

# Camera Self-calibration in the AUV Monocular Vision Navigation and Positioning

Jun-Chai Gao\*, Ming-Yong Liu

School of Marine Engineering, Northwestern Polytechnical University, Xi'an 710072, China,

Ph.: 15929319672

\*Corresponding author, e-mail: gaomuyou@126.com

## Abstract

Camera calibration is essential to obtain 3D information from 2D image, for underwater camera self-calibration, because the linear model can not accurately describe the imaging geometry of real cameras, the underwater camera nonlinear model and its calibration method are studied. According to the relationship between underwater and air in the camera focal length, principal point and no vertical factors, the underwater camera imaging geometry is modeled. Underwater camera nonlinear imaging geometry is modeled with the radial distortion. The constraint equation of the nonlinear model parameters are set up with linear imaging model and fundamental matrix of the matching points, which realize the camera self-calibration of nonlinear parameters. For the linear parameters of the camera, with the principal point and no vertical factor known, the self calibration of the focal length can be realized with two images, according to the Kruppa equation. Simulation and real image experiments show that the method given is feasible, and has certain practical value.

**Keywords:** camera nonlinear model, camera self-calibration, fundamental matrix, Kruppa equation

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## 1. Introduction

Camera calibration is to confirm the transformation from 2D images to 3D information in the field of computer vision, it has been widely used in 3D reconstruction, navigation, visual surveillance [1-2]. Camera calibration method can be divided into three kinds: traditional calibration method, calibration method based on active vision and self-calibration method. During the traditional calibration, calibration template is used with a known structure and high precision, the constrained relationship of the camera model parameters is established with the correspondence between space points and image points, and parameters are estimated [3-5]. the typical method are DLT (Direct Linear Transformation) method, two steps method of Tsai considering the distortion compensation, simple plane calibration method of Zhang Zhengyong and matrix template method of Wu Fuzhao. The advantages of traditional camera calibration methods is to get higher accuracy, its shortcoming is that the calibration process is complicated with high precision calibration template, while in practical the calibration block cannot be used in many cases, such as the robot of deep-sea or space, robot of dangerous environment and camera intrinsic parameters changing.

For visual navigation and localization of AUV, the specially designed underwater camera vision sensor is used. during the camera imaging underwater, refractivity is of different between air and water. Light would refract from water to air medium. Underwater camera imaging, target is in the object space of aqueous medium, while the imaging plane (CCD, film and so on) is in the image space of air medium. If lens of ordinary photography, camera is used, the angle, magnification and aberration of the objective lens would change, these parameters are to be calibrated under water, and the water quality have different effect on the calibration parameters, for mobile applications, it is required on-site camera calibration, so the self calibration method is employed.

## 2. The Camera Imaging Model of Underwater

A mathematical model of camera imaging firstly established before the camera calibration, which can make clear the camera calibration parameters [6-8]. The classical model

is the perspective projection or pinhole model in the computer vision, which is a linear model. This model is a simplified optical system of camera, which is based on that the object and its image are in the same homogeneous medium (air). In the small pinhole model, the linear imaging process can be described with the relationship as following [9]:

$$\lambda \begin{bmatrix} \bar{x} \\ \bar{y} \\ 1 \end{bmatrix} = k \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} \quad (1)$$

The matrix  $k$  is the intrinsic parameter matrix of camera linear model,  $f_u, f_v$  are the scale factor in the  $u, v$  axes of the image plane.  $(u_0, v_0)$  is the principal point coordinate, namely the pixel coordinates of optical axis and the image plane intersection.  $s$  is distortion factor, which is a coefficient of  $u, v$  axes not vertical in the image coordinates.  $(x_c, y_c, z_c)$  is the coordinates of space points in camera coordinate system.  $(\bar{x}, \bar{y}, 1)$  is the homogeneous coordinates of image points  $(\bar{x}, \bar{y})$ . Because the homogeneous coordinates of image point and the spatial points coordinates are the linear transformation in the formula, so the pinhole model is called linear model.

While the more general optical model must be used in the underwater camera, because object and its image are in different optical medium. Special camera lens of water contacting was studied outside of water by J.M.Lavest, and the conclusions as following [10]:

(1) the camera focal length ratio is the same as the refractive between the water and the air, i.e.:

$$f_{water} = \frac{n_{water}}{n_{air}} f_{air} \quad (2)$$

(2) The principal point position in image coordinates is unchanged.

(3) Assuming only the radial distortion, distortion ratio is the same as the refractive between the water and the air, i.e.:

$$u' + d_w(u') = \frac{n_{water}}{n_{air}} (u + d_a(u)) \quad (3)$$

Where  $u$  is distorted image of the target point in the air medium,  $d_a(u)$  is correction value of obtaining the ideal perspective projection.  $u'$  Water is distorted image of the target point in the water medium,  $d_w(u')$  is correction value of obtaining the ideal perspective projection.

According to the conclusion (1), (2), the camera linear imaging model underwater can be set up.

$$\lambda \begin{bmatrix} \bar{x} \\ \bar{y} \\ 1 \end{bmatrix} = k \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} f'_u & s' & u_0 \\ 0 & f'_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} \quad (4)$$

$$f'_u = \frac{n_{air}}{n_{water}} f_u \quad f'_v = \frac{n_{air}}{n_{water}} f_v \quad s' = \frac{n_w}{n_a} s$$

The general linear model can not accurately describe the cameras imaging geometry really, especially if using a wide angle lens, the image away from the center has a larger

distortion. Therefore, it has very important to study the camera self-calibration of nonlinear model. Compared with the camera linear model, generally camera nonlinear model is not obeying the pinhole model in the imaging process. which can be described with the following formula:

$$\begin{cases} \bar{x} = x + \delta_x(x, y) \\ \bar{y} = x + \delta_y(x, y) \end{cases} \quad (5)$$

According to the conclusion (3), linear imaging model of underwater camera can be set up:

$$\begin{cases} \bar{x} = x + \delta'_x(x, y) \\ \bar{y} = x + \delta'_y(x, y) \end{cases} \quad (6)$$

$$\begin{cases} \delta_x(x, y) = xk'_x(x^2 + y^2) \\ \delta_y(x, y) = yk'_y(x^2 + y^2) \end{cases} \quad \begin{cases} k'_x = \frac{n_w}{n_a}k_x \\ k'_y = \frac{n_w}{n_a}k_y \end{cases}$$

So, for the underwater camera,  $f'_u$ ,  $f'_v$ ,  $u_0$ ,  $v_0$ ,  $s'$  of the intrinsic parameters in linear model and  $k'_x, k'_y$  of the nonlinear parameters in nonlinear model are together constitute the entire camera intrinsic parameters.

nonlinear optimization algorithm is proposed to calibrate the general camera distortion parameters by Faig in [13], the calibration algorithm with assuming only radial distortion are given by Tsai in [14]. a double plane camera model is put forward by Wei Guoqing and Ma Songde in [15], however, high precision calibration reference object (block) is required during the intrinsic parameters calibration with all of these methods. This paper will discuss the nonlinear model self-calibration of underwater camera.

### 3. Intrinsic Parameters Self-calibration of Underwater Camera

#### 3.1. Calibration of Radial Distortion Coefficient

Firstly the constraint equations of nonlinear parameters is set up based on fundamental matrix of linear model and the relationship between the nonlinear model and linear model.

Set  $I_1, I_2, \dots, I_N$  are images of the same scene at  $N$  pieces of perspective in camera nonlinear model.  $\bar{I}_1, \bar{I}_2, \dots, \bar{I}_N$  are images in camera linear model.

If  $(x_{ij}, y_{ij}, 1) \in I_i$ ,  $i = 0, 1, 2, \dots, N$ ;  $j = 1, 2, \dots, M$  is a set of matching points of the nonlinear model,  $(\bar{x}_{ij}, \bar{y}_{ij}, 1) \in \bar{I}_i$  is a set of corresponding matching points of the linear model, from the Equation (1) of the relationship between the nonlinear and linear model, the equation can be changed as following:

$$\begin{cases} \bar{x}_{ij} = x_{ij} + \delta'_x(x_{ij}, y_{ij}) \\ \bar{y}_{ij} = y_{ij} + \delta'_y(x_{ij}, y_{ij}) \end{cases} \quad (7)$$

$i = 0, 1, 2, \dots, N$ ;  $j = 1, 2, \dots, M$

$F_i$  is the fundamental matrix of the image pair  $(\bar{I}_0, \bar{I}_i)$ , then:

$$\begin{bmatrix} \bar{x}_{ij} & \bar{y}_{ij} & 1 \end{bmatrix} F_i \begin{bmatrix} \bar{x}_{0j} \\ \bar{y}_{0j} \\ 1 \end{bmatrix} = 0 \quad (8)$$

$$i = 1, 2, \dots, N; j = 1, 2, \dots, M$$

The type (8) is substitution into (9):

$$\begin{bmatrix} x_{ij} + \delta'_x(x_{ij}, y_{ij}) & y_{ij} + \delta'_y(x_{ij}, y_{ij}) & 1 \end{bmatrix} F_i \begin{bmatrix} x_{0j} + \delta'_x(x_{0j}, y_{0j}) \\ y_{0j} + \delta'_y(x_{0j}, y_{0j}) \\ 1 \end{bmatrix} \quad (9)$$

So the nonlinear parameters will satisfy the constraint Equation (6). Let:

$$E_{ij}(F_i, k_x, k_y) = \left[ \begin{bmatrix} x_{ij} + \delta'_x(x_{ij}, y_{ij}) & y_{ij} + \delta'_y(x_{ij}, y_{ij}) & 1 \end{bmatrix} F_i \begin{bmatrix} x_{0j} + \delta'_x(x_{0j}, y_{0j}) \\ y_{0j} + \delta'_y(x_{0j}, y_{0j}) \\ 1 \end{bmatrix} \right]^2 \quad (10)$$

$$E(F, k_x, k_y) = \sum_{i=1}^N \sum_{j=1}^M E_{ij}(F_i, k_x, k_y) \quad (11)$$

Among them,  $F = (F(1), F(2), \dots, F(N))$ . So, solving nonlinear parameters is equivalent to solving the optimization problems as following:

$$\min_{(F, k_x, k_y)} S(F, k_x, k_y) \quad (12)$$

Minimum problems of type (12) is a nonlinear optimization problem, it need a nonlinear optimization technique to solve. In this paper, we use BFGS technology of the variable metric method to solve the type (12). BFGS technology is the research achievement of Broyden, Fletcher, Goldfarb and Shanno, which has great advantages in reliability and efficiency. For the sake of convenience, the type (11) changes as:

$$(X_0, X_1, \dots, X_N) = (x_{01}, x_{02}; x_{11}, x_{12}; \dots; x_{N1}, x_{N2}) \quad (13)$$

Where  $X_0 = (x_{01}, x_{02}) = (k_x, k_y)$  is the nonlinear parameters vector;

$X_i = (x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5}, x_{i6}, x_{i7}, x_{i8})$  is the vector elements of fundamental matrix

$$F_i = \begin{bmatrix} x_{i1} & x_{i2} & x_{i3} \\ x_{i4} & x_{i5} & x_{i6} \\ x_{i7} & x_{i8} & 1 \end{bmatrix} \text{ between the image pairs } (\bar{I}_0, \bar{I}_i), \text{ So the type (12) can be written}$$

into:

$$\min_{X_0, X_1, \dots, X_N} S(X_0, X_1, \dots, X_N) \quad (14)$$

### 3.2. Linear Model Self-calibration

After the self-calibration of nonlinear parameters, the image  $\bar{I}_i$  of linear model can be get from the actual image  $I_i$ . The linear intrinsic parameter matrix  $K$  can be calibrated with

hierarchical stepwise method or Kruppa equation from  $\bar{I}_i (i = 0, 1, 2, \dots, N)$ . If there is only, an unknown parameters of the focal length in the camera model, that reduces the difficulty of solving the Kruppa equation, so it is solved with Kruppa equation.

Let the image curve of space quadratic curve are respectively  $A_l, A_r$  in two views,  $e_l$  is the pole of left camera,  $e_r$  is the pole of right camera, and if the homography matrix of the plane that quadratic curves supports is  $H$ , the degradation quadratic curve imaged in left camera is  $A_{lab} = [e_l]_x A_l^{-1} [e_l]_x^T$ , in right camera is  $A_{rab} = [e_r]_x A_r^{-1} [e_r]_x^T$ , according to the projective transformation:

$$A_{rab} = H^{-T} A_{lab} H^{-1} \quad (15)$$

$$[e_r]_x A_r^{-1} [e_r]_x^T = H^{-T} [e_l]_x A_l^{-1} [e_l]_x^T H^{-1} \quad (16)$$

Because the fundamental matrix between two images is as following:

$$F = H^{-T} [e_l]_x \quad (17)$$

$$s[e_r]_x A_r^{-1} [e_r]_x^T = F A_l^{-1} F^T \quad (18)$$

And because the image curve of the absolute quadratic curve in two view is respectively  $\omega = K^{-T} K^{-1}$  and  $\omega' = K'^{-T} K'^{-1}$  which will be substituted into type (19), the constraint equations of DIAC, that the Kruppa equation is:

$$s[e_r]_x \omega'^* [e_r]_x^T = F \omega^* F^T \quad (19)$$

Then the SVD decomposition of  $F$  has the following form:

$$F = UDV^T = U \text{diag}(\sigma_1, \sigma_2, 0) V^T \quad (20)$$

Here,  $u_3 = e_r, v_3 = e_l$ , Equation (19) is substituted into type (21):

$$s[u_3]_x \omega'^* [u_3]_x^T = UDV^T \omega^* (UDV^T)^T \quad (21)$$

Kruppa equation is the constraint of the DIAC, it has nothing to do with the scene geometry structure, which depends only on the epipolar geometry of two views. Because the precision of the camera hardware made, the skew factor of camera is zero, pixel aspect ratio is 1, and the principal points is in the image plane center. The unknown parameters is focal length, and remained unchanged in the two views, i.e.  $\omega = \omega'$ . In the camera interior parameter matrix, although there is only one unknown parameter, because the epipolar geometry depends on two views, two images are still need to solve the focal length.

$$f^2 = -\frac{p'^T [e_r]_x \bar{I} F p p^T F^T p'}{p'^T [e_r]_x \bar{I} F \bar{I} F^T p'} \quad f'^2 = -\frac{p^T [e_l]_x \bar{I} F^T p' p'^T F p}{p^T [e_l]_x \bar{I} F \bar{I} F p} \quad (22)$$

$$\bar{I} = \text{diag}(1, 1, 0), p = (u_0, v_0, 1)^T, p' = (u'_0, v'_0, 1)^T$$

#### 4. Experimental Simulation

##### 4.1. Experiments with Simulated Data

In the experiments with simulated data, camera non-linear intrinsic parameters are solved with the basic matrix constraint. Nonlinear parameters theoretically is  $k_1 = k_2 = 1.0 \times 10^{-7}$ , the camera center coordinates of two view are  $O_1(0,0,0)$ ,  $O_2(801.6, 39.2, 596.6)$  mm, the angle of optical axis direction in  $O_1$  and  $O_2$  view is 8.66 degrees, the linear intrinsic parameter theoretically is image size is  $f'_u = f'_v = 1000$ ,  $u_0 = 320, v_0 = 240, s = 0$ , the size of image is 1280 x 1024 pixels. According to the imaging parameters given above, 12 spatial coordinates and pixel coordinates of linear imaging are used, the calculation value of pixel coordinates can be accurate to sub-pixel level, but because the imaging is pixel level, so it is represented with pixel level, as shown in Table 1. The pixel coordinates of its nonlinear imaging is represented with pixel level too, as shown in Table 2, the image plane center  $O(u_0, v_0)$  is the image plane origin.

Table 1. Spatial Points and Pixel Coordinates of Linear Imaging

NO.	spatial points coordinates			Imaging coordinates in $O_1$ view		Imaging coordinates in $O_2$ view	
	$x$	$y$	$z$	$u_1$	$v_1$	$u_2$	$v_2$
1	90.04	2652.43	11701.48	8	-227	177	-168
2	-822.78	-27.19	9020.62	-91	-3	106	77
3	499.51	-1023.97	10165.32	49	-101	222	-22
4	1118.47	1969.48	7852.10	142	251	308	331
5	64.15	352.40	3835.00	17	92	378	376
6	1235.90	234.03	6159.99	-201	-38	91	126
7	1655.05	92.78	9014.25	184	10	348	108
8	844.55	746.36	5571.14	-152	134	143	292
9	805.52	572.03	8381.66	96	68	276	165
10	1452.01	2580.49	11987.30	121	215	235	235
11	1049.06	1216.45	10204.98	103	119	248	175
12	688.20	735.30	8143.60	85	-90	287	34

Table 2. Space Points and Pixel Coordinates of Nonlinear Imaging

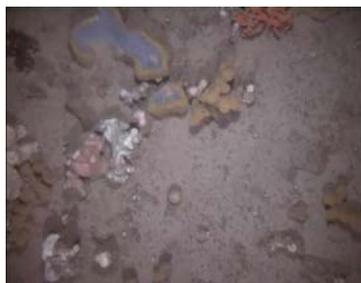
NO.	spatial points coordinates			Imaging coordinates in $O_1$ view		Imaging coordinates in $O_2$ view	
	$x$	$y$	$z$	$u_{1d}$	$v_{1d}$	$u_{2d}$	$v_{2d}$
1	90.04	2652.43	11701.48	8	-226	176	-167
2	-822.78	-27.19	9020.62	-91	-3	106	77
3	499.51	-1023.97	10165.32	49	-101	221	-22
4	1118.47	1969.48	7852.10	141	249	302	325
5	64.15	352.40	3835.00	17	92	368	366
6	1235.90	234.03	6159.99	-200	-38	91	126
7	1655.05	92.78	9014.25	183	10	344	107
8	844.55	746.36	5571.14	-151	133	142	289
9	805.52	572.03	8381.66	96	68	273	163
10	1452.01	2580.49	11987.30	120	214	232	232
11	1049.06	1216.45	10204.98	103	119	246	173
12	688.20	735.30	8143.60	85	-90	285	34

Generally camera optical center position and skew factor errors are relatively small, here the theoretical value is as the exact value, not to calibrate. In order to test the reliability of the algorithm, the focus length is estimated with the algorithm for the image data without round off error, it is  $f'_u = f'_v = 955.29$ . If the distortion is  $k_1 = k_2 = 1.0 \times 10^{-7}$ , the focus length is estimated with the algorithm for the image data with a rounding error, it is  $f'_u = f'_v = 1125.62$ , distortion coefficient is  $k = 0.89 \times 10^{-7}$ . The simulation results show

that, this algorithm can accurately estimate the camera parameters, calculation accuracy of pixel level will introduce larger error.

#### 4.2. Experiments with Real Images

Figure 1 is two images used for intrinsic parameters calibration of the camera, feature points with the SIFT algorithm are found as shown in Figure 2, based on the K-D tree, image matching points are selected in accordance with Euclidean distance ratio of the nearest and next-nearest neighbor matching the between points, which as shown in Figure 3.

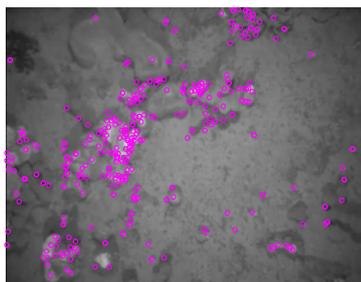


(a) The image of  $f_{k-1}$  frame

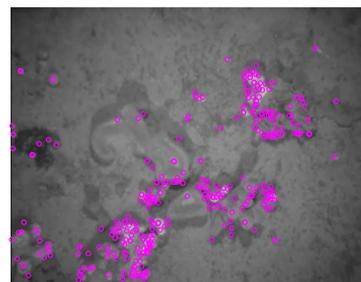


(b) The image of  $f_k$  frame

Figure 1. Two Consecutive Image Frames



(a) Feature point image of  $f_{k-1}$  frame



(b) Feature point image of  $f_k$  frame

Figure 2. Feature Point of the Consecutive Image Frames

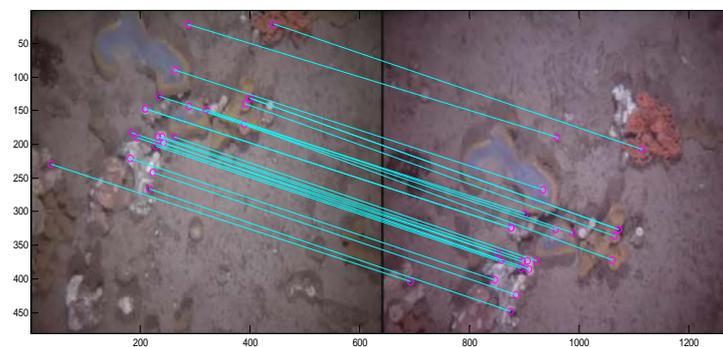


Figure 3. The Matching Feature Points of  $f_{k-1}$  and  $f_k$  Frames Image

Camera nonlinear distortion and interior parameter matrix are estimated by the self-calibration algorithm above with the matching points of two consecutive frames image,  $k = 3.84 \times 10^{-6}$ , the camera linear intrinsic parameters calculated is  $f'_u = f'_v = 549.74$ .

## 5. Conclusion

For the calibration of camera parameters underwater, mostly it is calibrated with accurate template in air in linear imaging model, and then amended based on the ratio of light refraction rate in two kinds of transmission medium, or directly in the water, about that there are a large number of research results. Because of the camera linear model cannot accurately describe the imaging geometry of real cameras, and precise template can not be effectively guaranteed in water. This paper studied the self-calibration algorithm of camera nonlinear model, simulated and real images experiments show that the method given is feasible, and has a certain practical value.

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