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# Simplified Gauss Hermite Filter Based on Sparse Grid Gauss Hermite Quadrature

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***Abstract***

*In order to improve estimation accuracy of nonliear system with linear measurement model, simplified gauss hermite filter based on sparse grid gauss hermite quadrature (SGHF) is proposed. Comparing to conventional Gauss-Hermite filter (GHF) based on tensor product gauss quadrature rule, simplified SGHF not only maintains GHF's advantage of precission controllable, high estimation accuracy, but also relieves the curse of dimension problem by reduce the number of gaussian intrgration points to the number of sigma points that scaled unscented transform uses. Theoretical analysis and experimental results show that estimation base on new filter performs significantly beter than extended kalman filter (EKF), and slightly better than unscented kalman filter (UKF) on estimation accuracy and convergence speed, and computational burden is significantly reduced comparing with traditional GHKF.*

***Keywords****: gasss-hermite quadrature, gauss filter, sparse grid gauss hermite quadrature, bayesian estimation*



## 1. Introduction

In bayesian estimation problem, integration of nonlinear function is intractable usually. Under the assumption that the state is of Gaussian distribution, the filter density may be approximated by Gaussian distribution parameteried by the conditional mean and covariance. Expectation values occurring in the time and measurement updates can be computed or estimated numerically. Expectations treated by truncated Taylor expansion, leading to the well known EKF. It can be shown that Gaussian filter (GF) is equivalent to an infinite Taylor expansion [3]. Another numerical integration method for computing expectations is the third degree spherical radical rule leading to the cubture kalman (CKF) filter of Arasaratnam I and Haykin S [1-2]. Closely related is the UKF [5] of Julier and Uhlmann which uses the unscented transform (UT) to estimate the expectation, difference filter (DF) of Nørgaard that uses strling interpolation, particle filter (PF) of Arulampalam m s, Maskell s, Gordon N that use mote-carlo Integration, gauss hermite filter (GHF) of Ito K and Xiong K that use gauss hermite quadrature (GHQ) [3-7]. Of all these digital filters, gauss hermite filter is simple in principle, gauss integration accuracy is esay to set according to the demand of filtering accuracy. However, gauss hermite filter based on tensor product has the problem of “curse of dimension” in high dimension state estimation. In order to break the curse of dimension of gauss hermite quadrature, Russia methematician Smolyak in the last 60s of the last century proposed sparse grid based gauss hermite quadrature (SGHQ). Heiss F, Winschel V incorporate SGHQ into Gaussian filter [2], B Jia and M Xin named the new filter SGHF [4]. SGHF not only maintained the superior characteristics of GHF, but also avoid the problem of “curse of dimension”.

In this paper, we propose a simplified SGHF algorithm that shows comparable performance to the UKF and computational cost much less than GHF for the estimation problem of nonlinear system with linear measurement model. The performance of the new filter is tested in the problem of strapdown inertial navgaiton system (SINS) nonlinear initial alignment, and the superiority of new filter is compararped with several other conventional algorithms [6-8].

The rest of this paper is organized as follows. Section 2 introduced the simplified SGHF. The navigation error propagation equation with big initial mis-alignment angles of SINS are briefly reviewed in Section 3. Section 4 gives the simulation results, illustrating the performance of simplified SGHF for the problem of SINS mis-alignment angles' estimation, and

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the perofrmance of simplified SGHF is comparied with the EKF and UKF. Some concluding remarks are given in section 5.

## 2. Simplified Sparse Gauss Hermite Quadrature Filter

Consider a nonlinear distrete system with additive process noise and measurement noise:

$$\begin{cases} \mathbf{x}_k = f\left(\mathbf{x}_{k-1}\right) + \mathbf{v}_{k-1} \\ \mathbf{y}_k = h\left(\mathbf{x}_k\right) + \mathbf{n}_k \end{cases} \tag{1}$$

Where $\mathbf{x}_k$ is the $n \times 1$ state vector and $\mathbf{y}_k$ is the $m_y \times 1$ measurement vector; $\mathbf{v}_{k-1}$ and $\mathbf{n}_k$ are independent zero mean white gaussian process noise and measurement noise with covariance $\mathbf{Q}_{k-1}$ and $\mathbf{R}_k$, respectively. Bayesian filter is only a conceptual solution, in the sense that (in most situations) it cannot be determined analytically. Thus one has to use approximations. Under the assumption of Gaussian distribution, bayesian filter can be simplified as Gaussian filter.

### 2.1. Gauss Hermite Quadrature Rule

Gauss hermite quadrature rule is used to solve the following gaussian integral:

$$I_n\left[f\right] = \int_{R^n} f\left(\mathbf{x}\right) N\left(\mathbf{x}, \mathbf{0}, \mathbf{I}\right) d\mathbf{x} \tag{2}$$

Where **x** is the integral vector, $f\left(\mathbf{x}\right)$ is nonlinear function, $N\left(\mathbf{x}, \mathbf{0}, \mathbf{I}\right)$ is the pdf of multidimension normal distribution, $R^n$ is n-dimensional Euclidean space, and $I_n\left[f\right]$ is integral to be solved.

#### 2.1.1. One Dimensional Gauss Hermite Quadrature

For the integral problem of formula (2), when the integration variable is one dimension. The gauss-hermite quadrature rule is given by:

$$I_1\left[f\right] = \int_{R^1} f\left(\mathbf{x}\right) N\left(x, 0, 1\right) dx \approx I_{1,L}\left[f\right] = \sum_{i=1}^{L} \omega_i f\left(\xi_i\right) \tag{3}$$

Where the equality holds for all polynomials of degree up to 2L-1, the quadrature points $\xi_i\left(i=1,2,\ldots,L\right)$ and the weights $\omega_i\left(i=1,2,\ldots,L\right)$ are determined as follows. Let J be the symmetric tridiagonal matrix with zero diagonals and $J_{i,i-1} = \sqrt{i/2}, i \le 1 \le L-1$. Then $\left\{\xi_i\right\}$ are the eigenvalues of J and $\omega_i$ equals to $\left|\left(v_i\right)_1\right|^2$ where $\left(v_i\right)_1$ is the first element of the ith normalize eigenvector of J.

#### 2.1.2. Tensor based Multi Dimensional Gauss Hermite Quadrature

Multi-dimensional gaussian numerical integration can be solved by multiple one dimension gaussian numerical integration, also known as tensor based gauss quadrature. Suppose n dimensional integral vector $\mathbf{x} = \mathbf{x}_{1:n} = \left[x_1 \quad \cdots \quad x_n\right]^{\mathrm{T}}$, tensor based multi-dimensional gauss hermite quadrature rue is given by:

$$\begin{aligned}
I_n[f] &= \int_{R^n} f(\mathbf{x}_{1:n}) N(\mathbf{x}_{1:n}, \mathbf{0}, I) dx_{1:n} \\
&\approx \int_{R^{n-1}} \sum_{i_1=1}^{L} \omega_{i_1} f(\xi_{i_1}, \mathbf{x}_{2:n}) N(\mathbf{x}_{2:n}, \mathbf{0}, I) d\mathbf{x}_{2:n} \\
&\approx \sum_{i_1=1}^{L} \cdots \sum_{i_n=1}^{L} \omega_{i_1} \cdots \omega_{i_n} f(\xi_{i_1}, \xi_{i_2}, \cdots \xi_{i_n}) \\
&= \sum_{i_1=1}^{L} \cdots \sum_{i_n=1}^{L} \left( \prod_{j=1}^{n} \omega_{i_j} \right) f(\xi_{i_1}, \xi_{i_2}, \cdots \xi_{i_n}) \\
&= I_{n,L}[f]
\end{aligned} \tag{4}$$

Where the equality holds for all polynomials $f(\mathbf{x}) = x_1^{i_1} x_2^{i_2} \cdots x_n^{i_n}$, $1 \le i_1 \le 2L-1, \cdots, 1 \le i_n \le 2L-1$, the multi-dimensional quadrature point $\begin{bmatrix} \xi_{i_1} & \cdots & \xi_{i_n} \end{bmatrix}^{\mathrm{T}}, 1 \le i_1 \le L, \cdots, 1 \le i_n \le L$ are determined astensor product of one dimensional quadrature point, and the corresponding weight for each quadrature point is $\prod_{j=1}^{n} \omega_{i_j}$ , $\omega_{i_j}$ is one dimension quadrature weight for jth dimension. As can be seen from Equation(4), the number of multi-dimension quadrature points is $n^L$, for each quadrature point, calculate $f(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_D}) \prod_{k=1}^{D} \omega_{i_k}$ once. The number of gauss quadrature points and computation cost grow exponentially with integral dimension which is the so called 'curse of dimensionality'. In order to reduce the number of multidimensional gaussian quadrature points, Russian mathematician first proposed sparse grid based multi-dimensional gaussian quadrature.

**2.1.3. Sparse Grid based Multi-Dimension Gauss Hermite Quadrature**

Suppose n dimensional integral vector $\mathbf{x} = \mathbf{x}_{1:n} = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}^{\mathrm{T}}$, multi-dimensional SGHQ rule is given by:

$$\begin{aligned}
I_n[f] &= \int_{R^n} f(\mathbf{x}_{1:n}) N(\mathbf{x}_{1:n}, 0, I) d\mathbf{x}_{1:n} \\
&\approx \sum_{q=L-n}^{L-1} \sum_{\Xi \in N_q^n} \sum_{\xi_{j_1} \in X_{j_1}} \cdots, \sum_{\xi_{j_n} \in X_{j_n}} f(\xi_{j_1}, \cdots, \xi_{j_n}) \left\{ (-1)^{L-n-q} C_{n-1}^{n-L-q} \prod_{p=1}^{n} \omega_{i_p} \right\} \\
&= I_{n,L}[f]
\end{aligned} \tag{5}$$

Where the equality holds for all polynomials $x_1^{i_1} x_2^{i_2} \cdots x_n^{i_n}$, $\sum_{j=1}^{n} i_j \le 2L-1$, L ($L \in \mathbb{N}$ , $\mathbb{N}$ is the set of natural numbers) is the accuracy level of multidimensional integral, n is the number of dimensions of multidimensional integral, q is auxiliary parameter which is an natural number and ranging from $L\text{-}n$ to $L\text{-}1$, $\xi_p$ is one dimensional quadrature point and $\omega_{i_p}$ is corresponding one dimensional weight for $\xi_p$, $\Xi \triangleq (i_1 \;\; \cdots \;\; i_n)$ is an acuracy level sequence of n natural numbers, $C_{n\text{-}1}^{n\text{-}L\text{-}q}$ is the binomial coefficient, $N_q^n$ is a set of accuracy lecel sequences definded by:

$$\begin{cases} N_q^n = \left\{ \Xi : \sum_{j=1}^{n} i_j = n + q \right\} & q \ge 0 \\ N_q^n = \varnothing & q < 0 \end{cases} \tag{6}$$

Where $\varnothing$ is empty set, the set of sparse grid points, $X_{n,L}$ is given by:

$$X_{n,L} = \bigcup_{q=L-n}^{L-1} \bigcup_{\Xi \in N_q^n} \left( X_{i_1} \otimes X_{i_2} \cdots \otimes X_{i_n} \right) \quad (7)$$

Where $\otimes$ denotes tensor product. $X_{i_j}, (1 \le j \le n)$, $X_{i_j}$ is the univariate point set with accuracy level $i_j$.

Comparing with tensor based gauss hermite integral, the number of integrate points of SGHQ is much less. For convenience, we give the general formaula for calculating the number of SGHQ points with level 2 and level 3. For level 2, when n=1, the SGHQ is not necessary, when $n \ge 2$, q=0 or 1, the set of sparse grid points $X_{n,2} = \bigcup_{q=2-n}^{1} \bigcup_{\Xi \in N_q^n} \left( X_{i_1} \otimes X_{i_2} \cdots \otimes X_{i_n} \right)$. When q=0, $N_0^n = \{(1,1,\cdots,1)\}$, generate one point (origin point). When q=1, $N_1^n = \{(2,1,\cdots,1),(1,2,\cdots,1)\cdots,(1,1,\cdots,2)\}$ generates 2n points (excluding the oringin point). Hense, for level 2, the total number of points is $2n+1$. Fro level 3, when n=1, the SGHQ is not nessary. When $n \ge 3$, q=0, 1 or 2, the set of sparse grid points $X_{n,3} = \bigcup_{q=3-n}^{2} \bigcup_{\Xi \in N_q^n} \left( X_{i_1} \otimes X_{i_2} \cdots \otimes X_{i_n} \right)$. When q=0, $N_0^n = \{(1,1,\cdots,1)\}$ generates the origin point. When q=1, $N_1^n = \{(2,1,\cdots,1),(1,2,\cdots,1)\cdots,(1,1,\cdots,2)\}$ generates 2n points, when q=2,
$N_2^n = \{(3,1,\cdots,1),(1,3,\cdots,1)\cdots,(1,1,\cdots,3),(2,2,1,\cdots,1,1),(2,1,2,\cdots,1,1)\cdots,(1,1,1,\cdots 2,2)\}$
generates $2n + 4C_n^2 = 2n + 2n + 2n(n-1) = 2n^2 + 2n$ new points. Hense for level 3 ($n \ge 3$), the total number of points is $2n^2 + 2n + 1$. When accuracy level $L \ge 3$, the method for counting quadrature point is similar. Based on the above discussion, the number of quadrature for SGHQ points and tensor product based GHQ points with accuracy levels 2 and 3 when $n \ge 3$ is summarized in Table 1.

Table 1. The Number of Quadrature Points for SGHQ and Tensor Product based GHQ

| L | Tensor product based GHQ | SGHQ |
|---|---|---|
| 2 | $2^n$ | $2n+1$ |
| 3 | $3^n$ | $2n^2+2n+1$ |

As can be see from

Table 1 the number of quadrature points for SGHQ is much less than quadrature points of tensor product based GHQ and equals to the sigma points of unscented transform [6]. For SGHQ, when accuracy level is 3, the number of quadrature points is a slight increase than accuracy level 2 of quadrature points' number.

### 2.2. Simplified Sparse Grid Gauss Hermite Filter

For system described in(1), Using SGHQ to solve gauss integral in gauss filter, we arrive at SGHF. For systems of linear measurement model, update of SGHF can be replaced by kalman filter update. The simplified sparse grid gauss filtering includes recursive prediction and update procedures.

**Prediction:**

Table 1. Compute the factorization of $\mathbf{P}_{k-1|k-1} = \sqrt{\mathbf{P}_{k-1|k-1}}\left(\sqrt{\mathbf{P}_{k-1|k-1}}\right)^{\mathrm{T}}$ using singular value decomposition and set $\boldsymbol{\eta}_{i,k-1|k-1} = \mathbf{S}\boldsymbol{\xi}_i + \hat{\mathbf{x}}_{k|k}$ , where i ( $i = 1,\ldots,\mathrm{n}$ ) is the point index, $\boldsymbol{\xi}_1$ is the origin point; $\xi_i\ (i = 1,\ldots,n)$ are the SGHQ points, $\{\omega_i\}$ are the corresponding weights.

Table 2. Caluclate the value of system function at sparse grid $\boldsymbol{\eta}_{i,k-1|k-1}, i = 1,\ldots,\mathrm{n}$ .

$$\mathbf{x}^{*}_{i,k|k-1} = f(\boldsymbol{\eta}_{i,k-1|k-1}) \tag{8}$$

Table 3. The corresponding propagated state vector value and covariance are given by the SGHQ algorithm,

$$\hat{\mathbf{x}}_{k|k-1} = \sum_{x^{*}_{i,k|k-1} \in X1_{n,L}} \omega_i \mathbf{x}^{*}_{i,k|k-1} \tag{9}$$

$$\mathbf{P}_{k|k-1} = \sum_{i=1}^{n} \omega_i \left(\mathbf{x}^{*}_{i,k|k-1} - \hat{\mathbf{x}}_{k|k-1}\right)^{\mathrm{T}} \left(\mathbf{x}^{*}_{i,k|k-1} - \hat{\mathbf{x}}_{k|k-1}\right) + \mathbf{Q}_{k-1} \tag{10}$$

**Update:**

$$\hat{\mathbf{x}}_{k,k} = \hat{\mathbf{x}}_{k,k-1} + \mathbf{L}_k \mathbf{v}_k \tag{11}$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{L}_k \mathbf{P}_{yy,k|k-1} \mathbf{L}_k^{\mathrm{T}} \tag{12}$$

Where:

$$\mathbf{L}_k = \mathbf{P}_k^{-} \mathbf{H}_k^{T} \mathbf{P}_{yy}^{-1} \tag{13}$$

$$\hat{\mathbf{y}}_{k,k-1} = \mathbf{H}_k \hat{\mathbf{x}}_{k,k-1} \tag{14}$$

$$\mathbf{v}_k = \mathbf{y}_k - \hat{\mathbf{y}}_{k,k-1} \tag{15}$$

$$\mathbf{P}_{yy} = \mathbf{H}_k \mathbf{P}_{k,k-1}^{-1} \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k \tag{16}$$

### 3. SINS Nonlinear Initial Alignment Model

Initial alignment is one of the most critical technology of SINS, because the performance of a SINS is largely determined by the accuracy of accuracy and rapidness of the alignment process [1, 8]. For simplicity resasons, nonlinear model for SINS is given directly as follows:

**(1) State model**

The inertial navigation error model derived above will be used in this paper for nonlinear alignment of SINS. The dynamic model can be writen as:

$$\dot{\mathbf{x}}(t) = f(\mathbf{x},t) + g(\mathbf{x},t)\boldsymbol{\omega}(t) \tag{17}$$

Where the state vector $x$ consists of attitude errors and velocity errors, process noise vector $\omega$ consists of sensor errors include the gyro noise and acclerometer noise. Then the state variable can be written as $\mathbf{x} = [\delta v_N \quad \delta v_E \quad \phi_N \quad \phi_E \quad \phi_D]^{\mathrm{T}}$ and

$\boldsymbol{\omega} = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \varepsilon_z & \nabla_x & \nabla_y \end{bmatrix}^{\mathrm{T}}$. Moreover, the subscripts N, E, and D denote the north, east, and down in the N-frame, and subscripts x, y, z denote the front, right, down components in the B frame. Moreover, $f(\mathbf{x},t)$ and disturbance matrix $g(\mathbf{x},t)$ is defined as follows:

$$f(\mathbf{x},t) = \begin{bmatrix} \begin{bmatrix} 1 & \sin\phi_x \tan\phi_y & \cos\phi_x \tan\phi_y \\ 0 & \cos\phi_x & -\sin\phi_x \\ 0 & \sin\phi_x \sec\phi_y & \cos\phi_x \sec\phi_y \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega}_{ip}^{p} - \mathbf{C}_n^p \boldsymbol{\omega}_{in}^{n} \end{bmatrix} \\ (\mathbf{C}_n^p - \mathbf{I})\mathbf{f}^n - (\boldsymbol{\omega}_{ie}^p + \boldsymbol{\omega}_{ip}^p) \times \delta\mathbf{v}^n + (\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{in}^n - \boldsymbol{\omega}_{ie}^p - \boldsymbol{\omega}_{ip}^p) \times \mathbf{v}^p + \delta\mathbf{g}^n \end{bmatrix} \tag{18}$$

$$g(\mathbf{x},t) = \begin{bmatrix} 1 & \sin\phi_x \tan\phi_y & \cos\phi_x \tan\phi_y & 0 & 0 & 0 \\ 0 & \cos\phi_x & -\sin\phi_x & 0 & 0 & 0 \\ 0 & \sin\phi_x \sec\phi_y & \cos\phi_x \sec\phi_y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\mathbf{C}_b^p & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_b^p \end{bmatrix} \tag{19}$$

$\boldsymbol{\omega}(t)$ is the white noise of the inertial sensors with zero mean and covariance $\mathbf{Q}(t)$.

**(2) Measurement model**

The velocity errors are taken as observations for the filter. It can be obtained from the velocity errors between the SINS and the GPS. That is:

$$\begin{bmatrix} \delta v_N \\ \delta v_E \end{bmatrix} = \begin{bmatrix} \delta v_N^{SINS} \\ \delta v_E^{SINS} \end{bmatrix} - \begin{bmatrix} \delta v_N^{GPS} \\ \delta v_E^{GPS} \end{bmatrix} \tag{20}$$

Therefore, the measurement model is linear and can be written as:

$$\mathbf{y}(t) = \mathbf{H}\mathbf{x}(t) + \boldsymbol{\eta}(t) \tag{21}$$

Where $\mathbf{H} = \begin{bmatrix} I_{2\times 2} & 0_{2\times 8} \end{bmatrix}$ is the observation matrix and $\boldsymbol{\eta}(t)$ is white noise with zero mean and covariance $\mathbf{R}(t)$.

## 4. Simulation and Analysis

To evaluate the performance of the proposed simplified SGHF algorithm against other nonlinear filter techniques, a numerical simulation of alignment of SINS is performed.

### 4.1. Simulation Parameter

Simulated SINS parameter are listed in Table 2.

Table 2. Simulation Parameters

| parameter | value |
|---|---|
| Initial horizontal mis-alignment angel | $\phi_x = \phi_y = 45^{\circ}$ |
| Initial azimuth mis-alignment angel | $\phi = 60^{\circ}$ |
| Gyroscope bias | $\varepsilon_{cx} = \varepsilon_{cy} = \varepsilon_{cz} = 0.1^{\circ} / h$ |
| Accelerometer bias | $\nabla_{rx} = \nabla_{ry} = 0.2mg$ |
| Gyroscope drift | $\varepsilon_{rx} = \varepsilon_{ry} = \varepsilon_{rz} = 0.01^{\circ} / h$ |
| Accelerometer drift | $\nabla_{rx} = \nabla_{ry} = 0.05mg$ |
| Local latitude | North latitude $45^{\circ}$ |

| | |
|---|---|
| Gyroscope and accelerometer sampling period | 10ms |
| Navigation solver cycle | 10ms |
| Filtering cycle | 0.5s |

The initial values of the filter are follows:

$$\mathbf{x}_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{P}(0) = \mathrm{diag}\left[ (0.1m/s)^2 \quad (0.1m/s)^2 \quad (45^\circ)^2 \quad (45^\circ)^2 \quad (60^\circ)^2 \quad (0.02mg)^2 \quad (0.02mg)^2 \quad (0.1^\circ/h)^2 \quad (0.1^\circ/h)^2 \quad (0.1^\circ/h)^2 \right]^T$$

$$\mathbf{Q} = \mathrm{diag}\left[ (0.05mg)^2 \quad (0.05mg)^2 \quad (0.01^\circ/h)^2 \quad (0.01^\circ/h)^2 \quad (0.01^\circ/h)^2 \right]^T$$

$$\mathbf{R} = \mathrm{diag}\left[ (0.1m/s)^2 \quad (0.1m/s)^2 \right]^T$$

**4.2. Simulation Results Analysis**

According to parameters in Table 2, initial alignment of SINS based on EKF, UKF and simplified SGHF of level 2 and level 4 are simulated 100 times, total alignment time is 600 seconds. Average estimation error of misalignment angles are shown in Figure 1~Figure 3.

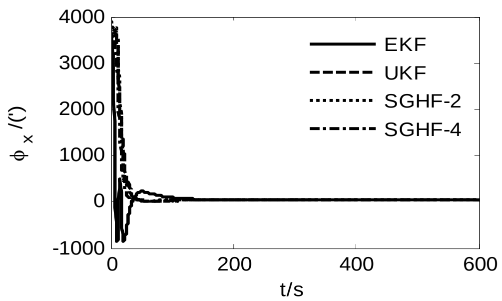


Figure 1. Estimation Error of North Angle

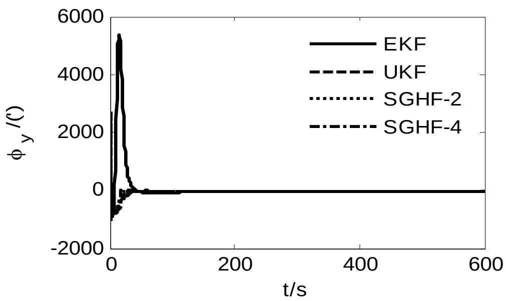


Figure 2. Estimation Error of East Angle

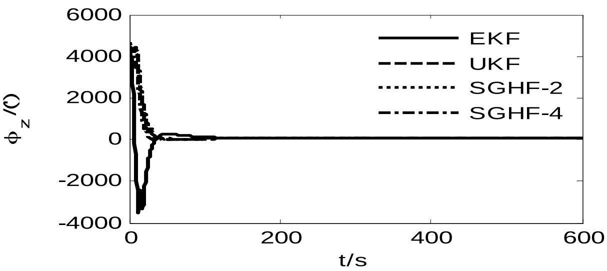


Figure 3. Estimation Error of Azimuth Angle

Table 3. Estimation Errors after 10 minutes

| | EKF | UKF | SGHF-2 | SGHF-4 |
|---|---|---|---|---|
| $\phi_x(')$ | 1.54 | 1.30 | 1.30 | 1.21 |
| $\phi_y(')$ | -1.37 | -1.21 | -1.20 | -1.14 |
| $\phi_z(')$ | 10.57 | 10.19 | 10.18 | 10.03 |

Estimation errors after ten minutes' alignment are summarized in Table 3.

It can be clearly seen from Figure 1 to Figure 3 and Table 3, in the estimation of mis-alignment angles, comparing with EKF, UKF and SGHF with accuracy level 2, SGHF with accuracy level 4 not only has property of fast convergence, but also has highest estimation accuracy.

## 5. Conclusion

In this paper, simplified gauss filter based on sparse grid gauss hermite quadrature is proposed. Comparing with conventional multidimensional tensor product based GHF, the simplified SGHF requires significantly fewer quadrature points while maintaining the performance of GHF.

The performance of simplified SGHF is demonstrated in a numerical simulation experiment of SINS initial alignment. The experiment results show that, comparing with other nonlinear filters such as EKF and UKF, simplified SGHF is easy to implement, especially simplified SGHF's filtering accuracy level is easy to tune according to demand, while EKF and UKF is difficult.

## References

[1] Fang, Jiancheng, Yang, Sheng. Study on Innovation Adaptive EKF for In-Flight Alignment of Airborne POS. *Instrumentation and Measurement, IEEE Transactions on.* 2011; 60(4): 1378-1388.

[2] Heiss, Florian, Winschel, Viktor. Likelihood approximation by numerical integration on sparse grids. *Journal of Econometrics.* 2008; 144(1): 62-80.

[3] Ito, Kazufumi, Xiong, Kaiqi. Gaussian filters for nonlinear filtering problems. *Automatic Control, IEEE Transactions on.* 2000; 45(5): 910-927.

[4] Jia, Bin, Xin, Ming, Cheng, Yang. Sparse Gauss-Hermite quadrature filter with application to spacecraft attitude estimation. *Journal of Guidance, Control, and Dynamics.* 2011; 34(2): 367-379.

[5] Julier, Simon J, Uhlmann, Jeffrey K. *New extension of the Kalman filter to nonlinear systems.* International Symposium on Aerospace/Defense Sensing, Simulation and Controls. Orlando. FL, USA. 1997: 121-126.

[6] Julier SJ, Durrant-Whyte HF. *Navigation and Parameter Estimation of High Speed Road Vehicles.* Robotics and Automation Conference. Nagoya. Japan. 1995: 101-105.

[7] DH Titterton, JL Weston. Strapdown Inertial Navigation Technology. London. UK: Institution of Electrical Engineers; 1997: 57-116.

[8] Wu, Yuanxin, Pan, Xianfei. Velocity/Position Integration Formula Part I: Application to In-Flight Coarse Alignment. *Aerospace and Electronic Systems. IEEE Transactions on.* 2013; 49(2): 1006-1023.