TELKOMNIKA, Vol.11, No.11, November 2013, pp. 6972~6977
e-ISSN: 2087-278X

A New Evidence Combination Method based on Consistent Strength

Changming Qiao, Shuli Sun*
Institute of Electronic Engineering, Hei Longjiang University (XU), Harbin, 150080, China,
Ph.: +86-13796204098
*Corresponding author, e-mail: hlju501@126.com

Abstract

A lot of practical applications show that DS (Dempster-Shafer) theory of evidence cannot deal with highly conflicted evidence. For this problem, Murphy proposed a method to solve it by modifying source model of evidence. But this method just calculated the mean of the conflicted evidence and didn't pay much attention on their correlation. Based on the combination method of Murphy, a new method to calculate the weighted average of the conflicted evidence using consistent strength is proposed by taking the correlation into account. The proposed combination method can combine highly conflicted evidences to be more satisfactory. A numerical example shows the effectiveness of the proposed method.

Keywords: *evidence theory, combination rule, evidence conflict, consistent strength*

1. Introduction

As an effective method of intelligent reasoning, the DS theory of evidence [1-2] has got more and more attention for many years and has achieved good results in many areas [3-4]. However, it is gradually discovered that there are some defects in the DS theory of evidence in practical applications [5-7]. On the one hand, a high conflict existing among evidences sometimes results in the failure of combination. On the other hand, it can cause an explosion of the number of focal elements and the computational cost is high. These two defects affect the application of the DS theory of evidence [8-10].

Considering these shortcomings, there have been two aspects to perfect the evidence theory. One is to modify the combination rule; the other is to modify the model of evidences at the source. The supporters of the former believe that the key to improve the evidence combination is the management of conflicts. The new combination rule needs to solve the conflict redistribution problem. So many methods have been proposed [11-13]. These rules can be seen as the special cases of reference [14]. One only needs to allocate different conflict in each subset to get different effects and have it work as a different combination rule. But the scholars who support the latter hold the opinion that the DS combination rule itself does not have errors, and the conflict of evidence should be preprocessed firstly when the degree of conflict is high, and then combined. In 2000, Murphy proposed a method to modify the source model of evidence [15]. This method calculates the mean of the basic probability assignment (BPA) of the evidence directly; and then combines iteratively with the Dempster combination rule. Compared with other methods, this combination rule can deal with conflicted evidences and converge quickly. But the shortcoming of the Murphy method is that the mean of the conflicted evidence is only calculated and the correlation among various evidences is not taken into account. So the conflict problem in some cases cannot be solved effectively. In reference [16], Deng Yong proposes an improved method which is mainly focused on the process of calculating the mean of evidence. This method firstly considers the correlation degree among various evidences; and then calculates the credibility of the evidence according to the distance among various evidences; finally, the credibility is used to calculate the weighted average of the evidence. Its experimental result shows that this method is more reasonable and more efficient. However, it needs to calculate a kind of distance among various evidences when handing the masses. Moreover, the calculation range includes the whole framework, so the computational complexity is relatively high. In reference [17], Zhang Jun put forward another method to reduce

Received June 4, 2013; Revised August 2, 2013; Accepted August 14, 2013

computational complexity when calculating the masses. However, these two methods allocate the masses of evidences due to the proportions of the distances of evidences and the sum of all distances. The changing range of weight coefficient is limited and the computational complexity of the algorithms is relatively high.

This paper also considers the problem from the view of modifying the source model of evidence based on the Murphy combination method. The method of calculating weighted average based on the consistent strength of the evidence is proposed. A new formula is applied to calculating the consistent strength of the evidence firstly; and then the support and credibility are calculated; at last, the weighted average of the evidence's BPA is calculated according to the credibility. The experimental result shows that this proposed method can deal with the conflicted evidences effectively and the combined result is further improved and very suitable for practical application.

2. DS Theory of Evidence

In 1967, Dempster first pointed out the definitions of upper and lower probabilities which do not satisfy the needs of addition. The opinion is more in line with the human habit of thinking. It simulates uncertainty by using a probability range rather than a single probability value. In 1968, Dempster discussed the generalization of statistical inference problems and presented the combined rule of two evidences. Based on the work of Dempster, Shafer proposed the evidence theory and extended it to the more general case. Their research results form the DS theory of evidence.

2.1. DS Combination Rule

The DS theory of evidence is based on a nonempty set Θ called the framework, and the elements in Θ meet the condition of incompatible. If $m: 2^{\Theta} \rightarrow [0,1]$ meets the following conditions:

$$\sum_{A \subset 2^{\Theta}} m(A) = 1 \qquad (1)$$

And $m(\Phi) = 0$, where Φ represents the empty set, $m(A)$ is called the BPA function. If $A \neq \Theta$, $m(A)$ represents the exact belief degree of the proposition A; if $A = \Theta$, $m(A)$ represents does not know how to allocate m; if $A \subset \Theta$ and $m(A) > 0$, A is called the focal element of m.

DS theory of evidence gives a useful combination rule to combine evidences provided by many evidence sources. Assuming that m_1 and m_2 are the BPA corresponding to the two evidences in the same framework, the focal elements are $A_1, A_2, \cdots, A_n$ and $B_1, B_2, \cdots, B_n$, respectively. Then the function $m: 2^{\Theta} \rightarrow [0,1]$ defined by the following formula is the BPA after the union of two evidences.

$$m(A) = \begin{cases} \dfrac{\sum_{A_i \cap B_j = A} m_1(A_i) m_2(B_j)}{1-K} & A \neq \Phi \\ 0 & A = \Phi \end{cases} \qquad (2)$$

Where, $K = \sum_{A_i \cap B_j = \Phi} m_1(A_i) m_2(B_j)$, it reflects the degree of conflict among various evidences.

2.2. Defects Analysis

It can be known from formula (2) that the closer K approach to 1, the greater the conflict among the evidences. When $K = 1$, the evidences are completely conflicted and cannot be combined. For example, the framework $\Theta = \{A, B, C\}$, the two evidences are m_1 and m_2 respectively, $m_1(A) = 0.99$, $m_1(B) = 0.01$, $m_2(B) = 0.01$, $m_2(C) = 0.99$. From an intuitive point of view, the degree of belief for focal element B is very low, so the result must not be the B, but the combined result of DS theory of evidence is $K = 0.9999$, $m(A) = m(C) = 0$, $m(B) = 1$, which is completely contrary to the result of human logic reasoning. Therefore, the DS theory of evidence cannot deal with the evidences with high conflict.

3. Improved Method

Assuming that the framework is Θ, the two evidences are E and F, and their BPA are m_i and m_j, respectively, A and B are focal elements, the consistent strength between E and F is defined by:

$$cs_{ij}(E,F) = \begin{cases} m_i(A)m_j(A) + m_i(B)m_j(B) & i \neq j \\ \begin{cases} m_i(A)m_j(A) + m_i(B)m_j(B) & m_i(A) \leq 0.5 \; or \; m_i(B) \leq 0.5 \\ 1 - m_i(A)m_j(A) + m_i(B)m_j(B) & m_i(A) > 0.5 \; or \; m_i(B) > 0.5 \end{cases} & i = j \end{cases} \quad (3)$$

Where, $cs_{ij}(E,F) \in (0,1]$.

The cs describe degree of consistency between evidences. Firstly, the correlation between evidences is considered when calculates cs; secondly, the case of $i = j$, i.e., the cs between one evidence and itself is analyzed in detail and it is the largest contribution of this formula. Generally, we may intuitively think that $cs_{ij}(E,F) = 1$ when $i = j$, but this way of thinking is not comprehensive enough. In this method, when $i = j$, we think that is similar to calculating the cs between evidence E and anther evidence F, only the BPA of E and F are equivalent. In this case, cs is a range of values rather than a constant, and its range is $0 < cs \leq 1$. Therefore, cs is more objective when describing the correlation between all evidences. So it will obtain more satisfactory results.

The implementation steps of the method based on cs adopted in this paper are as follows:

Step 1: calculate the consistent strength cs_{ij} between the evidences in accordance with the formula (3);

Step 2: all the consistent strengths cs_{ij} are expressed as a matrix form, that is,

$$CS = \begin{bmatrix} cs_{11} & cs_{12} & \cdots & cs_{1n} \\ cs_{21} & cs_{22} & \cdots & cs_{2n} \\ \vdots & \vdots & & \vdots \\ cs_{n1} & cs_{n2} & \cdots & cs_{nn} \end{bmatrix} \quad (4)$$

Step 3: the support of m_i is the sum of each row in the consistent strength matrix:

$$\mathrm{Sup}\,(m_i) = \sum_{j=1}^{n} cs_{ij}, \quad i = 1, 2, \cdots, n \quad (5)$$

Step 4: obtain the credibility of the evidence from the normalized support:

$$\mathrm{Crd}\,(m_i) = \frac{\mathrm{Sup}\,(m_i)}{\sum_{i=1}^{n}\mathrm{Sup}\,(m_i)} \quad (6)$$

Step 5: calculate the weighted average of evidence's BPA according to the credibility;

$$\mathrm{BPA}\,(m_i) = \sum_{i=1}^{n}\mathrm{Crd}\,(m_i) \times m_i\,(A) \quad (7)$$

Step 6: combine the evidences after weighted average with the DS combination rule. When the number of evidences is n, the times of combination are $n-1$.

4. Complexity Analysis

There are total 7 methods to be analyzed in this paper. The methods of Yager and Sun Quan belong to the way of modifying the combination rule, and the methods of Murphy, Deng Yong, Zhang Jun and the method proposed in this paper belong to the way of modifying the modeling of evidences at the source. So we mainly compare the complexity of the latter. Firstly, the method of Murphy only calculates the mean among various evidences, so the complexity is the lowest. But it cannot give the correct combination results when the number of evidences is few. Secondly, the method of Deng Yong considers the correlation between various evidences, so it needs to calculate the distances among various evidences. The distance got form this method is Jousselme distance [18]. The inner product of m_i, the inner product of m_j and the inner product between m_i and m_j are needed to be calculated when calculating the Jousselme distance d_{ij}, so the complexity is the highest. The method of Zhang Jun also needs to calculate the distance, but this kind of distance is the Euclidean distance. This method first calculates the mean of various evidences, and then calculates the Euclidean distance between each evidences and the mean. Although the complexity of this method is lower than Deng Yong, the computational cost is still relatively high. Finally, the method given in this paper also takes the correlation between various evidences into account when calculating the cs. Moreover, this method does not need to calculate any kinds of distance, and the formula (3) clearly shows that it only needs to perform a simple calculation.

5. Numerical Example

The numerical example includes 5 evidences and 3 identify targets, as shown in Table 1. $m(A)$, $m(B)$ and $m(C)$ represent the BPA of the target A, B and C. It can be seen from Table 1, for the target A, all evidences' BPA are high except evidence m_2, which is the typical problem of highly conflicted evidence. The combined result should be A according to the normal human thinking, but the result of the classical DS combination rule is that the belief of A is 0, which is in contradiction with the normal judgment.

Table 1. BPA of Three Targets for Four Evidences

	m_1	m_2	m_3	m_4	m_5
$m(A)$	0.5	0.0	0.55	0.55	0.55
$m(B)$	0.2	0.9	0.10	0.10	0.10
$m(C)$	0.3	0.1	0.35	0.35	0.35

Using the method of DS, Yager [11], Sun Quan [13], Murphy [15], Deng Yong [16], Zhang Jun [17] and the method proposed in this paper to combine the data in Table 1. The way to evaluate which combination method is better is mainly to see which method can make the belief of A higher as soon as possible. The results are shown in Table 2.

It can be seen from Table 2 that the method proposed in this paper significantly improves the belief of the focal element A, compared with other methods. Specifically, the DS method cannot get correct combination results. Yager combination rule gives the conflict to $m(X)$, so the belief of X is high, but others are very low. It will not cause an error of justice, but it also cannot help us to make the right judgment. The method of Sun Quan is a little better than that of Yager. The belief of A is not equal to 0, but the belief of X is still high. Moreover, with the increasing of the number of the evidences, the belief of A increases very slowly, so the combined results are not so good. When the number of evidences is 3, the method of Murphy cannot also provide the correct combination results, and the methods of Deng Yong and Zhang Jun give relatively correct combination results, but the belief of focal element A is not high. When four evidences are given, the methods of Murphy, Deng Yong and Zhang Jun give the correct combination results, but the beliefs of the focal element A are only 0.6027, 0.8060 and 0.8201. Observing the new method proposed in this paper, with the increasing of the number of evidences, we can find that combination result gradually increased from 0.3296 to 0.7161, 0.9662 and 0.9991; when all the 5 evidences are given, the belief of focal element B is 0. Thus, the method proposed in this paper is much better than the other six methods when dealing with the problem of evidences conflicted. The combination accuracy is greatly improved, especially when the number of evidences is more.

Table 2. Results of Different Combination Methods

	m_1, m_2	m_1, m_2, m_3	m_1, m_2, m_3, m_4	m_1, m_2, m_3, m_4, m_5
DS	$m(A)$=0 $m(B)$=0.8571 $m(C)$=0.1429	$m(A)$=0 $m(B)$=0.6316 $m(C)$=0.3684	$m(A)$=0 $m(B)$=0.3288 $m(C)$=0.6712	$m(A)$=0 $m(B)$=0.1288 $m(C)$=0.8772
Yager	$m(A)$=0 $m(B)$=0.18 $m(C)$=0.03 $m(X)$=0.79	$m(A)$=0 $m(B)$=0.0180 $m(C)$=0.0105 $m(X)$=0.9715	$m(A)$=0 $m(B)$=0.0018 $m(C)$=0.00368 $m(X)$=0.99452	$m(A)$=0 $m(B)$=0.00018 $m(C)$=0.00129 $m(X)$=0.99853
Sun Quan	$m(A)$=0.090 $m(B)$=0.377 $m(C)$=0.102 $m(X)$=0.431	$m(A)$=0.160 $m(B)$=0.201 $m(C)$=0.125 $m(X)$=0.486	$m(A)$=0.194 $m(B)$=0.160 $m(C)$=0.137 $m(X)$=0.509	$m(A)$=0.211 $m(B)$=0.138 $m(C)$=0.144 $m(X)$=0.507
Murphy	$m(A)$=0.1543 $m(B)$=0.7469 $m(C)$=0.0988	$m(A)$=0.3500 $m(B)$=0.5224 $m(C)$=0.1276	$m(A)$=0.6027 $m(B)$=0.2627 $m(C)$=0.1346	$m(A)$=0.7958 $m(B)$=0.0932 $m(C)$=0.1110
Deng Yong	$m(A)$=0.1543 $m(B)$=0.7469 $m(C)$=0.0988	$m(A)$=0.5816 $m(B)$=0.2439 $m(C)$=0.1745	$m(A)$=0.8060 $m(B)$=0.0482 $m(C)$=0.1482	$m(A)$=0.8909 $m(B)$=0.0086 $m(C)$=0.1005
Zhang Jun	$m(A)$=0.1543 $m(B)$=0.7469 $m(C)$=0.0988	$m(A)$=0.5767 $m(B)$=0.2522 $m(C)$=0.1711	$m(A)$=0.8201 $m(B)$=0.0380 $m(C)$=0.1419	$m(A)$=0.8984 $m(B)$=0.0049 $m(C)$=0.0967
This paper	$m(A)$= 0.3296 $m(B)$= 0.5116 $m(C)$= 0.1588	$m(A)$=0.7161 $m(B)$=0.1477 $m(C)$=0.1362	$m(A)$=0.9662 $m(B)$=0.0026 $m(C)$=0.0308	$m(A)$=0.9991 $m(B)$=0.0000 $m(C)$=0.0009

6. Conclusion

The classical DS theory of evidence cannot deal with the problem of highly conflicted evidences, especially when the degree of conflict is too high. The combined result is often contrary to the human reasoning. This paper proposes an improved combination method by modifying the source model of evidence. Based on the Murphy method, a new method of calculating weighted average with the consistent strength of the evidence is proposed. It has the advantage of simple calculation and high accuracy. The example shows that the combined results of the conflicting evidences are improved greatly compared with some existing methods. Moreover, the computation of the distances among various evidences is avoided, so it is more suitable for practical applications.

Acknowledgements

This work was supported in part by Natural Science Foundation of China under Grant NSFC-60874062, Program for High-qualified Talents under Grant Hdtd2010-03, and Electronic Engineering Province Key Laboratory.

References

[1] Dempster AP. Upper and Lower Probabilities Induced by a Multi-valued Mapping. *Annual Mach Statistics*. 1967; 38(4): 325-339.
[2] Shafer G. A Mathematical Theory of Evidence. *Princeton University Press, Princeton*. 1976; 121-138.
[3] He Luo, Shan-lin Yang, Xiao-jian Hu, et, al. Agent oriented intelligent fault diagnosis system using evidence theory. *Expert Systems with Applications.* 2012; 39(3): 2524-2531.
[4] Quan Wen, Wang Xiaodan, Wang Jian, etc. New combination rule of DST based on local conflict distribution strategy. A*cta Electronic Sinica*. 2012; 40(9): 1880-1884.
[5] Zadeh L. A simple view of the Dempster-Shafer Theory of evidence and its implication for the rule of combination. *Artificial Intelligence*. 1986; 7(2): 85-90.
[6] Han Deqiang, Han Congzhao, Deng Yong, etc. Weighted combination of conflicting evidence based on evidence variance. *Acta Electronica Sinica*. 2011; 39 (3A): 153-157.
[7] Nhan Nguyen-Thanh. Empirical distribution-based event detection in wireless sensor networks: An approach based on evidence theory. *IEEE Sensors Journal*. 2012; 12(16): 2222-2228.
[8] Deqiang Han, Yong Deng, Chongzhao Han. Weighted evidence combination based on distance of evidence and uncertainty measure. *Journal Infrared Millim. Waves*. 2011; 30(5): 396-400.
[9] Klirg J, Lewish W. Measuring Ambiguity in the Evidence Theory. *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*. 2008; 38(4): 995-999.
[10] Nhan Nguyen-Thanh, Insoo Koo. Evidence theory based cooperative spectrum sensing with efficient quantization method in cognitive radio. *IEEE Transaction on Vehicular Technology*. 2010; 60(1): 185-195.
[11] Yager RR. On the Dempster-Shafer framework and new combination rules. *Information Science*. 1989; 41(2): 93-137.
[12] Smets P. The combination of evidence in the transferable belief model. *IEEE Transaction on Pattern Analysis and Machine Intelligence*. 1990; 12(5): 447-458.
[13] Quan Sun, Xiuqing Ye, Weikang Gu. A new combination rules of evidence theory. *Acta Electronica Sinica*. 2000; 28(8): 117-119.
[14] Lefevree, Coloto, Vannoorenberghe P. Belief function combination and conflict management. *Information Fusion*. 2002; 3(3): 149-162.
[15] Murphy CK. Combining belief functions when evidence conflicts. *Decision Support Systems*. 2000; 29(1): 1-9.
[16] Yong Deng, WenKang Shi, ZhenFu Zhu. Combining belief functions based on distance of evidence, *Decision Support Systems*. 2004; 38(3): 489-493.
[17] Jun Zhang. Approaches to conflict evidence in D-S evidence theory and its applications. *Jiang Xi, Nan Chang University. China*. 2007.
[18] Jousselme AL, Grenier D, Bosse E. A new distance between two bodies of evidence. I*nformation fusion*. 2001; 2(1): 91-101.