$\overline{a}$ 

# **A Hybrid Structured Multistage Wiener Filter for GPS Interference Suppression**

# **Qing-Dong Huang\*, Guang-Yue Lu, Zhi-Qiang Bao, Yong-Zhi Zhai, Sheng-Li Pang**

School of Communication and information Engineering, Xi'an University of Posts and Telecommunications, Xi'an, Shaanxi, 710121, China, Ph./Fax: +086-02988166354 \*Corresponding author, e-mail: huangqingdong@xupt.edu.cn

# *Abstract*

*To suppress the influence of impulsive noise spikes (outliers) from the direction of arrival of the desired GPS signals, a combination of the Householder multistage Wiener filter (HMSWF) and the recently introduced Minimum Module Cascaded Canceller (MMCC) is employed to compose a hybrid structured HMSWF (HS-HMSWF) method. The enhanced algorithm is employed by space-time anti-jamming processing in GPS receiver, with the ability of natural protection against the impulsive noise spikes influence of weights calculation. In space signal processing, it can avoid the zero pitch formed at the direction of target signal, and prevent the desired signal be diminished. Simulation results demonstrated the favorable anti-jamming performance of the proposed algorithm.* 

*Keywords: array signal processing, global positioning system (GPS), anti-jamming (AJ), space-time adaptive processing (STAP)* 

#### *Copyright* © *2013 Universitas Ahmad Dahlan. All rights reserved.*

### **1. Introduction**

The GPS signals are so weak that the navigation receivers are very vulnerable to RF interference [1-3]. In adaptive signal processing, Werner [3] proposed the HMSWF and successfully used it in GPS space-time (ST) anti-jamming (AJ) processor, with advantages of low computational complexity and favorable performance. Iterative algorithms, such as the recursive least squares (RLS) algorithms and normalized least-mean-squares (NLMS), consist of a joint iterative optimization through the conjugate gradient (CG) algorithm [4]. These methods use the minimum mean-squared error (MMSE) criterion, or, in other words, they can be considered as the realization of the sample matrix inversion (SMI) methods. It was found that the nonstationary impulsive noise spikes (outliers) often occurred in detecting the desired signal under the complex electromagnetic or intentional jamming environment. Sample covariance matrices are excessively sensitive to the outliers from desired direction, which always produces a deleterious effect on convergence performance or forms a zero pitch in target signal direction, and makes the processor unable to work.

According to the problem of outliers' effect, Picciolo et al. proposed a Median Cascaded Canceller (MCC) methods [5, 6], and Q.D. Huang et al. presented sample selection HMSWF (SSHMSWF) [7] and MMCC methods [8, 9]. These methods achieve some good effects. But when the number of outliers is large (such as above the 1/2 of the training data contain outliers), the above methods will be effected and unable to work properly.

 A hybrid structured HMSWF (HS-HMSWF) method is proposed in this paper, which combined the HMSWF with MMCC method. It can be used in space-time (ST) GPS receiver's anti-jamming processor. The HS-HMSWF exhibits a mixture of useful characteristics from two separate processors. Optimal rank reduction capability from the HMSWF portion is retained, which is robust to outliers. Fitting for non-stationary data capability is attributed to the MMCC portion. Simulation results indicate the good performance, even in large number of outliers condition.

#### **2. ST Processors for AJ**

Assume the ST processor has *M* antenna elements and *L* filter coefficients per antenna, so the ST processor will have  $(M \times L-1)$  degrees of freedom. By stacking the samples from each antenna at time instant *n* into vector  $X(n) \in \mathbb{R}^{(M \times 1)}$  we get:

$$
\mathbf{X}(n) = \sum_{k=1}^{K_{out}} \mathbf{a}(\theta_k) s_k(n) + \sum_{j=1}^{K_{jum}} \mathbf{a}(\theta_j) r_j(n) + \mathbf{n}(n)
$$
\n(1)

where  $\mathbf{a}(\theta_i)$  and  $\mathbf{a}(\theta_i)$  denote the steering vectors of the signal and interference sources,  $\theta_k$  and  $\theta_i$  are the respective directions of arrivals (DOAs),  $s_k(n)$  is the transmitted signal from satellite  $k$ ,  $r(n)$  is the *j* th interfering signal, and  $n(n)$  is an additive circular white Gaussian noise vector with  $E [ n(n) n^H(n) ] = \sigma^2 I$ . In the paper,  $\left[ \right]^*$  denotes complex conjugate,  $\left[ \right]^{T}$ denotes transpose operation,  $\begin{bmatrix} 1 & H \end{bmatrix}$  denotes the Hermitian or conjugate-transpose operation. In the case of a uniform linear array with element spacing of half the wavelength, the steering vector has the particular simple form given by  $a(\theta) = [1, e^{j\pi \sin \theta}, \dots, e^{j(M-1)\pi \sin \theta}]^T$ . The ST snapshot vector  $\bar{\mathbf{X}}(n) \in \mathbb{D}^{(ML \times 1)}$  is obtained by stacking *L* consecutive spatial samples of  $\mathbf{X}(n)$ 

$$
\overline{\mathbf{X}}(n) = [\mathbf{X}^{\mathrm{T}}(n), \mathbf{X}^{\mathrm{T}}(n-1), ..., \mathbf{X}^{\mathrm{T}}(n-L+1)]^{\mathrm{T}}
$$
\n(2)

The AJ processor can be modeled as an  $(ML\times 1)$  array weight vector  $\overline{W}$  given by  $\bar{\mathbf{W}} = [\mathbf{W}_{1}^{T}, \mathbf{W}_{2}^{T}, ..., \mathbf{W}_{L}^{T}]^{T}$ , where  $\mathbf{W}_{i} = [w_{i,1}, w_{i,2}, ..., w_{i,M}]^{T}$  is the weight vector for the *i* th filter taps across the array. Note that the desired GPS signals can be considered narrowband. The AJ processor output is given by  $\hat{s}(n) = \overline{\mathbf{W}}^H \overline{\mathbf{X}}(n)$ . The AJ is used to form array beams so that we have unit gains towards the desired satellites and all the other signals are suppressed. In AJ it is reasonable to assume that the DOAs from the GPS satellites are known.

The output signal  $\hat{s}(n)$  is finally processed by the GPS receiver. Only the azimuth angle is considered here. The single-constraint beamformer can be changed into an unconstrained generalized sidelobe canceller (GSC) [3] based structure as illustrated in Figure 1  $d(n)$  are calculated as  $d(n) = a^H(\theta) X(n)$ , and

$$
\overline{\mathbf{X}}_{\perp}(n) = \begin{bmatrix} [\mathbf{B}_a \mathbf{X}(n)]^T & \mathbf{X}^T(n-1) & \cdots & \mathbf{X}^T(n-L+1) \end{bmatrix}^T
$$
 (3)

Where **B**<sub>a</sub> is a  $(M-1)\times M$  blocking matrix for which  $B_a a(\theta_b) = 0$ .



Figure 1. The Single-constraint GSC-based Structure

#### **3. The HS-HMSWF Method**

Though, HMSWF has the characteristic of optimal, and reduced rank capability, the convergence performance of HMSWF is vulnerable to target outliers. The weight calculation of MMCC algorithm depend on the statistic property, which different from HMSWF and a single sample point is used to calculate random variable weight. Then the MMCC weight is got from

the minimum module of the random variable weight. Hence, the method can effectively reduce the target outliers influence to weight calculation.

The HMSWF has the identical structure as HS-HMSWF (depicted in Figure 2) in crosscorrelation form. In the following, we assume that all signals are zero-mean, jointly stationary, complex Gaussian random processes. The residual error vector is  $e_{i-1}(n) = u_{i-1}(n) - \hat{w}_i^* e_i(n)$ (estimates denoted by  $\land$ ), where  $\land$  denotes complex conjugate,  $n = 1, 2, \dots, K$ , and *i* denotes the recursion stage. This minimization results in the two-input Wiener weight is estimated as:

$$
\widehat{w}_i = \widehat{R}_i^{-1} \widehat{r}_{i,i-1} \tag{4}
$$

where the estimates of the scalar auto and cross-correlation are, respectively,  $R_i = \frac{1}{n} \sum_{i} |e_i(n)|^2$  $\frac{1}{K}\sum_{n=1}^{K} |e_i(n)|$  $\widehat{R}_i = \frac{1}{K} \sum_{n=1}^{K} |e_i(n)|$ 

and  $\hat{r}_{i,i-1} = \frac{1}{K} \sum_{n=1}^{K} e_i(n) u_{i-1}^*(n)$  $\hat{r}_{i,i-1} = \frac{1}{K} \sum_{n=1}^{K} e_i(n) u_{i-1}^{\dagger}(n)$ , and  $\hat{w}_i$  is the Wiener filter for estimating the scalar  $u_{i-1}(n)$  from the scalar  $e_i(n)$ .

To derive the two-input adaptive weight algorithm, the solution of (4) for the case of a single point average (i.e.,  $K = 1$ ) is considered, which is  $\hat{w}_i(1) = [u_{i-1}(1)/e_i(1)]^*$ . This remains the single sample solution even for cost functions (4) having samples other than two. It is noted that the quantity  $\hat{w}_i(n) = [u_{i-1}(n) / e_i(n)]^*$  is a random variable for different values of *n*. By itself,  $\hat{w}_i(n)$ zeros the output of a two-input canceller if calculated and applied at each single point *n* . For adaptive applications, it is shown that the sample complex expectation of  $\hat{w}_i(n)$ ,  $n = 1, 2, \dots, K$ , converges to the optimal weight  $w_{i, \text{out}}$  in the mean square sense, as  $K \rightarrow \infty$  [5]. The adaptive weight for the two-input MMCC algorithm is calculated as follows. Take the minimum module sample weight within the random variable  $\{\hat{w}_i(n)\}$  as the new optimal complex weight estimate  $\widehat{w}_{i,\text{MMCC}}$  . The two-input MMCC complex weight is estimated as:

$$
\widehat{w}_{i,\text{MMCC}} = \min_{\to 0,n} \{ \widehat{w}_i(n) \} = \min_{\to 0,n} \{ u_{i-1}^*(n) / e_i^*(n) \}
$$
\n(5)

Where  $\min_{\to 0,n}$  denotes finding the complex weight with minimum module in  $\{\widehat{w}_i(n)\}\.$  To explain more clearly, the weight is  $\hat{w}_{i,\text{MMCC}} = \hat{w}_i(m), m \in \{1,2,\dots,K\}$ , where  $|\hat{w}_i(m)| \leq |\hat{w}_i(n)|$ ,  $n \neq m$ ,  $n \in \{1, 2, \dots, K\}$ . It has been proved that  $\hat{w}_{i,\text{MMCC}}$  is linearly close to  $w_{i, opt}$  as  $K \to \infty$ , in ideal stationary Gaussian noise condition [8]. That is,  $\hat{w}_{i,\text{MMCC}}$  is equivalent to the MMSE solution of (4) as  $K \rightarrow \infty$ , and there is no influence on output SINR ratio.

The MMCC can naturally avoid the influence of target outliers on weight estimation, and offer natural protection against the desired signal cancellation when the weight training data contains target components. Although  $\hat{w}_{i,\text{MMCC}}$  is linearly close to  $w_{i,opt}$ , the close extent relates to sample numbers and the value  $|w_{i,out}|$ . We successfully combined HMSWF with MMCC to form HS-HMSWF processor. As we estimate the weights in Figure 2, we use formula (5) (the MMCC method) to estimate the first two stage weight  $w_1$  and  $w_2$ , which reduces the influence of target outliers on weights estimation and avoids performance degradation of the processor in the presence of power leakage from the first stage target outliers. Formula (4) (the Wiener method) is used for the other stages, which improves AJ performance in finite sample solution. Hence, we can not only avoid deleterious effect of the outliers on HMSWF weights, but also form zero pitch zones more accurately in resisting narrowband and barrage jamming on spatial and ST applications. Compare with the IHMSWF method in paper [9], the difference is that IHMSWF use formula (5) to calculate all the weights, but HS-HMSWF only use formula (5) to calculate the first two stage weights, and formula (4) for the other stages weights.

■ 6925

The HS-HMSWF method employs the cross correlation based structure, which is showed in Figure 2. The scalar  $u_0(n) = d(n)$  denotes the desired-signal. The vector  $X_0(n) = \overline{X}_1(n)$  denotes the space time samples, which contains interference, noise and other satellite signals. The algorithm is given as follows:

**Initialization:**  $u_0(n) = d(n)$ ,  $X_0(n) = \overline{X}_1(n)$ **Forward recursion:** For  $i = 1, 2, \dots, D$  $\mathbf{v}_i = \mathbf{p}_i - c_i \delta_i \mathbf{u}_1; \quad \mathbf{u}_1 = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}^T;$  $\mathbf{p}_i = E\left[\mu_{i-1}^*(n)\mathbf{X}_{i-1}(n)\right] / \left\|E\left[\mu_{i-1}^*(n)\mathbf{X}_{i-1}(n)\right]\right\|_2;$  $\delta_i = \left\| E \left[ u_{i-1}^*(n) \mathbf{X}_{i-1}(n) \right] \right\|_2;$  $c_i = \pm p_{i,1} / |p_{i,1}|$ ,  $p_{i,1}$  is the first element of vector **p**<sub>i</sub>;  $\beta_i = -1/(c_i^* \delta_i v_{i,1}), v_{i,1}$  is the first element of the vectro  $\mathbf{v}_i$ ;  $\begin{bmatrix} \tilde{u}_i(n) \\ \mathbf{X}_i(n) \end{bmatrix} = \mathbf{X}_{i-1}(n) - \beta_i \mathbf{v}_i \mathbf{v}_i^{\mathbf{H}} \mathbf{X}_{i-1}(n); u_i(n) = c_i^* \tilde{u}_i(n);$ **Backward recursion**:  $e_n(n) = u_n(n)$ ,  $\hat{w}_{i} = \min_{n \to 0,n} \{ u_{i-1}^{*}(n) / e_{i}(n) \};$ For  $i = D, D - 1, \dots, 1;$ if  $i \leq 2$  else  $\hat{w}_{i} = \mathbb{E}\left[u_{i-1}^{*}(n)e_{i}(n)\right] / \mathbb{E}\left[\left|e_{i}(n)\right|^{2}\right];$ end

 $e_{i-1}(n) = u_{i-1}(n) - \widehat{w}_i^* e_i(n);$ 

Where  $n = 1, 2, \dots, K$ ; *D* denotes the recursion stages;



Figure 2. Cross-correlation based HS-HMSWF.

# **4. Simulation Results**

In the GPS AJ simulations, a uniform linear ST array is used with  $M = 7$  antennas, half a wavelength sapced. Each antenna had a tapped delay line with  $L=7$  coefficients providing a total of  $ML-1=48$  degrees of freedom for jammer cancellation. Nyquist Sampling rate in time domain was employed in the receiver. The navigation signals were coarse /acquisition (C/A) code signals of the GPS system with the signal-to-noise ratio (SNR) -22dB. Navigation data was received from one satellite with a DOA -10 deg and a randomly chosen C/A code. A total of  $K = 200$  snapshots were observed. Five narrowband jammers with DOAs  $[-50, -30, 0, 30, 50]$  deg respectively were presented. Their normalized frequencies (with respect to GPS L1 bandwidth of 2.046MHz) were  $[0.1,0.25,0.55,0.75,0.9]$  where 0.5 corresponds to the carrier frequency. The jammerto-noise ratios (JNRs) were all 20dB. In addition, there were four wideband jammers covering the whole GPS L1 frequency band with 20dB above the noise level (JNR 20dB). The DOAs of the wideband jammers were  $[-40, -20, 0.20]$  deg. Beampatterns produced by the ST array beamformer obtained via HS-HMSWF processing are plotted in Figure 3, where the array received signals contained narrowband jammers, wideband jammers and desired signal.

Narrow band jammers are marked by white "¤.", and the desired signal by white "□". Nulls associated with wideband jammers can be seen as canyons parallel to the frequency axis, and the nulls associated with narrowband jammers appear as deep and narrow pits in the plot. All the jammers are reliably nulled. Figure 4 and 5 show the cross-correlations of the SSHMSWF, IHMSWF [9] and HS-HMSWF methods with different phase offsets on the x-axis. Figure 4(a)-(c) show those three methods with prior mentioned 5 narrow band and 4 wide band jammers occur, also 3 outliers with JNR 30dB in desired signal samples  $d(n)$ . Figure 5(a)-(c) show these methods with the same narrow band and wide band jammers as Figure 4, also contains 105 outliers with JNR 30dB in desired signal samples  $d(n)$ . It can be seen that the HS-HMSWF method can effectively cancel the jammers and allow the receiver to obtain a proper synchronization to the GPS signal in either condition, and slightly better than the IHMSWF method in paper [9]. But SSHMSWF loses the synchronization when there are 105 outliers occurs. The HS-HMSWF method can achieve good performance for even more outliers occurs but the SSHMSWF method loses the effectiveness, especially with a large number of outliers (more than half of the snapshots).

In space signal processing, Figure 6 shows gain pattern of liner array with halfwavelength and  $N = 12$  sensor elements. In the simulation, there has one  $+40\text{dB}$  target outlier in desired-signal  $u_0(n)$  located at 5deg. One narrowband sidelobe barrage jammer has a power equal to 40dB above noise level, located at 20deg. It shows that a zero pitch is located at target signal direction (5deg) in HMSWF method's gain pattern. For HS-HMSWF, the narrowband jammer is attenuated, and the desired signal receives a unity gain.



Figure 3. Beam Pattern of the ST Array Via HS-**HMSWF** 



Figure 5. Cross-correlation after ST Processor (contains 105 outliers with JNR 30dB)







Figure 6.Comparison of Gain Pattern while Single Strong Target Outlier Added

# **5. Conclusion**

In this paper we proposed a hybrid structured HS-HMSWF method, which offers natural protection against the desired signal cancellation while containing target outliers in desired signal. The method is used in GPS receivers AJ ST processor, it exhibits a mixture of useful characteristics from the HMSWF and MMCC processor. The optimal rank reduction capability is retained from the HMSWF portion, and the robustness to the outliers is retained from the MMCC portion. The HS-HMSWF is effective even when the desire signal contains large number of outliers (more than half of the snapshots). In space signal processing, this method can efficiently avoid the forming of zero pitch on the desire signal direction when the desire signal contains big outliers. Simulations demonstrate that the HS-HMSWF performance is superior to the IHMSWF method, and more robust to the SSHMSWF.

# **Acknowledgements**

This work was supported in part by theProgram for New Century Excellent Talents in University of China (NCET-08-0891), the Natural Science Foundation of Shaanxi Province, China (Grant No. 2010JQ80241), the Research Foundation of Education Bureau of Shaanxi Province, China (Grant No. 11JK0929, No. 11JK0925).

### **References**

- [1] J Baek, S Yoo, SY Kim. Jamming Effect Analysis of Two Chinese GNSS BeiDou-II Civil Signals. *International Journal of Electrical and Computer Engineering*. 2012; 2(6): 840-845.
- [2] YR Wang, H Zhang, QF Zhou. Adaptive integrated navigation filtering based on accelerometer calibration. *TELKOMNIKA Indonesian Journal of Electrical Engineering*. 2012; 10(7): 1869-1878.
- [3] S Werner, M With, V Koivunen. Householder multistage Wiener filter for space-time navigation receivers. *IEEE Transactions on Aerospace and Electronic Systems*. 2007; 43(3): 975-988.
- [4] RC de Lamare, R Sampaio-Neto. Adaptive reduced-rank processing based on joint and iterative interpolation, decimation, and filtering. *IEEE Transactions on Signal Processing*. 2009; 57(7): 2503- 2514.
- [5] ML Picciolo, K Gerlach. Median cascaded canceller for robust adaptive array processing. *IEEE Transactions on Aerospace and Electronic Systems*. 2003; 39(3): 883-900.
- [6] ML Picciolo, K Gerlach. *An adaptive multistage median cascaded canceller*. IEEE Conference Proceeding IEEE Radar Conference 2002. Long Beach. 2002; 22: 22-25.
- [7] QD Huang, LR Zhang, C Wang, B Zhang. Interference Suppression Method of Sample Selected Householder Multistage Wiener Filter for Space Time Navigation Receivers. *Acta Electronic Sinica.* 2011; 39(6): 1368-1372.
- [8] QD Huang, LR Zhang, GY Lu. Minimum module value cascaded canceller. *Journal of Xidian University*. 2010; 37(2): 204-209.
- [9] QD Huang, LR Zhang, GY Lu, D Shen. An Improved Householder Multistage Wiener Filter Method for Interference Suppression of Space-time Navigation Receivers. *Journal of Electronics & Information Technology*. 2011; 33(10): 2526-2530.