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A study on uncertain fixed charge bi-objective 4-dimensional transportation problems under budget constraints

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ABSTRACT

Generally, in traditional transportation problems, single items are transported from a set of origins to a set of destinations in such a way that the transportation cost is minimized. To face the challenges that arise in modelling transportation problem various parameters are to be considered. In this work to provide an optimal transportation policy to the decision maker, we have studied a fixed-charge bi-objective multi-product 4-dimensional transportation problem with vehicle speed under budget constraints under uncertain environment. The objective of the proposed model is to have the maximum profit with the minimum time taken of transporting the goods. An equivalent deterministic model of the proposed model is obtained to deal the uncertain variable using expected value-chance constraint properties on uncertainty theory and its compromise solution is obtained by using the goal programming technique. A numerical example is discussed to provide more clarity on the proposed model.

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1. INTRODUCTION

Transportation problems are important in transportation planning for organizations. It is essential for businesses to find ways to be profitable and efficient in the global market. Also, during transit, given the availability of numerous distinct transfer methods, moving two or more objects at once can be more advantageous and transforms a transportation problem (TP) into solid transportation (STP) problem. Taking into account different routes and modes of transportation might open up more alternatives and chances for the organization to run a successful, profitable operation. In general, the distance between origin and destination is not taken into consideration in TPs as the cost and time will remain constant for a given route. But in real life, more often than not, we have multiple routes to choose from between origin and destination. The choice of routes will often play a big role, as some parts may be smooth while some may be uneven, affecting the operation of vehicles and inducing the breakability factor. Thus, when the route is considered, the TP is called a four-dimensional TP (4DTP).

In situations wherein complete data is not available, the existing TP models in various environments fail. Liu [1] proposed the uncertainty theory to handle imprecise data. Uncertainty theory is one of the areas of mathematics for modelling the degree of belief and is used in many mathematical models such as uncertain programming, logic, statistics, graphs, and finance. The level of belief that an uncertain event will occur is measured by an uncertain measure. Liu [1] also introduced the use of random uncertainty variables and random measures to handle uncertainty and randomness at the same time. Liu [1] then presented insecure random programming for modelling optimization problems involving multiple random variables. Gao [2] continuously proposed certain properties based on uncertainties. Ghasemi and Safi [3] introduced an uncertain linear fractional programming problem and presented several ways to transform an uncertain optimization problem into an equivalent deterministic problem.

When multiple objectives are present in an STP, it is called a multi-objective STP (MOSTP). Many researchers have made contributions in this field, like Lee and Moore [4]. Gao *et al.* [5] had built new types of uncertain programming models like the limited budget model and the model where the fixed load cost and transportation cost are non-deterministic.

As per the knowledge of the authors, the fixed charge bi objective four-dimensional transportation problem with budget constraints in an uncertain environment has not yet been researched or addressed yet. In order fill the gap in the research, we sought to propose an uncertain fixed charge bi-objective 4-dimensional transportation problem under budget constraints (UFCBO4DTPBC). The ultimate aim of this model is to maximise the profit and minimise the transportation charge. The objective function of the above model considered fixed costs, destination budgets, speeds of various means of transportation, and slowdowns due to poor road conditions for the first time ever.

The primitive transportation problem (TP) was introduced by Hitchcock [6]. It is well known as an optimization problem that includes a lot of real-life applications. Some of the other optimization problems like the TP are traffic control problems and the house purchase recommendation system [7], [8]. The primitive TP involves only two constraints, the demand constraint and the availability constraint. The objective of the TP is minimizing the transportation cost. In fact, according to TP, different types of TP have been studied and solved by numerous researchers from different perspectives [9], [10]. When there is more than one mode of transportation available, a new transport constraint is introduced and the problem becomes 3 dimensional (3D) TP or solid TP (STP). A more advanced solution for STP was given by Arsham and Kahn [11] in 1962. Further research and advancements were made as a variety of models in both crisp and fuzzy environments were presented [12], [13].

When multiple goods with STPs are to be transported, it becomes a multi-item STP (MISTP), which was worked upon by many researchers like Dalman *et al*. [14], Dalman [15], and Gupta *et al.* [16]. Various types of TPs have been solved by researchers in different environments like uncertain, fuzzy, crisp, rough, and random. Zimmerman [17] proposed the fuzzy programming method for solving MOTP, which was extended to multi-objective solid TP by Bit *et al.* [18]. Ohja *et al*. [19] applied a genetic algorithm by taking objective coefficients as fuzzy numbers. The best candidate method to obtain the optimal solution of mixed constraint TP was presented by Pathade *et al*. [20]. Rani and Gulati [21] presented a method for obtaining the best compromise for the completely fuzzy, multi-object, multi-object fixed transport problem (FFMOMISTP).

Liu and Chen [22] proposed the uncertainty goal programming method for solving multi-objective uncertain programming problems. The techniques to solve multi-level uncertain programming problems were developed by Liu and Yao [23]. Zhou *et al.* [24] and Zhong *et al*. [25] proposed the interactive satisfied methods and compromise programming models with uncertain multi-objective problems. An expected constrained model to solve the uncertain STP was given by Cui and Sheng [26]. Guo *et al*. [27] later extended the TP by studying the situation where the cost was uncertain and the supplies are random. Yang *et al*. [28] solved the fixed charge STP by using the type-2 uncertain optimization method. Dalman *et al.* [14] proposed the technique to solve the multi-objective multi-item STP under an uncertain environment (UE).

Table 1 contains observations made on the existing literature on TP under uncertain environments based on their various types. In this investigation, we observed that, the four-dimensional TPs have not been studied very much by researchers yet. From Table 1, a clear gap can be observed in terms of developing a model on fixed charge bi objective 4-diemensional transportation problem with the consideration of the parameters such as selling price, purchase price, warehousing cost, procurement cost, cost of fixed charge, various conveyances, different routes, the distance between the origins and destinations, budget, capacity of the vehicle, rate of breakability of the product, speed of vehicles, rate of disturbances due to the road conditions and loading and unloading time for goods. The most important aspect is that this study considers all the parameters except the rate of breakability, the distance between the various routes, speed of the vehicles and rate of disturbances the rate of breakability as uncertain variables.

The research work is listed as shown in: in section 1, we have presented an introduction and literature review. We have reviewed some necessary definitions and theorems on uncertain variables, notations utilized within the article, the mathematical model of UFCBO4DTPBC, and the deterministic model of the proposed model are given in section 2. Section 3 provides the methods for solving UFCBO4DTPBC. The result discussion and conclusion have been considered in sections 4 and 5 respectively.

Table 1. Existing model and proposed model

Reference	Different kinds of TP			Item		Objective			Factor		Kind of environment
	2	3	4	single	Multi	single	multi	fixed	Vehicle speed	Budget constraint	
Hitchcock [6]	✓	×	×	✓	×	×	×	×	×	×	Crisp
Hirsch and Dantzig [29]	✓	×	×	✓	×	✓	×	✓	×	×	Crisp
Verma *et al*. [30]	✓	×	×	✓	×	×	✓	×	×	×	Fuzzy
Ojha *et al*. [19]	✓	×	×	×	✓	×	✓	×	×	✓	Fuzzy
Yang and Liu [13]	×	✓	×	✓	×	✓	×	✓	×	×	Fuzzy
Gupta *et al*. [16]	×	✓	×	×	✓	✓	×	✓	×	×	Fuzzy
Halder *et al*. [31]	×	×	✓	×	✓	✓	×	✓	×	✓	Rough
Pramanik *et al.* [32]	✓	×	×	✓	×	✓	×	✓	×	×	Gaussian type 2 fuzzy
Dalman *et al.* [14]	×	✓	×	×	✓	×	✓	×	×	×	Fuzzy
Das *et al*. [33]	×	✓	×	×	×	×	✓	✓	×	×	Type-2 fuzz
Bera *et al*. [34]	×	×	✓	×	✓	✓	×	×	×	×	Rough
Sahoo *et al*. [35]	×	×	✓	×	✓	×	✓	×	×	×	Type-1 uncertain variable
Sahoo *et al*. [36]	×	×	✓	×	✓	✓	×	✓	×	×	Type-2 uncertain variable
Proposed work	×	×	✓	×	✓	×	✓	✓	✓	✓	Uncertain variable

2. RESEARCH METHOD

2.1. Basic concepts

This section contains some concepts on uncertainty theory that have been used in the research work. Uncertain measures, uncertainty distribution, Independent uncertain variable and normal uncertain variable definitions are ordered to provide context to the solution of this proposed model. Also, few theorems on expected value of the uncertain variable are given.

2.1.1. Definition

Adlakha and Kowalski [9] Let $\mathcal{L}$ be a σ- algebra of collection of events Λ of a universal set Γ. A set function M is said to be an uncertain measure (UM) defined on the σ- algebra where M{Λ} indicates the belief degree with which we believe that the event will happen and satisfies the as shown in four axioms:

a. For the universal set Γ, we have:

$$\mathrm{M}\{\Gamma\} = 1 \tag{1}$$

b. For any event Λ, we have:

$$\mathrm{M}\{\Lambda\} + \mathrm{M}\{\Lambda^c\} = 1 \tag{2}$$

c. For every countable sequence of events Λ_1, Λ_2,... we have:

$$\mathrm{M}\left\{\cup_{j=1}^{\infty} \Lambda\right\} \leq \sum_{j=1}^{\infty} \mathrm{M}\left\{\Lambda_j\right\} \tag{3}$$

d. Let $(\Gamma_j, \mathcal{L}_j, \mathrm{M}_j)$ be uncertainty spaces for. The product UM measure is a UM holds:

$$\mathrm{M}\left\{\prod_{j=1}^{\infty} \Lambda_j\right\} = \wedge_{j=1}^{\infty} \mathrm{M}\left\{\Lambda_j\right\} \tag{4}$$

where $\Lambda_j \in \mathcal{L}_j$ for $j = 1,2, \dots, \infty$.

The uncertainty distribution[UD] $\rho(y)$ of a UV ξ for any real number y is defined by:

$$\rho(y) = \mathrm{M}\{\xi \leq y\} \tag{5}$$

for any RUD ρ(y) of an UV ξ, ρ^{-1}(y) is called an inverse uncertainty distribution (IUD) of ξ and it exists on (0, 1).

2.1.2. Definition

Adlakha and Kowalski [9] the UV ξ_t $(t = 1,2,\dots,T)$ are said to be independent if;

$$\mathrm{M}\{\cap_{t=1}^{T}(\xi_t \in B_t)\} = \wedge_{t=1}^{T}\mathrm{M}(\xi_t \in B_t) \quad (6)$$

where B_t $(t = 1,2,..,T)$ are the Borel sets with real numbers.

2.1.3. Theorem

The RUD of independent UV ξ_t $(t = 1,2,\dots,T)$are ρ_t $(t = 1,2,\dots,T)$ respectively [9]. If the function $h(y_1, y_2, \dots, y_t)$ is strictly increasing and strictly decreasing with respect to $y_1, y_2, \dots, y_s$ and $y_{s+1}, y_{s+2}, \dots, y_t$ respectively then the uncertain variable $\xi = h(\xi_1, \xi_2, \dots, \xi_s, \dots, \xi_t)$ has an IUD:

$$\rho^{-1}(\gamma) = h(\rho_1^{-1}(\gamma), \rho_2^{-1}(\gamma), \dots, \rho_s^{-1}(\gamma), f(\rho_{s+1}^{-1}(1-\gamma), \rho_{s+2}^{-1}(1-\gamma), \dots, \rho_t^{-1}(1-\gamma)) \quad (7)$$

the expected value of UV ξ is given by:

$$E(\xi) = \int_0^{\infty}\mathrm{M}\{\xi \geq y\}\,dy - \int_{-\infty}^{0}\mathrm{M}\{\xi \leq y\}\,dy \quad (8)$$

this is valid only if at least one of the integrals is finite.

2.1.4. Theorem

Let $\rho_t(t = 1,2,\dots,T)$ be RUD of independent ξ_t $(t = 1,2,\dots,T)$ with respectively [37]. If the function $h(y_1, y_2, \dots, y_t)$ is strictly increasing and strictly decreasing w.r.to $y_1, y, \dots, y_s$ and $y_{s+1}, y_{s+2}, \dots, y_t$ respectively, then:

$$E(\xi) = \int_0^1 h(\rho_1^{-1}(\gamma), \dots, \rho_s^{-1}(\gamma), \rho_{s+1}^{-1}(1-\gamma), \dots, \rho_t^{-1}(1-\gamma))d\gamma \quad (9)$$

from this theorem, we have,

$$E(\xi) = \int_0^1 \rho^{-1}(\gamma)d\gamma \quad (10)$$

here ξ is a UV with RUD ρ.

2.1.5. Definition

The distribution function of a normal uncertain variable [NUV] is [9]:

$$\rho(x) = \left[1 + exp^{\left[\frac{\pi(\mu-x)}{\sigma\sqrt{3}}\right]}\right]^{-1}, x \geq 0 \quad (11)$$

and represented by $N(\mu, \sigma)$; $\mu, \sigma \in R\ with\ \sigma > 0$. The IUD and the expected value of N(μ,σ) are given as:

$$\rho^{-1}(\gamma) = \mu + \frac{\sigma\sqrt{3}}{\pi}ln\frac{\gamma}{1-\gamma} \quad (12)$$

$$E[\xi] = \mu \quad (13)$$

2.2. Nomenclature

The proposed model was developed using notations:
m indexed for origins.
d indexed for destinations.
w indexed for the mode of transport.
v indexed for transportation routes.
g indexed for goods.
$\tilde{Z}_n$ uncertain objective functions, where n=1,2.
$\tilde{P}_{mg}$ the unit purchasing price of g^{th}- good at m^{th}- origin.
$\tilde{S}_{dg}$ the unit selling price of g^{th}- good at d^{th}- destination.
$\widetilde{W}_{mg}$ the warehousing cost of g^{th}- good at m^{th}- origin.

$\widetilde{Pro}_{mg}$ the unit procurement cost of g^{th}- good at m^{th}- origin.
$\tilde{C}_{mdwvg}$ the unit transportation cost of g^{th} good from m^{th}- origin to d^{th}- destination by w^{th}- transport via v^{th}- road per unit distance.
β_{mdwvg} the rate of breakability per unit distance of g^{th} good from m^{th}- origin by w^{th}- transport to d^{th}- destination via v^{th}- road.
D_{mdv} distance from m^{th}- origin to d^{th}- destination via v^{th}- road.
V_w speed of w^{th}- transport
γ_{mdv} the rate of disturbance of the speed due to v^{th} road from m^{th}origin to d^{th} destination.
$\tilde{\delta}_{mdwg}$ loading and unloading time of g^{th} good with respect to the transportation activity from m^{th}origin to d^{th} destination by w^{th}- transport.
$\tilde{a}_{mg}$ quantity of g^{th} good available at d^{th}origin.
$\tilde{b}_{dg}$ the requirement of g^{th} good at d^{th}destination.
$\tilde{e}_w$ the capacity of w^{th}- transport.
$\tilde{f}_{mdwv}$ the fixed charge, which must be paid when the transportation activity happens from m^{th}origin to d^{th} destination via v^{th}- road by w^{th}- transport.
$\widetilde{Bud}_d$ total budget at the d^{th}destination.
e_{mdwvg}, y_{mdwv} binary indicator.
N_n^- negative deviational value.
P_n^+ positive deviational value.

2.3. Mathematical formulation of UFCBO4DTPBC

The proposed model UFCBO4DTPBC is formulated as follows. Let there be M origins O_m(m=1,2,…M), N demands D_d (d=1,2,……N), R roads Q_v (v=1,2….R), G goods P_g (g=1,2,….G), W conveyances E_w (w=1,2,….W). The model aims to have the maximum profit with the minimum time taken on transportation. TP cannot be applied in real life as there are a lot of uncertainties involved like raw material availability, variation in demand at different destinations and fluctuation in transportation cost and unavailability of proper data on the above factors. The following are the drawbacks that don't allow traditional transportation problems to be extended further as stated by the authors Dimoka *et al*. [38] and Carlton [39].

a. In the consideration of the time factor, the product's availability may be uncertain.
b. Decision makers [DM] do not always know the cost of transportation.
c. Market demand can be uncertain.
d. The cost of a fixed charge that must be paid may be indeterminate when a certain quantity of product is transferred from m^{th}- origin to d^{th}- destination using transport via v^{th}- road.
e. The decision-maker may be unaware of the possibility of shipping goods being damaged in transit.

In the current situation, estimating the exact set of related parameters is not so easy. Therefore, by considering all the parameters that may be included in the actual situation as UV, we formulate the UFCBO4DTPBC model at a vehicle speed within the budget and under budget constraints.

Here, we have introduced the model by involving both the breakability rates and fixed charge as:

$$\text{Max } \bar{Z}_1 = \sum_{m,d,w,v,g}[(1-\beta_{mdwvg}\ d_{mdv})\tilde{S}_{dg} - \widetilde{W}_{mg} - \tilde{P}_{mg} - \widetilde{Pro}_{mg} - \tilde{C}_{mdwvg}\, d_{mdv}]x_{mdwvg} - \sum_{m,d,w,v,g}(\tilde{f}_{mdwv}\, e_{mdwvg})$$

$$\text{Min } \bar{Z}_2 = \sum_{m,d,w,v} \frac{d_{mdv} \cdot y_{mdwv}}{V_w\,(1-\gamma_{mdv})} + \sum_{m,d,w,v,g}(\tilde{\delta}_{mdwg} \cdot x_{mdwvg})$$

where,

$$y_{mdwv} = \begin{cases} 1 \text{ if } \sum_g x_{mdwvg} > 0 \\ 0 \text{ if } \sum_g x_{mdwvg} = 0 \end{cases}$$

$$e_{mdwvg} = \begin{cases} 1 \text{ if } \sum_g x_{mdwvg} > 0 \\ 0 \text{ if } \sum_g x_{mdwvg} = 0 \end{cases}$$

subject to

$$\sum_{d,w,v} x_{mdwvg} \le \tilde{a}_{mg}, \forall \text{ m} = 1 \text{ to M } and \text{ g} = 1 \text{ to G}.$$

$\sum_{m,w,v}(1-\beta_{mdwvg}\,.\,d_{mdv}))x_{mdwvg} \geq \tilde{b}_{dg}, \forall$ d = 1 to D and g = 1 to G .

$\sum_{m,d,v,g} x_{mdwvg} \leq \tilde{e}_w, \forall$ w = 1 to W .

$\sum_{d,w,v}\left(\tilde{P}_{mg}+\widetilde{W}_{mg}+\widetilde{Pro}_{mg}+\tilde{C}_{mdwvg}\right)x_{mdwvg}+\sum_{m,d,w,v,g}\tilde{f}_{mdwv}\,.\,y_{mdwv} \leq \widetilde{Bud}_d\ , \forall\ d=1$ to D.

$$x_{mdwvg} \geq 0, \forall\ m,d,w,v,g. \quad (17)$$

2.3.1. Definition

A feasible solution $Y^* = \{y^*_{mdwvg}\} \in S$ is an efficient (no dominated) solution for under budget constraints if there UFCBO4DTPBC does not exist another $\mathrm{Y} = \{\mathrm{y}_{\mathrm{mdwvg}}\} \in \mathrm{S}$ such that $\mathrm{Z_n(Y)} \leq \mathrm{Z_n(Y^*)}, 1 \leq \mathrm{n} \leq \mathrm{N}$ and $\mathrm{Z_n(Y)} \neq \mathrm{Z_n(Y^*)}$ for some l,1 ≤ l ≤ N.

2.4. Equivalent and formulation of deterministic model of UFCBO4DTPBC

This section explains the equivalent deterministic model for UFCBO4DTPBC. Suppose that $\widetilde{W}_{mg}$, $\tilde{P}_{mg}$, $\widetilde{Pro}_{mg}$, $\tilde{C}_{mdwvg}$, $\tilde{f}_{mdwv}$, $\tilde{a}_{mg}$, $\tilde{b}_{dg}$, $\tilde{e}_w$, $\widetilde{Bud}_d$ are uncertain variables with RUD θ_{mg}, γ_{mg}, η_{mg}, λ_{mdwvg}, ρ_{mdwv}, ψ_{mg}, ϕ_{bg}, χ_{mg} and φ_d respectively.

Using the expected–chance constraint method for NUVs and their properties, the equivalent deterministic model of UFCBOMP4DTPS under budget constraints is given in (18):

$$\text{Max } Z_1 = E\big[\textstyle\sum_{m,dw,v,g}[(1-\beta_{mdwvg}\ d_{mdv})\tilde{S}_{dg}-\widetilde{W}_{mg}-\tilde{P}_{mg}-\widetilde{Pro}_{mg}-\tilde{C}_{mdwvg}\,d_{mdv}]x_{mdwvg} - \textstyle\sum_{m,d,w,v,g}(\widetilde{f}_{mdwv}\,e_{mdwvg})\big]$$

$$\text{Min } Z_2 = E[\textstyle\sum_{m,d,w,v}\frac{d_{mdv}\cdot \boldsymbol{y_{mdwv}}}{V_w\,(1-\gamma_{mdv})}+\sum_{m,d,w,v,g}\delta_{mdwg}\ \ .\,x_{mdwvg}]$$

where,

$$e_{mdwvg} = \begin{cases} 1\ if\ \sum_g x_{mdwvg} > 0 \\ 0\ if\ \sum_g x_{mdwvg} = 0 \end{cases}$$

$$y_{mdwv} = \begin{cases} 1\ if\ \sum_g x_{mdwvg} > 0 \\ 0\ if\ \sum_g x_{mdwvg} = 0 \end{cases}$$

subject to,

$\sum_{d,w,v} x_{mdwvg} \leq \psi^{-1}_{mg}\,(1-\alpha_1), \forall$ m = 1 to M and g = 1 to G

$\sum_{m,w,v}(1-\beta_{\ mdwvg}\,.\,d_{mdv}))x_{mdwvg}] \geq\ \phi^{-1}_{dg}\,(\alpha_2), \forall$ d = 1 to D and g = 1 to G

$\sum_{m,d,v,g} x_{mdwvg} \leq \chi^{-1}_w(1-\alpha_3),, \forall$ w = 1 to W

$\sum_{d,w,v,g}\left(\phi^{-1}_{mg}\,(\alpha_3)+\theta^{-1}_{wg}\,(\alpha_4)+\eta^{-1}_{mg}\,(\alpha_5)+\lambda^{-1}_{mdwvg}\,(\alpha_6)\right)x_{mdwvg}+\rho^{-1}_{mdwv}\,(\alpha_7)\,.\,e_{mdwvg} \leq \varphi^{-1}_d(\alpha_8), \forall\ d = 1$ to D

$$x_{mdwvg} \geq 0, \forall\ m,d,w,v,g. \quad (18)$$

here, α_q, q=1 to 9, are predetermined chance levels and $0 \leq \alpha_q \leq 1$.

2.4.1. Deterministic model with NUV definition

By using the definition (2.10) on (18), we have:

$$Max\ Z_1 = E\big[\textstyle\sum_{m,dw,v,g}[(1-\beta_{mdwvg}\ d_{mdv})\tilde{S}_{dg}-\widetilde{W}_{mg}-\tilde{P}_{mg}-\widetilde{Pro}_{mg} - \tilde{C}_{mdwvg}\,d_{mdv}]x_{mdwvg}\ - \textstyle\sum_{m,d,w,v,g}\tilde{f}_{mdwv}\,e_{mdwvg}\big]$$

$$\text{Min } \boldsymbol{Z_2} = E[\sum_{m,d,w,v} \frac{d_{mdv} y_{mdwv}}{V_w\,(1-\gamma_{mdv})} + \sum_{m,d,w,v,g} \delta_{mdwg} \;.\, x_{mdwvg}]$$

where,

$$e_{mdwvg} = \begin{cases} 1 \; if \; \sum_g x_{mdwvg} > 0 \\ 0 \; if \sum_g x_{mdwvg} = 0 \end{cases}$$

$$y_{mdwv} = \begin{cases} 1 \; if \; \sum_g x_{mdwvg} > 0 \\ 0 \; if \sum_g x_{mdwvg} = 0 \end{cases}$$

subject to,

$$\sum_{d,w,v} x_{mdwvg} \le e_{mg} + \frac{\sigma_{mg}}{\pi} * \sqrt{3}\; \log \frac{1-\alpha_{mg}}{\alpha_{mg}} \;, \forall \text{ m} = 1 \text{ to M } and \text{ g} = 1 \text{ to G} .$$

$$\sum_{m,w,v} (1 - \beta_{\,mdwvg} \,.\, d_{mdv}))x_{mdwvg}] \ge \; e_{dg} + \frac{\sigma_{dg}}{\pi} * \sqrt{3}\; \log \frac{\alpha_{dg}}{1-\alpha_{dg}} \;, \forall \text{ d} = 1 \text{ to D and g} = 1 \text{ to G.}$$

$$\sum_{m,d,v,g} x_{mdwvg} \le e_{mg} + \frac{\sigma_{mg}}{\pi} * \sqrt{3}\; log \frac{1-\alpha_{mg}}{\alpha_{mg}} \;, \forall \text{ w} = 1 \text{ to W.}$$

$$\sum_{d,w,v,g} \left(\phi_{mg}^{-1}(\alpha_3) + \theta_{wg}^{-1}(\alpha_4) + \eta_{mg}^{-1}(\alpha_5) + \lambda_{mdwvg}^{-1}(\alpha_6) \right) x_{mdwvg} + \rho_{mdwv}^{-1}(\alpha_7) \,.\, e_{mdwvg} \le \varphi_d^{-1}(\alpha_8), \forall d$$

$$x_{mdwvg} \ge 0, \forall\, m, d, w, v, g. \quad (19)$$

here, α_q, q=1 to 9, are predetermined chance levels and $0 \le \alpha_q \le 1$.

3. METHOD

3.1. Goal programming approach

In situations involving multiple objectives, the Charnes and Cooper [40] proposed the GPT to obtain a satisfactory solution. The GPT has been further investigated and developed by many authors such as Chang [41], and Mohamed [42] proposed the fuzzy GP approach for solving MOTP, which was used later by Zangiabadi and Maleki [43], [44] to solve MOTP with linear membership functions as well as nonlinear membership functions. Minimizing the distance between $Z = (Z_1, Z_2, Z_3, \dots \dots Z_n)$, and aspiration (or) target level $\bar{Z} = (\bar{Z}_1, \bar{Z}_2, \bar{Z}_3, \dots \dots \bar{Z}_n)$, which are set by the DM, is the objective of GP. We introduce the negative and positive deviational variables to apply GP here.

$$P_n^+ = \max\,(0, Z_n - \bar{Z}_n)$$

$$N_n^- = \max\,(0, \bar{Z}_n - Z_n)$$

To minimize the distance between $Z_n$and $\bar{Z}_n$, we have to minimize either P_n^+, N_n^- or $P_n^+ + N_n^-$.When we have to maximize Z_n, $h_n(P_n^+, N_n^-) = N_n^-$ while, when we have to minimize Z_n, $h_n(P_n^+, N_n^-) = P_n^+$. When we desire $Z_n = \bar{Z}_n$, $h_n(P_n^+, N_n^-) = P_n^+ + N_n^-$. Apart from the solution, to reflect the satisfaction of the DM, the membership functions are defined as:

$$\mu_n\,(Z_n) = \begin{cases} 1 & if\; Z_n \le L_{Z_n} \\ 1 - \frac{Z_n - L_{Z_n}}{U_{Z_n} - L_{Z_n}} & if\; L_{Z_n} < Z_n < U_{Z_n} \\ 0 & if\; Z_n \ge U_{Z_n} \end{cases} \quad (20)$$

$\mu_n\,(Z_n)$ represents the DM’s satisfaction. Hence, it must be maximized i.e:

$$\max\,(\,\big(\mu_1(Z_1(x)), \mu_2(Z_2(x)), \mu_3(Z_3(x)), \dots \dots \dots .. \mu_n(Z_n(x))\big).$$

here U_{Z_n} and L_{Z_n} are the greatest acceptable, aspired level of performance for $\tilde{Z}_n, (n = 1,2)$ objective function.

We reduce its negative deviation from 1 to bring them as close to 1 as feasible in order to maximise any of the membership functions since the membership function's maximum value cannot be more than one. The LPP can be formulated as:

$$\min\left(\max\left(h_n(P_n^+, N_n^-)\right)\right)$$
i.e, Min S

subject to,

$$\frac{U_{Z_n} - Z_n}{U_{Z_n} - L_{Z_n}} + N_n^- - P_n^+ = 1,$$

$$S \geq N_n^-, \text{where } n - 1,2.$$

$$P_n^+ . N_n^- = 0. \quad (21)$$

$N_n^-, P_n^+ \geq 0, 0 \leq S \leq 1.$ and with given constraints. We have taken into account the UFCBO4DTPBC problem type here; GPT will be the best way for obtaining the most palatable compromise solution.

3.2. Algorithm for solving UFCBO4DTPBC

The proposed model UFCBO4DTPBC is solved using the procedures listed in:

Step 1: Formulate the uncertain model of UFCBO4DTPBC with the given data as of (17).

Step 2: Convert the UFCBO4DTPBC model into the deterministic model by applying the properties of the expected-chance constraint model as of (19).

Step 3: Compute the time objective and profit functions Z_n, (n = 1,2) individually to each of the demand, and supply constraints including breakability, budgets, and conveyance constraints.

Step 4: Obtain the values of each objective function Zn, with (n = 1,2) at each solution obtained in step 3.

Step 5: Obtain the upper U_{Z_n} and lower L_{Z_n} bounds for each objective function from the set of solutions calculated from step 3. Here U_{Z_n} and L_{Z_n} are the highest acceptable and aspired level of achievement for Z_n, (n = 1,2) objective function.

Step 6: For the given UFCBO4DTPBC, use the GPT to obtain the following LPP model under the budget constraints model.

$$Min\ S$$

Subject to,

$$\frac{U_{Z_n} - Z_n}{U_{Z_n} - L_{Z_n}} + N_n^- - P_n^+ = 1,$$

$$S \geq N_n^-, \text{ where } n - 1,2.$$

$$P_n^+ . N_n^- = 0.$$

$$N_n^-, P_n^+ \geq 0, 0 \leq S \leq 1 \quad (22)$$

Alongside constraints of the respective models.

Step 7: By applying the generalized reduced gradient technique [GRG] (LINGO-18.0 Suite Solver) and solve the model found in step 6 to obtain the compromise solution.

4. RESULTS AND DISCUSSION

4.1. Numerical example

To explain the effectiveness and efficiency of the proposed UFCBO4DTPBC model, a numerical example is given in this section, whose parameters are NUV. Two different customers (destinations), sources (origins), conveyances, different goods, and roads each are considered in this model. i.e m=d=w=v=g=2. Data on availabilities of goods in the sources are presented in Table 2. Table 3 shows the data for goods demand at the destination. The data for the good's Selling price is shown in Table 4. Table 5 consists the purchase price, Ware housing cost and procurement cost of the goods. The unit transportation costs of various goods and goods' breakability rate per unit distance are in Tables 6 and 7 respectively. Table 8 contains the distance between the different origins and different destinations via different routes. Table 9 shows the fixed charges paid to transport a certain quantity of goods from m^{th} origin to the destination of d^{th} via the v^{th} road by using w^{th}- transport. Table 10 contains the budget amount of each destination, capacity and speed of the vehicles. The data related to the rate of disturbance of the vehicles are given in Table 11. Table 12 contains the loading and unloading times of different goods.

Table 2. Availability of stock in each origin

M	$\tilde{a}_{m1}$	$\tilde{a}_{m2}$
1	(390,8)	(310,9)
2	(280,11)	(297,10)

Table 3. Destination's demand

d	$\tilde{b}_{d1}$	$\tilde{b}_{d2}$
1	(32,1.5)	(25,1)
2	(15,2)	(20,.8)

Table 4. Good's selling price

d	$\tilde{S}_{d1}$	$\tilde{S}_{d2}$
1	(87,1)	(115,2)
2	(112,1.5)	(102,2.2)

Table 5. The purchase price, Ware housing cost and procurement cost of the goods

m	$\tilde{P}_{m1}$	$\tilde{P}_{m2}$	$\widetilde{W}_{m1}$	$\widetilde{W}_{m2}$	$\widetilde{Pro}_{m1}$	$\widetilde{pro}_{m2}$
1	(14,1	(13,2)	(3,1)	(4,2)	(25,2)	(13,1.5)
2	(11,3)	(12,4)	(8,1)	(6,2.5)	(25,.5)	(10,1.9)

Table 6. Unit transportation cost

m	d	w	$\tilde{C}_{mdw11}$	$\tilde{C}_{mdw12}$	$\tilde{C}_{mdw21}$	$\tilde{C}_{mdw22}$
1	1	1	(.34,.1)	(.34..2)	(.36,.3)	(.3,.25)
		2	(.33,.32)	(.23,.1)	(.485,.2)	(.2,.3)
	2	1	(.4,.45)	(.22,.1)	(.42,.1)	(.26,.15)
		2	(0.4,.15)	(.34,.2)	(.5,.3)	(.47,.22)
2	1	1	(.44,.2)	(.235,.1)	(.46,.2)	(.26,.3)
		2	(.4,.1)	(.3,.15)	(.48,.2)	(.48,.1)
	2	1	(.42,.2)	(.106,.3)	(.46,.21)	(.22,.22)
		2	(.405,.21)	(.3,.3)	(.5,.2)	(.2,.2)

Table 7. Rate of breakability

m	d	w	β_{mdw11}	β_{mdw12}	β_{mdw21}	β_{mdw22}
1	1	1	0.014	0.024	0.014	0.015
		2	0.009	0.01	0.024	0.014
	2	1	0.015	0.025	0.024	0.015
		2	0.014	0.014	0.011	0.012
2	1	1	0.014	0.024	0.024	0.024
		2	0.019	0.019	0.016	0.009
	2	1	0.012	0.016	0.014	0.01
		2	0.012	0.016	0.012	0.011

Table 8. Distance between the origins and destinations

m	D_{m11}	D_{m12}	D_{m21}	D_{m22}
1	33	45	45	35
2	43	40	56	45

Table 9. Cost of fixed charge

m	D	$\tilde{f}_{md11}$	$\tilde{f}_{md12}$	$\tilde{f}_{md21}$	$\tilde{f}_{md22}$
1	1	(23,.5)	(24,1.2)	(16,1.4)	(7,.5)
	2	(12,2.1)	(27,3)	(28,1)	(14,2)
2	1	(15,1.5)	(16,2)	(27,2.5)	(14,3)
	2	(12,.5)	(14,1)	(55,2)	(47,2.5)

Table 10. Budget amount, Capacity and vehicle's speed

$\widetilde{\mathbf{Bud}_1}$	$\widetilde{Bud_2}$	$\tilde{e}_1$	$\tilde{e}_2$	V_1	V_2
(5500,100)	(4350,200)	(398,15)	(415,25)	35	25

Table 11. Rate of disturbances of different vehicles

m	γ_{m11}	γ_{m12}	γ_{m21}	γ_{m22}
1	0.0122	0.03	0.013	0.022
2	0.03	0.025	0.011	0.04

Table 12. Time for loading and unloading the goods

m	d	$\tilde{\delta}_{md11}$	$\tilde{\delta}_{md12}$	$\tilde{\delta}_{md21}$	$\tilde{\delta}_{md22}$
1	1	(0.36,.1)	(.36,.2)	(o.45,3)	(0.29,.4)
	2	(0.36,.21)	(0.39,.23)	(0.33,.41)	(0.33,.2)
2	1	(0.6,.3)	(0.66,.1)	(0.42,.2)	(0.36,.3)
	2	(0.86,.23)	(0.22,.2)	(0.37,.12)	(0.39,.3))

Applying the above-developed algorithm for the problem taken, the steps are:

Step 1: from the above data, the deterministic problem of the considered UFCBO4DTPBC model is obtained using the expected-chance constrained model [EC] as of (17) and solved.

Step 2: considering the objectives separately and solving them, we get $Z_1 = 5182.5$ and $Z_2 = 54.8345$. By using these solutions, the value of each objective function is found as:

$Z_1(X_1) = 5182.5$ and $Z_1(X_2) = 2856.7632$

$Z_2(X_1) = 74.71162$ and $Z_2(X_2) = 54.8345$

The boundary values of Z_1 and Z_2 are given as;

$U_{Z_1} = 5182.5, L_{Z_1} = 2856.7632$

$U_{Z_2} = 74.71162, L_{Z_2} = 54.8345$

Step 3: using the GPT, the GP expected-chance constrained method for the proposed model is as shown in;

$Min\ S$

subject to,

$$E\left[\sum_{m,dw,v,g}\left[\left(1-\beta_{mdwvg}\ d_{mdv}\right)\tilde{S}_{dg} - \widetilde{W}_{mg} - \tilde{P}_{mg} - \widetilde{Pro}_{mg} - \tilde{C}_{mdwvg}\, d_{mdv}\right]x_{mdwvg} - \sum_{m,d,w,v,g}(\widetilde{f}_{mdwv}\ e_{mdwvg})\right] + 2325.73(N_1^- - P_1^-) - 2856.76 = 2325.7$$

$$74.711 - E\left[\sum_{m,d,w,v}\frac{d_{mdv}\cdot \mathbf{y}_{mdwv}}{V_w\,(1-\gamma_{mdv})} + \sum_{m,d,w,v,g}\delta_{mdwg}\ .\,x_{mdwvg}\right] + 19.87712(N_2^- - P_2^-) = 19.8771$$

where,

$$e_{mdwvg} = \begin{cases} 1\ if\ \sum_g x_{mdwvg} > 0 \\ 0\ if\ \sum_g x_{mdwvg} = 0 \end{cases} \quad y_{mdwv} = \begin{cases} 1\ if\ \sum_g x_{mdwvg} > 0 \\ 0\ if\ \sum_g x_{mdwvg} = 0 \end{cases}$$

$\sum_{d,w,v} x_{mdwvg} \leq e_{mg} + \frac{\sigma_{mg}}{\pi} * \sqrt{3}\ \log\frac{1-\alpha_{mg}}{\alpha_{mg}}$, ∀ m = 1 to M *and* g = 1 to G .

$\sum_{m,w,v}(1 - \beta_{mdwvg} . d_{mdv}))x_{mdwvg}] \geq e_{dg} + \frac{\sigma_{dg}}{\pi} * \sqrt{3}\ \log\frac{\alpha_{dg}}{1-\alpha_{dg}}$, ∀ d = 1 to D and g = 1 to G.

$\sum_{m,d,v,g} x_{mdwvg} \leq e_{mg} + \frac{\sigma_{mg}}{\pi} * \sqrt{3}\ log\frac{1-\alpha_{mg}}{\alpha_{mg}}$, ∀ w = 1 to W.

$\sum_{d,w,v,g}\left(\phi_{mg}^{-1}(\alpha_3) + \theta_{wg}^{-1}(\alpha_4) + \eta_{mg}^{-1}(\alpha_5) + \lambda_{mdwvg}^{-1}(\alpha_6)\right)x_{mdwvg} + \rho_{mdwv}^{-1}(\alpha_7).$
$e_{mdwvg} \leq \varphi_d^{-1}(\alpha_8), \forall d.$

$x_{mdwvg} \geq 0, \forall m, d, w, v. g$

$S \geq N_n^-$, where $n = 1,2$.

$P_n^+ . N_n^- = 0.0$

$N_n^-, P_n^+ \geq 0, 0 \leq S \leq 1$ (22)

Step 4: with the use of the GRG technique(LINGO-18.0 Suite Solver), we obtain the efficient value of S = 0.46 and the corresponding transportation plan is $P_1^+ = 0, N_1^- = 0.46$, $P_2^+ = 0$, $N_2^- = 0.46$,Max $Z_1 = 4195.276$, Min $Z_2 = 64.01$, $x_{11211} = 48.103, x_{11212} = 70.803, x_{12221} = 28.32, x_{22122} = 38.125$, $e_{11212} = e_{12222} = e_{22122} = 1, y_{1121} = y_{1121} = y_{1121} = 1$ and the value of the other decision variables is 0. We can observe DM's objectives are achieved to a satisfactory level.

In our work, we have obtained the compromise solution of UFCBO4DTPBC by goal programming expect-chance constrained method. The efficient solution of the proposed model UFCBO4DTPBC is obtained using goal programming techniques and are given in Step 4. Thus, the goal programming technique is a fitting method for dealing with multi-objective transportation problems. Depending on the condition that is given larger weight as preferred by the decision maker as deduced from the solutions, we can achieve different suitable solutions in the EC constrained model, leading to optimistic and pessimistic results. Here, the decision maker could obtain the maximum profit with the minimum time taken even in case of considering several parameters.

5. CONCLUSION

In this work, the fixed charge bi-objective 4-dimensional TP with vehicle speed under budget constraints [UFCBO4DTPBC] consisting of uncertain variable parameters has been presented. We have considered the cost of fixed charge, budget constraints, and the variance in vehicle speed based on the condition of the road, unlike other transportation models for the first time. We have obtained the equivalent deterministic models for UFCBO4DTPBC by using EC constrained methods followed by a compromise solution which is obtained by applying GPT. The proposed model is very easy to use, understandable and economically advantageous for the firm as it increases profits and significantly reduces shipping time. Therefore, better managerial decisions can be made by the DM. The ease of application of this method's effectiveness in solution for UFCBO4DTPBC has been demonstrated in the numerical example. The proposed work can be extended as a multi level bi objective 4-diemensional TP with vehicle speed under budget constarints further in future.

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