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Three Dimensional Electromagnetic Inversion based on Electric Dipole Source Multiple Locations Excitation

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Abstract

In this paper, we use integral equation and damped least-squares method to invert three dimensional abnormal body's electromagnetic field through horizontal electric dipole source multiple locations excitation. Multiple groups electromagnetic field data in different excitation and receiving points to be uniform consideration in once inversion, the Jacobian matrix is obtained and divided into linear terms and nonlinear terms. At last, we use the forward simulation data fit the measured data, and gradually modify geoelectricity model parameter values, ultimately achieve optimal fitting, gain three dimensional abnormal body's resistivity. Model test shows that the inversion algorithm has a fast convergence speed, less dependents on the initial value; the inversion result is accurate and reliable. It is an effective solution to the inversion failure caused by insufficient amount of data.

Keywords: *integral equation, inversion, multiple locations*

1. Introduction

With the rapid development of computer technology, integral equation method was introduced into geophysical electromagnetic methods. Hohmann. GW first calculated the electromagnetic field of a three-dimensional anomalous body in a homogeneous earth, on this basis, Wannamaker PE calculated the electromagnetic field in the layered earth [1, 2]. Different from finite element method and finite difference method which require a full space subdivision [3-5], integral equation method can be carried out only in the anomaly region, less calculation, especially for the calculation of three dimensional electromagnetic forward and inversion.

Now the common inversion methods of three dimensional electromagnetic field are the conjugate gradient method [6-11], least square method [12-14], α center method [15], neural network method [16] and so on. However under the influence of excitation source strength and receiving devices, these methods usually use only one group measurement data in once inversion, when the data is insufficient, easily lead to inversion fails.

In this paper, the integral equation method combination with the least square method, multiple groups measurement data in different excitation and receiving point to be uniform consideration in once inversion,which is an effective solution to the inversion failure caused by insufficient amount of data.

2. Basic Theory of Integral Equations

According to Maxwell's equations, integral equation theory and tensor green function of the electric and magnetic fields, integral equations for three dimensional abnormal body in homogeneous earth is [1]:

$$\mathbf{F}(\mathbf{r}) = \mathbf{F}_1(\mathbf{r}) + \int_V \Delta\sigma \tilde{\mathbf{G}}(\mathbf{r},\mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')dv \quad (1)$$

In the formula, $\mathbf{F}(\mathbf{r})$ is the total electric field or magnetic field at $\mathbf{r}$; $\mathbf{F}_1(\mathbf{r})$ is the primary electric field or magnetic field at $\mathbf{r}$ in the earth, $\Delta\sigma = \sigma - \sigma_b$ is the difference of three dimensional

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abnormal body conductivity σ and homogeneous earth conductivity σ_{b}, $\tilde{\mathbf{G}}(\mathbf{r},\mathbf{r}')$ is electric or magnetic tensor Green's function, $\mathbf{E}(\mathbf{r}')$ is the electric field inside the abnormal body.

For purposes of calculation, the three dimensional body is divided into N small cubic unit,and assuming that the resistivity is uniformly distributed in each of the split unit , also the electromagnetic field value within each unit value is approximately equal to the value of the center point ,then each split unit electric field can be expressed as:

$$\mathbf{E}(\mathbf{r}_m) = \mathbf{E}_1(\mathbf{r}_m) + \sum_{n=1}^{N} \Delta\sigma_n \int_{V_n} \tilde{\mathbf{G}}^{\mathrm{E}}(\mathbf{r}_m, \mathbf{r}') dv \cdot \mathbf{E}(\mathbf{r}_n) \quad m = 1, 2, ..., N \tag{2}$$

In above formula, $\mathbf{E}(\mathbf{r}_m)$ is the total electric field of the m-th unit of three dimensional abnormal body, $\mathbf{E}_1(\mathbf{r}_m)$ is the primary electric field of the m-th unit of three dimensional abnormal body, $\Delta\sigma_n = \sigma_n - \sigma_b$ is the difference of the n-th unit of three dimensional abnormal body conductivity σ_n and homogeneous earth conductivity σ_{b}, $\tilde{\mathbf{G}}^{E}(\mathbf{r}_m, \mathbf{r}')$ is electric tensor Green's function, $\mathbf{E}(\mathbf{r}_n)$ is the total electric field of the n-th small unit of three dimensional abnormal body. Solution of equations (2), to obtain the split unit's total electric field $\mathbf{E}(\mathbf{r}_m)$ of three dimensional abnormal body, then use split form of formula (1):

$$\mathbf{F}(\mathbf{r}) = \mathbf{F}_1(\mathbf{r}) + \sum_{n=1}^{N} \Delta\sigma_n \int_{V_n} \tilde{\mathbf{G}}(\mathbf{r}, \mathbf{r}') dv \cdot \mathbf{E}(\mathbf{r}_n) \tag{3}$$

We can obtain any point's electric or magnetic fields within the space.

3. Inversion Method Overview

This paper uses the method of literature given by [14], the partial differential coefficient sensitivity matrix is divided into linear and nonlinear, so reduce the amount of computation, improve accuracy, then by classical damped least-squares method, using the forward simulation data fit the measured data, and gradually modify geoelectricity model parameter values, ultimately achieve optimal fitting, gain three dimensional abnormal body' s resistivity.

The inversion process can be simply described as follows:

We use $\mathbf{f}_{si}$ represent the measured field values, $\mathbf{F}_i(\mathbf{r})$ represent the theory forward values, $\bar{X}$ represent the array resistivity parameters for each unit in abnormal body, the fitting degree between theoretical and measured values expressed as a relative deviation $\delta_i(\bar{X})$ therefore:

$$\delta_i(\bar{X}) = [\mathbf{f}_{si} - \mathbf{F}_i(\mathbf{r})] / \mathbf{f}_{si} \tag{4}$$

So the inversion fitting error of m points is:

$$\phi(\bar{X}) = \sum_{i=1}^{m} [\delta_i(\bar{X})]^2 \tag{5}$$

In the formula, the measured field and theoretical field have three components, and subscript i= 1,2…m,represents each observation location point or frequency point.

Since the forward function is non-linear, thus the deviation function and the fitting error is non-linear too, in order to overcome the difficulties of solving nonlinear equations, deviation function needs to be dealt with for the linear approximation. First the model parameters given a set of initial value of $\bar{X}^0$, then to Taylor $\delta_i(\bar{X})$ in $\bar{X}^0$ place, and ignore the order more than the second-order partial derivative, so:

$$\delta_i(\bar{X}) \approx \delta_i(\bar{X}^0) + \sum_{k=1}^{n} \frac{\partial \delta_i(\bar{X}^0)}{\partial x_k} \Delta x_k \tag{6}$$

In above formula, subscript k is the k-th model parameter, $\Delta x_k = x_k - x_k^0$ is model modification. Set $p_{ik} = \frac{\partial \delta_i(\bar{X}^0)}{\partial x_k}$, the fitting error expression is:

$$\phi(\bar{X}) = \sum_{i=1}^{m}[\delta_i(\bar{X}^0) + \sum_{k=1}^{n} p_{ik} \cdot \Delta x_k]^2 \tag{7}$$

At this time, fitting error is expressed as a multiple function of conductivity model's variables $\Delta x_1, \Delta x_2, ..., \Delta x_n$, conditions for its minimum value is:

$$\frac{\partial \phi(\bar{X})}{\Delta x_j} = 2\sum_{i=1}^{m}[\delta_i(\bar{X}^0) + \sum_{k=1}^{n} p_{ik} \cdot \Delta x_k] \cdot p_{ij} = 0 \tag{8}$$

Further deduced get that:

$$\sum_{i=1}^{m}\sum_{k=1}^{n} p_{ij} p_{ik} \Delta x_k = -\sum_{i=1}^{m} \delta_i(\bar{X}^0) \cdot p_{ij} \tag{9}$$

So are $j = 1, 2, ..., n$, linear equations can be derived to solve the model modification as follows:

$$\left(P^T P + \lambda\right) \cdot \Delta X = S \tag{10}$$

In above formula, P is Jacobian matrix,its elements are p_{ik} , $\Delta X = (\Delta x_1, \Delta x_2, ..., \Delta x_n)^T$, S is right-side vector , its elements are $s_j = -\sum_{i=1}^{m} \delta_i(\bar{X}) \cdot p_{ij}$, λ represents damping factor, which is a positive constant. We use (10) calculate resistivity modification value ΔX of abnormal body,and take $\bar{X} = \bar{X}^0 + \Delta X$ as a new model of initial parameters, recalculate the fitting error. So many iterations are taken until the fitting error is less than a small pre-given positive number ε , and $\bar{X}$ is the inversion result. The key to this method is how to obtain the Jacobian matrix P and right-side vector S in the condition of the source multiple locations excitation.

4. How to Obtain Jacobian Matrix and Right-side Vector in the Condition of Source Multiple Locations Excitation

Assuming a total of N group measurements, the number of receiving points is m in each group, and the locations of the source and receiving point are not identical. In accordance with the method described above, Jacobian matrix P_1 , P_2 , …, P_N of every measurement is obtained, they are all the matrix of m rows and N columns. The total Jacobian matrix Q of multiple measurements can be expressed by every measurement's Jacobian matrix as follows:

$$Q = \begin{pmatrix} P_1 \\ P_2 \\ \cdots \\ P_N \end{pmatrix} \tag{11}$$

Above formula, Q is a matrix of N · m rows and n columns.Total right-side vector after multiple measurements is $T = (t_1, t_2 ..., t_n)$,its element as follows:

$$t_j = -\sum_{i=1}^{N\cdot m} \delta_i(\bar{X}) \cdot p_{ij} \tag{12}$$

Q , T are substituted $P^T P$ 、 S in formula (10), then:

$$\left(Q^T Q + \lambda\right) \cdot \Delta X = T \tag{13}$$

Use formula (13) instead of (10), obtains a model parameter modifier, so N group measurements data are used in one inversion.

The element of Jacobian matrix in formula (13) is:

$$q_{ik} = \frac{\partial \delta_i(\bar{X}^0)}{\partial x_k} \tag{14}$$

Take formula (4) into (14), so:

$$q_{ik} = -\frac{1}{f_{si}} \frac{\partial \mathbf{F}_i}{\partial x_k} \tag{15}$$

And through (3):

$$\frac{\partial \mathbf{F}_i}{\partial \Delta\sigma_k} = \frac{1}{\partial \Delta\sigma_k} \left(\sum_{n=1}^{N} \Delta\sigma_n \left(\int_{V_n} \tilde{\mathbf{G}}(\mathbf{r}_i, \mathbf{r}') dv' \right) \cdot \mathbf{E}(\mathbf{r}_n) \right) \tag{16}$$

Let $\mathbf{p}_{in} = \left(\int_{V_n} \tilde{\mathbf{G}}(\mathbf{r}_i, \mathbf{r}') dv' \right) \cdot \mathbf{E}(\mathbf{r}_n)$, then above formula change into:

$$\frac{\partial \mathbf{F}_i}{\partial \Delta\sigma_k} = \mathbf{p}_{ik} + \left(\sum_{n=1}^{N} \Delta\sigma_n \frac{\partial \mathbf{p}_{in}}{\partial \Delta\sigma_k} \right) \tag{17}$$

Solving above equation need to first obtain the partial derivatives of the abnormal body unit at the center point of the electric field on the conductivity of each unit, and then superimposed find the partial derivatives of the total field on the conductivity of each unit. The previous $\mathbf{F}_i$, $\mathbf{p}_{in}$ has x, y, z three components, expand:

$$\frac{\partial (\mathbf{p}_{in})_x}{\partial \Delta\sigma_k} = \frac{\partial E_x(\mathbf{r}_n)}{\partial \Delta\sigma_k} \Gamma_{xixn} + \frac{\partial E_y(\mathbf{r}_n)}{\partial \Delta\sigma_k} \Gamma_{xiyn} + \frac{\partial E_z(\mathbf{r}_n)}{\partial \Delta\sigma_k} \Gamma_{xizn} \tag{18}$$

$$\frac{\partial (\mathbf{p}_{in})_y}{\partial \Delta\sigma_k} = \frac{\partial E_x(\mathbf{r}_n)}{\partial \Delta\sigma_k} \Gamma_{yixn} + \frac{\partial E_y(\mathbf{r}_n)}{\partial \Delta\sigma_k} \Gamma_{yiyn} + \frac{\partial E_z(\mathbf{r}_n)}{\partial \Delta\sigma_k} \Gamma_{yizn} \tag{19}$$

$$\frac{\partial (\mathbf{p}_{in})_z}{\partial \Delta\sigma_k} = \frac{\partial E_x(\mathbf{r}_n)}{\partial \Delta\sigma_k} \Gamma_{zixn} + \frac{\partial E_y(\mathbf{r}_n)}{\partial \Delta\sigma_k} \Gamma_{ziyn} + \frac{\partial E_z(\mathbf{r}_n)}{\partial \Delta\sigma_k} \Gamma_{zizn} \tag{20}$$

Above formula, $\frac{\partial E_x(\mathbf{r}_n)}{\partial \Delta\sigma_k}$, $\frac{\partial E_y(\mathbf{r}_n)}{\partial \Delta\sigma_k}$, $\frac{\partial E_z(\mathbf{r}_n)}{\partial \Delta\sigma_k}$ are partial derivatives of the abnormal body's n-th unit at the center point of the electric field on the conductivity of abnormal body's k-th unit, $\Gamma_{xixn} = \int_{V_n} \tilde{\mathbf{G}}_{xix'}(\mathbf{r}_i, \mathbf{r}') dv'$, $\Gamma_{xiyn} = \int_{V_n} \tilde{\mathbf{G}}_{xiy'}(\mathbf{r}_i, \mathbf{r}') dv'$, $\Gamma_{xizn} = \int_{V_n} \tilde{\mathbf{G}}_{xiz'}(\mathbf{r}_i, \mathbf{r}') dv'$ are x component of electric (or magnetic) tensor Green's function of abnormal body's n-th unit on ground i;

$\Gamma_{yixn} = \int_{V_n} \tilde{\mathbf{G}}_{yix'}(\mathbf{r}_i, \mathbf{r}')dv'$, $\Gamma_{yiyn} = \int_{V_n} \tilde{\mathbf{G}}_{yiy'}(\mathbf{r}_i, \mathbf{r}')dv'$, $\Gamma_{yizn} = \int_{V_n} \tilde{\mathbf{G}}_{yiz'}(\mathbf{r}_i, \mathbf{r}')dv'$ are y component of electric (or magnetic) tensor Green's function of abnormal body's n-th unit on ground i; $\Gamma_{zixn} = \int_{V_n} \tilde{\mathbf{G}}_{zix'}(\mathbf{r}_i, \mathbf{r}')dv'$, $\Gamma_{ziyn} = \int_{V_n} \tilde{\mathbf{G}}_{ziy'}(\mathbf{r}_i, \mathbf{r}')dv'$, $\Gamma_{zizn} = \int_{V_n} \tilde{\mathbf{G}}_{ziz'}(\mathbf{r}_i, \mathbf{r}')dv'$ are z component of electric (or magnetic) tensor Green's function of abnormal body's n-th unit on ground i.

The method according to formula (2), partial derivative matrix equation can be expressed as follows:

$$\frac{\partial \mathbf{E}(\mathbf{r}_m)}{\partial \Delta\sigma_k} = \left(\int_{V_k} \tilde{\mathbf{G}}^E(\mathbf{r}_m, \mathbf{r}')dv'\right)\cdot \mathbf{E}(\mathbf{r}_k) + \sum_{n=1}^{N} \Delta\sigma_n \left(\int_{V_n} \tilde{\mathbf{G}}^E(\mathbf{r}_m, \mathbf{r}')dv'\right)\cdot \frac{\partial \mathbf{E}(\mathbf{r}_n)}{\partial \Delta\sigma_k} \tag{21}$$

So by solving the above formula, we can get $\frac{\partial E_x(\mathbf{r}_n)}{\partial \Delta\sigma_k}$, $\frac{\partial E_y(\mathbf{r}_n)}{\partial \Delta\sigma_k}$, $\frac{\partial E_z(\mathbf{r}_n)}{\partial \Delta\sigma_k}$, and last by using formula (17)-(20), we can get a partial derivative matrix of observation points field value at any point on the earth's surface to the conductivity of each three dimensional abnormal body's unit.

5. Calculation Examples of Inversion

Resistivity inverse problem of three dimensional abnormal body as showed in Figure 1. A resistivity anomaly body in homogeneous earth, its central point coordinate is (0, 0,150); volume is $200\,m \times 200\,m \times 50\,m$, working frequency is 0.01 Hz, 0.1 Hz, 1 Hz, 10 Hz; horizontal electric dipole moment is $10\,\mathrm{A}\cdot m$; surrounding rock resistivity $\rho_0 = 100\,\Omega\cdot m$; true resistivity of abnormal body $\rho = 10\Omega\cdot m$; abnormal body is divided into $4\times4\times1$ small units, assuming resistivity is uniformly distributed in each unit and approximately equal to the value of the center point.

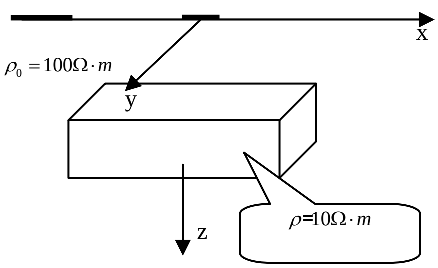

Figure 1. Three Dimensional Resistivity Model in Homogeneous Earth

First, the inversion uses only one group measurement data, set the source coordinate (-500, 0, 0), the receiving points are 20, initial resistivity values of the inversion model is 50 $\Omega\cdot m$, damping factor is 1, damping factor scaling by a factor of 10. Calculation of each small unit resistivity according to above algorithm, after 17 iterations, the inversion results as show in Figure 2, the fitting error is 1.63×10^{-7}, greater than a given threshold of 1×10^{-7}, and fitting error no longer decreases with iterations increases, the unit inverted resistivity value and true value quite different, inversion failure.

Analyses indicate that inversion fails due to lack of data. If the inversion with two groups measurement data, set the sources coordinates (-500, 0, 0) and (-300, 0, 0), each measurement are 20 points, a total of 40 points, the remaining parameters are the same as defined above. After 8 iterations, the inversion results as show in Figure 3, the fitting error is 1.98×10^{-8}, smaller than a given threshold of 1×10^{-7}, and unit inverted resistivity value converges to the true value, inversion success.

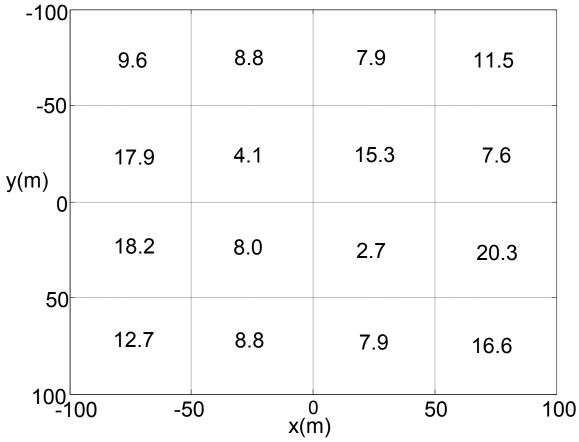

Figure 2. Resistivity Inversion Results of Abnormal Body using One Group Measurement Data

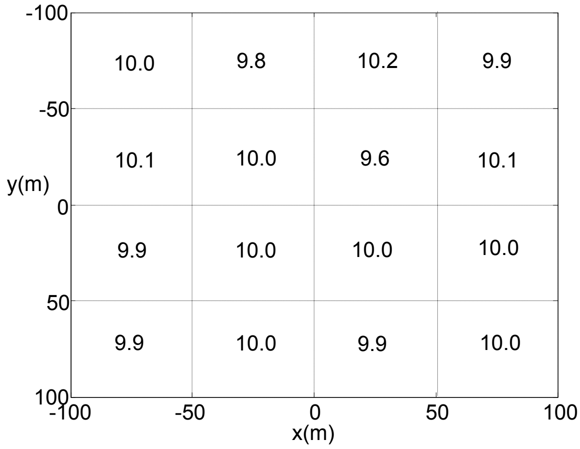

Figure 3. Resistivity Inversion Results of Abnormal Body using Two Groups Measurement Data

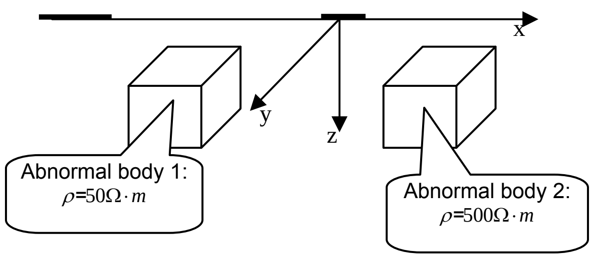

Figure 4. Two Three Dimensional Resistivity Models in Homogeneous Earth

The inversion algorithm given in this paper is applicable not only to individual resistivity anomaly body, but also to multiple abnormal bodies. In this case, model of the design as showed in Figure 4. Two resistivity anomaly bodies in homogeneous earth, their central point coordinates are (-200, 0,100) and (200, 0,100); true resistivity of abnormal bodies are $50\Omega \cdot m$ and $500\Omega \cdot m$;volume are all 200 $m \times$ 200 $m \times$ 100 m ; set the sources coordinates (-100, 0, 0) and

(100, 0, 0); working frequency is 0.01 Hz, 0.1 Hz, 1 Hz, 10 Hz; horizontal electric dipole moment is 10 $A \cdot m$; surrounding rock resistivity ρ_0 =100 $\Omega \cdot m$; abnormal bodies are divided into $2 \times 2 \times 1$ small units, assuming resistivity is uniformly distributed in each unit and approximately equal to the value of the center point; damping factor is 1, damping factor scaling by a factor of 10. Inversion using two groups measurement data (40 points). After 6 iterations, the fitting error is 4.47×10^{-8}, smaller than a given threshold of 1×10^{-7}, and unit inverted resistivity value converges to the true value, inversion success.The inversion results as show in Figure 5 and Figure 6 .

Through the above examples can be seen, in view of the complex situation of several abnormal bodies underground, the algorithm in this paper is still applicable, and the initial resistivity can converge to the true value quickly.

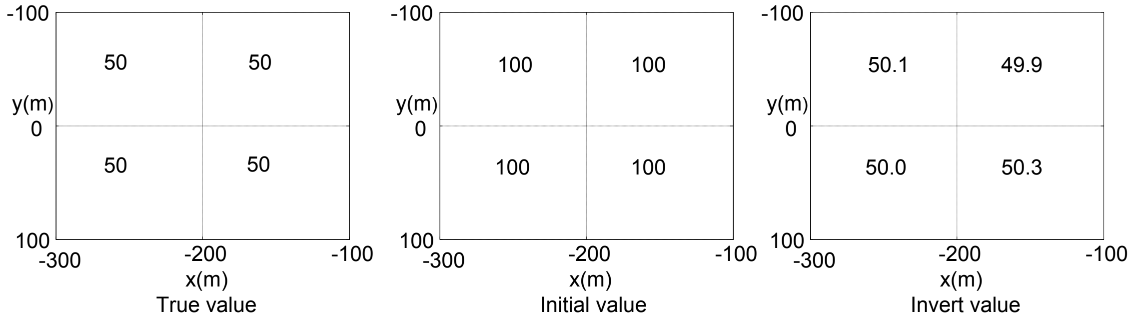

Figure 5. Resistivity Inversion Results of Abnormal Body 1

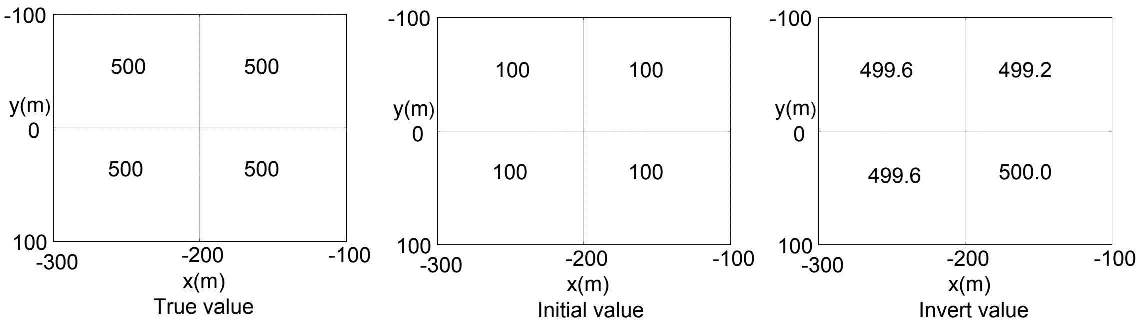

Figure 6. Resistivity Inversion Results of Abnormal Body 2

6. Conclusion

This paper has proposed an inversion algorithm of three dimensional resistivity anomaly body under the condition of horizontal electric dipole source multiple locations excitation in homogeneous earth. By model calculations, we have drawn the following conclusions:

(1) Integral equation is calculated only for abnormal body, more efficiency, and lays the foundation for the application of three dimensional resistivity inversion.

(2) Multiple groups of electromagnetic field data in different excitation and receiving point to be used in one inversion, successfully solve the parameter non-convergence problem caused by lack of data.

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