is poorer than that of the SRCKF algorithm on the Y-axis. The estimation precision of the SRCKF algorithm has obvious advantages on the Y-axis. But the estimation precision of the SRCKF algorithm is poorer on the X-axis.

The estimation precision of the SRCKF algorithm is only better than that of the EKF algorithm. The estimation precision of the AISRCKF algorithm is basically the same as the estimation precision of the SRCKF algorithm on the Y-axis. The estimation precision of the method has obvious advantages on the X-axis. Under the same time, the estimation precision of the AISRCKF algorithm most increase 2m. To sum up, the error of the EKF-SLAM is largest. The estimation precision of the UKF-SLAM algorithm is basically the same as the estimation precision of the SRCKF-SLAM algorithm. During the whole exploring process of the robot, the localization estimation precision of the AISRCKF-SLAM algorithm is the highest. The error of the AISRCKF-SLAM algorithm is lesser and the numerical stability of the AISRCKF-SLAM algorithm is also better. So, the effective and superior of the AISRCKF-SLAM algorithm is verified.

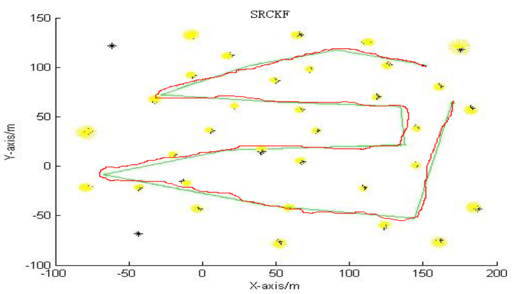


Figure 1. Results of SRCKF-SLAM (the green line is the true path, the red line is SLAM path, the black point is the real position of the landmark, the yellow point is the estimated position of the landmark)

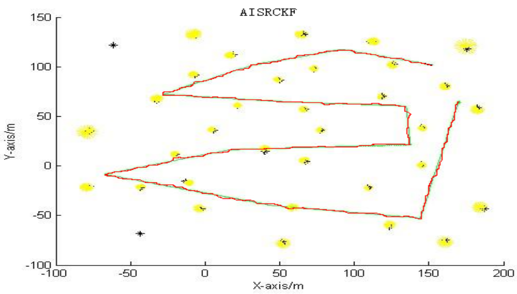
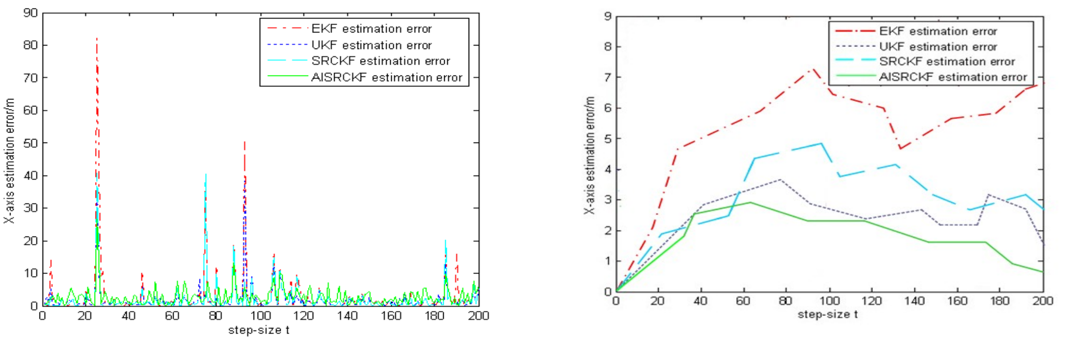


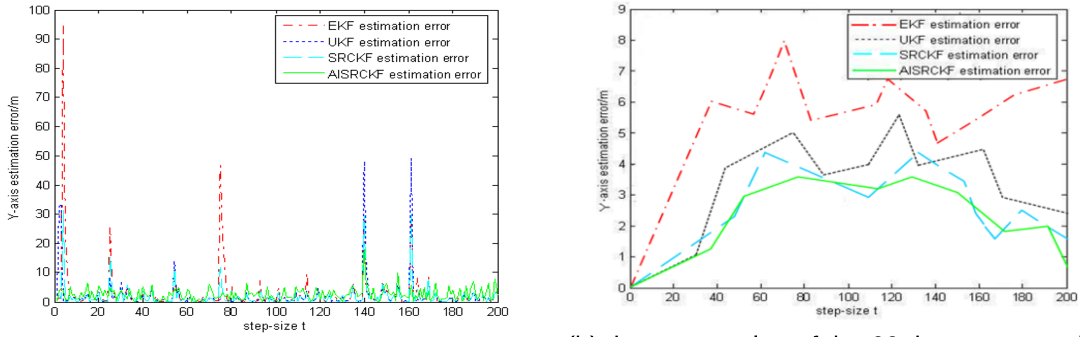
Figure 2. Results of AISRCKF-SLAM (the green line is the true path, the red line is SLAM path, the black point is the real position of the landmark, the yellow point is the estimated position of the landmark)



(a) the single experimental results

(b) the mean value of the 60 times repeated experiment results

Figure 3. The SLAM Algorithm Error Comparison on the X-axis under the Gaussian Noise



(a) the single experimental results

(b) the mean value of the 60 times repeated experiment results

Figure 4. The SLAM Algorithm Error Comparison on the Y-axis under the Gaussian Noise

The data contrast of the system noise and the observation noise under the gaussian white noise condition is shown in the table2. From the complexity contrast, the complexities of four kinds of algorithms all are the same. But from the running time contrast, the time consumption of the EKF, UKF and SRCKF algorithm are basically the same. Because joined the iterative algorithm in the AISRCKF algorithm, the time consumption of the AISRCKF algorithm is a little longer than those other algorithms. The estimation precision contrast is from theoretical derivation. The experiment result verifies it. The accuracy of the map estimation are compared. The results show that the map estimation error is smaller than the path estimation error. During the process of the estimation error contrast, the error of the EKF algorithm is the biggest. The error of the SRCKF algorithm is smaller than that of the UKF algorithm. The error of the AISRCKF algorithm is smaller than that of the UKF algorithm and the SRCKF algorithm. So, the validity of the AISRCKF-SLAM algorithm is verified.

Table 2. The Data Contrast of System Noise and Observation Noise under the Condition of Gaussian White Noise

SLAM	complexity	Run time	precision	G_m/t
EKF	O(n2)	4.51s	First-Order Accurate	5.2541
UKF	O(n2)	4.97s	Second-Order Accurate	3.6352
SRKF	O(n2)	4.43s	Second-Order Accurate	4.1425
AISRKF	O(n2)	5.47s	Second-Order Accurate	3.3256

Here, Gm/t is the root mean square error of the map estimation respectively.

4. Conclusion

A strategy which improves the state estimation accuracy of the SRCKF-SLAM algorithm is proposed in this paper. For the problem which the state estimation error range of the SRCKF algorithm is bigger, to improve the innovation covariance and the cross-covariance, the latest measurement is iteratively used in the measurement update. It effectively improves the accuracy of the system state estimation. The algorithm also used adaptive iterating estimation restricted by the iterative sentencing guideline to adjust the proportion of the observation and dynamic model, to make the estimated square root of the error covariance more accurate and reasonable. By the demonstrating of simulation experiment, it shows that compared with the current several kinds of SLAM algorithm, the paper proposed the AISRCKF-SLAM algorithm to make the state estimation of the system to better converge to near the real value, at the same time satisfying the requirement of real time. The proposed method provides a new train of thought for the mobile robot SLAM in unknown environment.

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