An Adaptive Fuzzy Control Method for Spacecrafts Based on T-S Model

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Abstract

A model reference adaptive control method is proposed for uncertain nonlinear characteristics of spacecraft attitude control system. This method combines fuzzy control methodology and nonlinear feedback linearization methodology, which made the closed-loop system stable and the state of fuzzy system track the state of reference model according to the parallel distributed compensation theory and the rational design of the fuzzy state feedback control law. The nonlinear closed-loop system was linearized by selecting fuzzy state feedback parameters and fuzzy membership function. Then an adaptive control law was designed by Lyapunov function. As a result the system can be adaptive to all kinds of parameter uncertainties and robust to modeling inaccuracy and external disturbance. Meanwhile, the simulation results indicate that the control law can quickly guarantee the stability of the spacecraft attitude and be robust to model perturbations and external disturbances.

Keywords: spacecraft attitude control, T-S model, model reference adaptive control, parallel distributed compensation

1. Introduction

Higher requirements have been put forward for the spacecraft attitude control system with the more complicated structure of a new generation of spacecrafts and the increasing modern space missions, such as the spacecraft attitude capture, the reentry attitude adjustment, spacecraft space docking and so on. Because of the working environment and the structure characteristics of the spacecraft, the nonlinear system structure, the uncertain parameters and so forth, the design of the spacecraft control system suffers huge difficulties. Therefore, it requires that the modern spacecraft control system not only can finish various space missions, but also have strong robustness, to ensure good dynamic characteristics and steady-state quality with uncertain parameters, external disturbances, structure perturbations and other various uncertain factors' effects [1] .

In the traditional nonlinear control method, the controller is designed directly based on the non-linear model of the controlled object, by feedback linearization and sophisticated linear system theory. Fuzzy control method based on T-S model is not directly based on the traditional non-linear model, but based on the fuzzy model built by rules of the state equation. Compared with the traditional nonlinear control, the biggest advantage of this method is the traditional control theory can be applied to the fuzzy system stability analysis and controller design. The controller also can be designed through the introduction of uncertain T-S model when the object has bounded uncertainty. So the nonlinear control method based on T-S model provides a new way for the modeling and control of complex nonlinear multivariable system [2].

From the control effects, the effects of these methods depend on the approximation degree of T-S model to the nonlinear object, and the approximation accuracy is improved at the cost of increasing the number of fuzzy local models [3-6]. In order to reduce the number of fuzzy local models without affecting the control performance of the system, a series of fuzzy adaptive control algorithms with robust performance to the modeling errors and the uncertainty has been researched [7-9].

In this paper, a model reference adaptive control method combing fuzzy control methodology and nonlinear feedback linearization methodology is proposed for the spacecraft attitude control system with uncertain nonlinear characteristics, as T-S model describes. This

method designed a rational fuzzy state feedback control law according to the parallel distributed compensation theory, which made the closed-loop system stable and the state of fuzzy system track the state of reference model. The nonlinear closed-loop system was linearized by selecting fuzzy state feedback parameters and fuzzy membership function. Then according to the error equation, an adaptive control law was designed by Lyapunov function method. As a result the system can be robust to uncertainties and modeling inaccuracy. Meanwhile, the simulation results indicate that the control law can quickly guarantee the stable control on the attitudes of the aircraft, and be robust to model perturbations and external disturbances.

2. Description of the Spacecraft Attitude Stability Control

In order to achieve high-precision attitude control of the spacecraft with uncertain nonlinear characteristics, the T-S model is established and spacecraft attitude control law is designed based on the given spacecraft dynamics and kinematics equation to realize high-precision attitude of the spacecraft control.

2.1. Description of the Spacecraft Attitude Control Model

The dynamic equation of the rigid spacecraft attitude can be written as [10-11]:

$$\dot{\omega} = \boldsymbol{J}^{-1}S(\omega)J\omega + \boldsymbol{J}^{-1}M_c + \boldsymbol{J}^{-1}M_d \tag{1}$$

Kinematics equation based on Rodrigues parameters is:

$$\dot{\rho} = H(\rho)\omega \tag{2}$$

J for spacecraft inertia matrix, $\omega \in R^3$ for spacecraft angular velocity matrix, $M_c \in R^3$ for control torque vector, $M_d \in R^3$ for disturbance vector, $\rho \in R^3$ for Rodrigues parameter vector, $S(\omega)$ for skew symmetric matrix , and $H(\rho)$ for third-order matrix.

$$S(\omega) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix} \tag{3}$$

$$H(\rho) = \frac{1}{2}\left[I_3 - S(\rho) - \rho\rho^T \right] \tag{4}$$

ω can be considered as the virtual control input of subsystem (2), so we design control law to stabilize this subsystem.

$$\omega_d = -k_1\rho \ , \ k_1 > 0 \tag{5}$$

Error vector e is introduced,

$$e = \omega - \omega_d = \omega + k_1\rho \tag{6}$$

Thus, the attitude control system (1) and (2) is rewritten as:

$$\dot{e} = \left\{ J^{-1}S(e - k_1\rho)J + k_1H(\rho) \right\} e$$
$$-k_1\left\{ J^{-1}S(e - k_1\rho)J + k_1H(\rho) \right\}\rho + J^{-1}M_c + J^{-1}M_d \tag{7}$$

$$\dot{\rho} = H(\rho)e - k_1H(\rho)\rho \tag{8}$$

For the system composed by (7)-(8), choose $k_1 = 0.5$ and compose $e_i, \rho_i \in [-2, 2]$, $i = 1, 2, 3$. Select the state quantity $x = [e_1, e_2, e_3, \rho_1, \rho_2, \rho_3]^T = [x_1, x_2, x_3, x_4, x_5, x_6]^T$, and define $x_e = [x_1, x_2, x_3]^T$, $x_\rho = [x_4, x_5, x_6]^T$, so the system composed of (7) - (8) can be rewritten as:

$$\dot{x} = A(x)x + Bu + d \tag{9}$$

In the formula,

$$A(x) = \begin{bmatrix} J^{-1}S(x_e - 0.5x_\rho)J + 0.5H(x_\rho) & -0.5\{J^{-1}S(x_e - 0.5x_\rho)J + 0.5H(x_\rho)\} \\ H(\rho) & -0.5H(\rho) \end{bmatrix}$$

$$B = \begin{bmatrix} J^{-1} \\ 0_3 \end{bmatrix} \text{, } u = M_c \text{, } d = \begin{bmatrix} J^{-1}M_d \\ 0_3 \end{bmatrix}.$$

The control system model above is nonlinear model. The corresponding T-S mode must be established to design the controller by traditional control theory.

2.2. Description of the Spacecraft Attitude Control T-S Model
In order to transform the dynamic model of spacecraft attitude into T-S model, $A(x)$ can be divided into nine operating points.

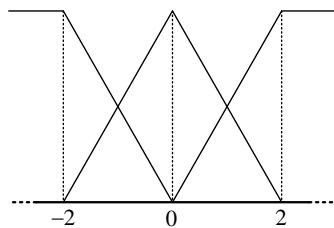$$[x_{ei}, x_{\rho i}] = [0,0], [0, 2(-2)], [2,0], [2, 2(-2)], [2, 0], [-2, 2(-2)]$$
$$i = 1, 2, 3$$

Figure 1. Membership Function Curve

The following rules can be drawn,

Rule 1: If x_1 is F_{11} (close to 0), x_2 is F_{12} (close to 0), x_3 is F_{13} (close to 0), x_4 is F_{14} (close to 0), x_5 is F_{15} (close to 0), and x_6 is F_{16} (close to 0), $\dot{x} = A_1 x + B_1 u + d$.

……

Rule 9: If x_1 is F_{91} (close to -2), x_2 is F_{92} (close to -2), x_3 is F_{93} (close to -2), x_4 is F_{94} (close to -2), x_5 is F_{95} (close to -2), and x_6 is F_{96} (close to -2), $\dot{x} = A_9 x + B_9 u + d$.

Considering the uncertainties in the system and the external disturbances, the spacecraft attitude dynamics T-S model can be described as follows:

$$R_i: \text{ if } z_1(t) \text{ is } F_{i1} \text{ and } \cdots\cdots z_p(t) \text{ is } F_{ip}$$
$$\text{then } x(t) = (A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + d(t) \tag{10}$$
$$i = 1, 2, 3, \cdots, q$$

In the formula, $x(t) = [x_1(t), \cdots, x_n(t)]^T \in R^n$ is the system state variable, $u(t) \in R^m$ is the control input variable, $z(t) = [z_1(t), \cdots, z_p(t)]^T \in R^p$ is the fuzzy antecedent variable.

$F_{ij} (i = 1, \cdots, q; j = 1, \cdots, p)$ is the fuzzy set, q is the number of fuzzy rules, (A_i, B_i) is a matrix with the corresponding dimension of the i-th system, $(\Delta A_i, \Delta B_i)$ is a structure uncertainty matrix with the corresponding dimension of the i-th system, and $d(t)$ is the system modeling errors and external interferences. $(\Delta A_i, \Delta B_i)$ and $d(t)$ are unknown. Using the singleton fuzzification, product inference and the weighted average defuzzification, with the given $(x(t), u(t))$, the output of the fuzzy system is the weighted average of all the outputs in each subsystem. That is,

$$
\dot{x}(t) = \frac{\sum\limits_{i=1}^{q} a_i(z)[(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t) + d(t)]}{\sum\limits_{i=1}^{q} a_i(z)} \tag{11}
$$

$$
= \sum\limits_{i=1}^{q} \mu_i(z)[(A_i + \Delta A_i)x(t) + (B_i + \Delta B_i)u(t)] + d(t)
$$

$\alpha_i(z) = \prod\limits_{j=1}^{p} F_{ij}(z_j(t))$, $F_{ij}(z_j(t))$ represent that $z_j(t)$ belongs to the membership of the fuzzy set. F_{ij}, and $\alpha_i(z)$ is the membership function. $\mu_i(z) = \dfrac{\alpha_i(z)}{\sum\limits_{i=1}^{q} \alpha_i(z)}$, and $\sum\limits_{i=1}^{q} \mu_i(z) = 1$.

$\alpha_i(z) \geq 0$, so $0 \leq \mu_i(z) \leq 1$.

Assuming that the matrix (A_i, B_i) $i = 1, 2, \cdots, q$ is controlled, namely system (10) with T-S model is locally controllable, so the matrix (A_i, B_i) can be transformed to controllable canonical form by linear state transformation.

Therefore, without loss of generality, the matrix (A_i, B_i) is set as follows.

$$
A_i = \begin{bmatrix} a_n^i & a_{n-1}^i & \cdots & a_2^i & a_1^i \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, B_i = \begin{bmatrix} b^i \\ 0 \\ \vdots \\ 0 \end{bmatrix}, d(t) = \begin{bmatrix} d_1(t) \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{12}
$$

Assuming in the formula (12), the symbol of b^i remains unchanged, that is to say b^i is a positive or negative constant. Assuming $d_1(t)$ represents bounded modeling errors and external interferences, namely $|d_1(t)| \leq d_{\max}$.

Thereby the spacecraft attitude control T-S model is established and the adaptive fuzzy controller is designed.

3. Model Reference Adaptive Design based on T-S Model
3.1. Model Reference Feedback Control Law Design
According to the theory of parallel distributed compensation, fuzzy state feedback controller is designed by fuzzy control law as follows:

$$
\begin{aligned}
&R_i: \ if \ z_1(t) \ is \ G_{i1} \ and \ \cdots\cdots z_p(t) \ is \ G_{ip} \\
&then \ u(t) = -K_i x(t) + l_i r(t) \quad i = 1, 2, \cdots, q
\end{aligned} \tag{13}
$$

$G_{ij} (i = 1, \cdots, q; j = 1, \cdots, p)$ is the fuzzy set, q is the number of fuzzy rules, K_i is the state feedback gain, $K_i = [k_n^i, k_{n-1}^i, \cdots, k_1^i]$.

The whole state feedback control law is:

$$u(t) = \frac{\sum_{i=1}^{q} \beta_i(z)[-K_i x(t) + l_i r(t)]}{\sum_{i=1}^{q} \beta_i(z)} = \sum_{i=1}^{q} v_i(z)[-K_i x(t) + l_i r(t)] \tag{14}$$

$\beta_i(z) = \prod_{j=1}^{p} G_{ij}(z_j(t))$, $G_{ij}(z_j(t))$ represent $z_j(t)$ belongs to the membership of the fuzzy set G_{ij}, and $\beta_i(z)$ is the membership function. $v_i(z) = \dfrac{\beta_i(z)}{\sum_{i=1}^{q} \beta_i(z)}$, and $\sum_{i=1}^{q} v_i(z) = 1$.

$\beta_i(z) \geq 0$, so $0 \leq v_i(z) \leq 1$.

The closed-loop system can be obtained by substituting Equation (14) into Equation (11).

$$\dot{x}(t) = \sum_{i=1}^{q} \sum_{j=1}^{q} \mu_i(z) v_j(z)\{[(A_i + \Delta A_i) - (B_i + \Delta B_i)K_j]x(t) + (B_i + \Delta B_i)l_j r(t)\} + d(t) \tag{15}$$

Ignoring parameter uncertainties and external interferences, namely $\Delta A_i = 0$, $\Delta B_i = 0$, $d(t) = 0$, Equation (15) can be simplified as:

$$\dot{x}(t) = \sum_{i=1}^{q} \sum_{j=1}^{q} \mu_i(z) v_j(z)[(A_i - B_i K_j)x(t) + B_i l_j r(t)] \tag{16}$$

The following lemma is given without proof.

Lemma: If the fuzzy state feedback control law is taken as (14) and (17) to (19), the fuzzy closed-loop system (16) can be transformed into linear system (20).

$$k_s^i = \frac{1}{b^i}(a_s^i - a_s^m) \qquad s = 1, 2, \cdots, n \tag{17}$$

$$l_i = \frac{1}{b^i} b^m \qquad i = 1, 2, \cdots, q \tag{18}$$

$$G_{ij}(z_j) = \sqrt[p]{|b^i|} F_{ij}(z_j) \quad i = 1, 2, \cdots, q \qquad j = 1, 2, \cdots, p \tag{19}$$

$$\dot{x}(t) = A_m x(t) + B_m r(t) \tag{20}$$

$$A_m = \begin{bmatrix} a_n^m & a_{n-1}^m & \cdots & a_2^m & a_1^m \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, \quad B_m = \begin{bmatrix} b^m \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

This lemma shows that when the system does not exist parameter uncertainties, external disturbances and all the parameters are precisely known, the states $x(t)$ of closed-loop fuzzy system (16) can track the states of linear system (20) through fuzzy state feedback control law (14). As the reference model, the linear system (20) is globally asymptotically stable with desired dynamic performance. Then the states of closed-loop fuzzy system (16) can completely track the states of system (20).

From the feedback control law (14) and (17)-(19), the values of A_i and B_i in formula (12) must be obtained accurately, that is, the accurate values of $a_j^i, b^i (i = 1, 2, \cdots, q; j = 1, 2, , n)$, in order to track the reference model accurately. However, for practical systems, there are various uncertain parameters $\Delta A_i, \Delta B_i$ inside and external interference $d(t)$, and these parameters vary with time, so we must estimate the parameters online by the adaptive algorithm.

3.2. Fuzzy Adaptive Law Design

Because the parameters A_i and B_i in the system can not be obtained accurately due to the structural uncertainty, the values of A_i and B_i must be estimated on-line by the adaptive algorithm, and then replace the coefficients in the fuzzy state feedback control law with the estimated parameters. In addition, there also exist modeling errors and external interferences $d(t)$, so a control law must be designed to compensate the interferences. Then we get the combination control law:

$$u(t) = u_f(t) + u_s(t) \tag{21}$$

In the formula, $u_f(t)$ is the state feedback control part and $u_s(t)$ is the adaptive control part.

$$u_f(t) = \frac{\sum_{i=1}^{q} \hat{\beta}_i(z)[-\hat{K}_i x(t) + \hat{l}_i r(t)]}{\sum_{i=1}^{q} \hat{\beta}_i(z)} \tag{22}$$

$$u_s(t) = -\frac{\sum_{i=1}^{q} \alpha_i(z)}{\sum_{i=1}^{q} \alpha_i(z)\hat{b}_i} \hat{d}_{max} \, \text{sgn}[e^T(t) P_1] \tag{23}$$

In the formula, $\hat{\beta}_i(z), \hat{K}_i, \hat{l}_i$ are the estimated values of $\beta_i(z), K_i, l_i$ respectively. By the formulas (17)-(19), $\hat{A}_i$ and $\hat{B}_i$ can be estimated through the unknown parameters A_i and B_i to obtain the estimated values of $\beta_i(z), K_i, l_i$.

$$\hat{K}_i = \frac{1}{\hat{b}^i}[\hat{a}_n^i - a_n^m, \hat{a}_{n-1}^i - a_{n-1}^m, \cdots, \hat{a}_1^i - a_1^m] \qquad i = 1, 2, \cdots, q \tag{24}$$

$$\hat{l}_i = \frac{1}{\hat{b}^i} b^m \qquad i = 1, 2, \cdots, q \tag{25}$$

$$\hat{\beta}_i(z) = \alpha_i \left| \hat{b}^i \right| \qquad i = 1, 2, \cdots, q \tag{26}$$

From formula (11),

$$\dot{x}(t) = \frac{\sum_{i=1}^{q} \alpha_i(z) \left[A_i x(t) + B_i u(t) \right]}{\sum_{i=1}^{q} \alpha_i(z)} + d(t)$$

$$= A_m x(t) + \frac{\sum_{i=1}^{q} \alpha_i(z)\left[(A_i - A_m)x(t) + B_i u(t)\right]}{\sum_{i=1}^{q} \alpha_i(z)} + d(t) \tag{27}$$

Construct the estimated value $\hat{x}(t)$ of $x(t)$ to satisfy:

$$\dot{\hat{x}}(t) = A_m \hat{x}(t) + \frac{\sum_{i=1}^{q} \alpha_i(z)\left[(\hat{A}_i - A_m)x(t) + \hat{B}_i u_f(t)\right]}{\sum_{i=1}^{q} \alpha_i(z)} \tag{28}$$

Substitute the formula (22) and (24) to (26) into the above formula.

$$\dot{\hat{x}}(t) = A_m \hat{x}(t) + B_m r(t) \tag{29}$$

Compare the formula (29) with the reference model.

$$\hat{x}(t) = x_m(t) \tag{30}$$

Thus, the estimated error $e_1(t) = x(t) - \hat{x}(t)$ is the same as the tracking error $e_2(t) = x(t) - x_m(t)$.

The formula (27) subtracts formula (28) to get the unified error equation.

$$\dot{e}(t) = A_m e(t) + \frac{\sum_{i=1}^{q} \alpha_i(z)\left[\tilde{A}_i x(t) + \tilde{B}_i u(t)\right]}{\sum_{i=1}^{q} \alpha_i(z)} + \frac{\sum_{i=1}^{q} \alpha_i(z)\hat{B}_i}{\sum_{i=1}^{q} \alpha_i(z)} u_s(t) + d(t) \tag{31}$$

$\tilde{A}_i = A_i - \hat{A}_i$, $\tilde{B}_i = B_i - \hat{B}_i$ represent the estimated parameter errors. Formula (31) can be rewritten as follows.

$$\dot{e}(t) = A_m e(t) + \frac{\sum_{i=1}^{q} \alpha_i(z)[\tilde{\bar{a}}_i, \bar{0}, \cdots, \bar{0}]^T x(t)}{\sum_{i=1}^{q} \alpha_i(z)} + \frac{\sum_{i=1}^{q} \alpha_i(z)[\tilde{b}_i, 0, \cdots, 0]^T u(t)}{\sum_{i=1}^{q} \alpha_i(z)}$$

$$+ I_{n \times 1} \frac{\sum_{i=1}^{q} \alpha_i(z)\hat{b}_i}{\sum_{i=1}^{q} \alpha_i(z)} u_s(t) + d(t) \tag{32}$$

In the formula, $\bar{a}_i = [a_n^i, a_{n-1}^i, \cdots, a_1^i]^T$, $\tilde{\bar{a}}_i = \bar{a}_i - \hat{\bar{a}}_i$, $\tilde{b}_i = b_i - \hat{b}_i$.

$\hat{a}_j^i, \hat{b}_j^i$ in the estimated parameters $\hat{A}_i$ and $\hat{B}_i$ can be obtained by the adaptive law as follows.

$$\dot{\hat{\bar{a}}}_i = \gamma_{1i} \frac{\alpha_i(z)}{\sum_{i=1}^{q} \alpha_i(z)} x(t) e^T(t) P_1 \tag{33}$$

$$\dot{\hat{b}}_i = \gamma_{2i} \frac{\alpha_i(z)}{\sum\limits_{i=1}^{q} \alpha_i(z)} e^T(t) P_1 u(t) \tag{34}$$

γ_{1i}, γ_{2i} are the positive constants. $P = [P_1, P_2, \cdots, P_n]$ is a positive definite symmetric matrix and satisfies the solution of Lyapunov equation $PA_m + A_m^T P = -Q$ (Q is a positive definite symmetric matrix). Because the reference model is asymptotically stable, the solution of Lyapunov equation must exist.

Also, because the modeling errors and the upper bound $d_{\max}$ of external disturbances $d_1(t)$ are difficult to obtain accurately for the actual system, as it changes with the running state of the system, the estimated value of $d_{\max}$ can be got by the adaptive algorithm of formula (35).

$$\dot{\hat{d}}_{\max} = \gamma_3 \left| e^T(t) P_1 \right| \tag{35}$$

And $\tilde{d}_{\max} = d_{\max} - \hat{d}_{\max}$ is defined as the estimated error of the upper bound $d_{\max}$.

3.3. Proof of the Control System Stability

For the nonlinear uncertain system (10) based on T-S model, if the unknown parameters are adjusted by adaptive law (33), (34) and (35) according to the model reference adaptive control law of (21), the closed-loop system is globally asymptotically stable. The proof is the following.

Select the quasi-Lyapunov equation.

$$V(t) = e^T(t) Pe(t) + \sum_{i=1}^{q} \frac{\tilde{a}_i^T \tilde{a}_i}{\gamma_{1i}} + \sum_{i=1}^{q} \frac{\tilde{b}_i^2}{\gamma_{2i}} + \frac{\tilde{d}_{\max}^2}{\gamma_3} \tag{36}$$

Derivative of $V(t)$ can be obtained along the trajectory of the error system (32).

$$\dot{V}(t) = \dot{e}^T(t) Pe(t) + e^T(t) P\dot{e}(t) + 2\sum_{i=1}^{q} \frac{\tilde{a}_i^T \dot{\tilde{a}}_i}{\gamma_{1i}} + 2\sum_{i=1}^{q} \frac{\tilde{b}_i \dot{\tilde{b}}_i}{\gamma_{2i}} + 2\frac{\tilde{d}_{\max} \dot{\tilde{d}}_{\max}}{\gamma_3} \tag{37}$$

Substitute error equation (32), and we obtain:

$$\dot{V}(t) = -e^T(t) Qe(t) + 2\frac{\sum\limits_{i=1}^{q} \alpha_i(z) \tilde{a}_i^T x(t) e^T(t) P_1}{\sum\limits_{i=1}^{q} \alpha_i(z)} + 2\frac{\sum\limits_{i=1}^{q} \alpha_i(z) \tilde{b}_i e^T(t) P_1 u(t)}{\sum\limits_{i=1}^{q} \alpha_i(z)}$$
$$+ 2e^T(t) P_1 \left[\frac{\sum\limits_{i=1}^{q} \alpha_i(z) \hat{b}_i}{\sum\limits_{i=1}^{q} \alpha_i(z)} u_s(t) + d_1(t) \right] - 2\sum_{i=1}^{q} \frac{\tilde{a}_i^T \dot{\tilde{a}}_i}{\gamma_{1i}} - 2\sum_{i=1}^{q} \frac{\tilde{b}_i \dot{\tilde{b}}_i}{\gamma_{2i}} - 2\frac{\tilde{d}_{\max} \dot{\tilde{d}}_{\max}}{\gamma_3} \tag{38}$$

Then substitute the compensation control (23).

$$\dot{V}(t) = -e^T(t) Qe(t) + 2\frac{\sum\limits_{i=1}^{q} \alpha_i(z) \tilde{a}_i^T x(t) e^T(t) P_1}{\sum\limits_{i=1}^{q} \alpha_i(z)} + 2\frac{\sum\limits_{i=1}^{q} \alpha_i(z) \tilde{b}_i e^T(t) P_1 u(t)}{\sum\limits_{i=1}^{q} \alpha_i(z)}$$
$$- 2\left| e^T(t) P_1 \right| \hat{d}_{\max} + 2e^T(t) P_1 d_1(t) - 2\sum_{i=1}^{q} \frac{\tilde{a}_i^T \dot{\tilde{a}}_i}{\gamma_{1i}} - 2\sum_{i=1}^{q} \frac{\tilde{b}_i \dot{\tilde{b}}_i}{\gamma_{2i}} - 2\frac{\tilde{d}_{\max} \dot{\tilde{d}}_{\max}}{\gamma_3} \tag{39}$$

Because $|d_1(t)| \leq d_{\max}$, we can obtain:

$$\dot{V}(t) \leq -e^T(t)Qe(t) + 2\sum_{i=1}^{q} \tilde{\bar{a}}_i^T \left[\frac{\alpha_i(z)x(t)e^T(t)P}{\sum_{i=1}^{q} \alpha_i(z)} - \frac{\dot{\hat{\bar{a}}}_i}{\gamma_{1i}} \right]$$

$$+2\sum_{i=1}^{q} \tilde{b}_i \left[\frac{\alpha_i(z)e^T(t)P_1 u(t)}{\sum_{i=1}^{q} \alpha_i(z)} - \frac{\dot{\hat{b}}_i}{\gamma_{2i}} \right] + 2\tilde{d}_{\max} \left[\left| e^T(t)P_1 \right| - \frac{\dot{\hat{d}}_{\max}}{\gamma_3} \right]$$

(40)

Substitute the adaptive law (33), (34) and (35).

$$\dot{V}(t) \leq -e^T(t)Qe(t) \leq 0$$

(41)

When $t \to \infty$, the systematic error $e(t) \to 0$. So the closed-loop fuzzy system is asymptotically stable. QED.

4. Simulation Result

The spacecraft attitude control law can be obtained by the method of fuzzy adaptive control law based on T-S model, and the effectiveness of the control law can be verified by the nonlinear model of the spacecraft as a controlled object.

Digital simulation is made by building SIMULINK control block diagram in MATLAB. The main moment of inertia J_x, J_y, J_z is respectively 2043, 3044 and 1989(kgm2), and the main moment of inertia perturbation $\Delta J_x, \Delta J_y, \Delta J_z$ are all $5\sin t$ kgm2. Assume plus interference is $0.05\sin t$ rad/s2, and the initial condition of simulation is $\omega_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, $\rho_0 = \begin{bmatrix} 1.23 & 0.55 & 2.17 \end{bmatrix}^T$.

The simulation results are shown in Figure 2-4.
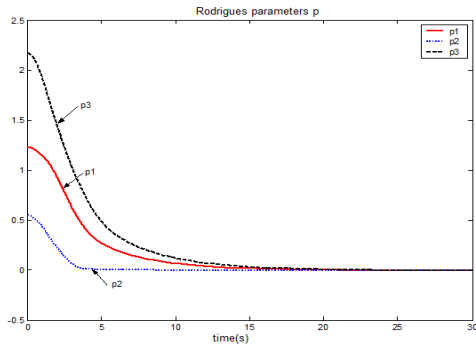

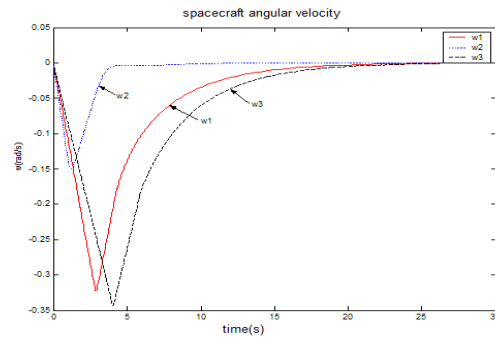Figure 2. Rodrigues Parameter ρ Change Curve
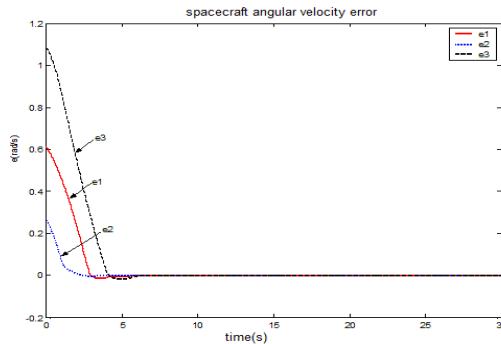

Figure 3. ω Change Curve

Figure 4. e Change Curve

From the simulation results, the control law proposed can quickly control the nonzero initial attitude of spacecraft to the balance position. At the same time, certain robustness has been shown to model perturbations and external disturbances.

5. Conclusion

For uncertain nonlinear characteristics of spacecraft attitude control system, according to the parallel distributed compensation theory, fuzzy control methodology and nonlinear feedback linearization methodology are combined to realize the linearization of the closed-loop system nonlinear feedback by appropriate choices of the fuzzy state feedback coefficient and fuzzy membership function. On this basis, combined with Lyapunov's function, an adaptive law is designed, such that the closed-loop system is stable, and the fuzzy system state can track the state of the reference model to realize adaptive control. The method can adapt to all kinds of parameter uncertainty, and has good robustness to modeling errors and uncertainty. The simulation results show that the designed control law can realize the spacecraft attitude control effectively and has good robustness to the uncertainty, model perturbations and external disturbances. It is fully shown that the T-S model has a broad application prospect to handle uncertain nonlinear controlled object.

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