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Linear Unbiased Optimal Filter for Discrete-Time Systems with One-Step Random Delays and Inconsecutive Packet Dropouts

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Abstract

This paper is concerned with the linear unbiased minimum variance estimation problem for discrete-time stochastic linear control systems with one-step random delay and inconsecutive packet dropout. A new model is developed to describe the phenomena of the one-step delay and inconsecutive packet dropout by employing a Bernoulli distributed stochastic variable. Based on the model, a recursive linear unbiased optimal filter in the linear minimum variance sense is designed by the method of completing the square. The solution to the linear filter is given by three equations including a Riccati equation, a Lyapunov equation and a simple difference equation. A sufficient condition for the existence of the steady-state filter is given. A simulation shows the effectiveness of the proposed algorithm.

Keywords: *linear unbiased filter, random delay, inconsecutive packet dropouts, steady-state filter*

1. Introduction

In recent years, the research on networked systems and sensor networks has gained lots of interests due to wide applications in communication, control and signal processing [1-3]. In networked systems, the time delays and packet dropouts are unavoidable in data transmission through unreliable communications from sensors to a processing center. The data available in the processing center may not be real time due to the delays or packet dropouts. So estimation and control in the networked systems are very challenging [4].

In wireless networks, the systems with stochastic delays, packet dropouts and missing measurements can be described by a stochastic parameter system [5-8]. Yaz et al. [5] designs the filtering problem in the least mean square sense. However, the filters are not optimal since a colored noise induced by augmentation is treated as a white noise. The estimation problem for systems with missing measurements is studied in [6], where sensor data are only the measurement noises at some samples. Ray et al. [7] presents a linear unbiased minimum variance state estimator to accommodate the effects of random delays in data arrival at the controller. In [8], the state estimation for discrete-time linear systems with stochastic parameters is treated. A recursive least-squares linear estimator is designed for random delays by the covariance information approach in [9]. Studying the robust H-infinite filter for systems with random delays and missing measurements [10]. The optimal H_2 filtering for systems with random delays, packet dropouts and uncertain observations is presented based on a unified stochastic parameterized model in [11]. For systems with infinite and finite packet dropouts, the optimal linear estimators are developed in the linear minimum variance sense by an innovation analysis approach in [12] and [13], respectively. However, the random delays are not taken into consideration in [12, 13]. Investigates the optimal linear estimation problem for systems with random delays and packet dropouts, however, which may bring network congestion since a sensor packet is sent several times to avoid loss [14]. Studies the optimal linear filter for systems with one-step random delay and compensation of packet dropouts [15].

In this paper, we consider the linear unbiased optimal filtering problem for systems with the possible one-step random delay and inconsecutive packet dropout. A model is developed to describe the phenomena by a Bernoulli random variable with a known probability. A sensor packet is only sent once to avoid the network congestion, and the packet dropout is inconsecutive. A recursive linear unbiased optimal filter is obtained by the method of completing the square. The solution is given in terms of one Riccati, one Lyapunov and one simple

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difference equation. The steady-state property is analyzed. A sufficient condition for the existence of the steady-state filter is given.

2. Problem Formulation

Consider the discrete time-invariant linear stochastic system:

$$x(t+1) = \Phi x(t) + Bu(t) + \Gamma w(t) \tag{1}$$

$$z(t) = Hx(t) + v(t) \tag{2}$$

Where $x(t) \in R^n$ is the state, $u(t) \in R^h$ is the input, $z(t) \in R^m$ is the measured output, $w(t) \in R^r$ and $v(t) \in R^m$ are the process and measurement noises, respectively, and Φ, B, Γ and H are constant matrices with suitable dimensions.

In networked systems, the measurement $z(t)$ of a sensor is sent to a processing center through the unreliable communications with random delays and losses. To avoid network congestion, we assume that a packet is only sent once. Here, we only deal with one-step delay and inconsecutive packet dropout. We adopt the following model for the measurement received by the processing center.

$$y(t) = \xi(t)z(t) + (1-\xi(t))(1-\xi(t-1))z(t-1) \tag{3}$$

Where $\xi(t)$ is a Bernoulli random variable with the probabilities $\mathrm{Prob}\{\xi(t)=1\} = \alpha$ and $\mathrm{Prob}\{\xi(t)=0\} = 1-\alpha$ with $0 \le \alpha \le 1$, and is uncorrelated with other random variables. Table 1 shows the case of data transmission:

Table 1. Data Transmission in Network

t	1	2	3	4	5	6	7	8	9	10
ξ(t)	1	0	1	0	0	1	0	0	0	1
y(t)	z(1)	0	z(3)	0	z(4)	z(6)	0	z(7)	z(8)	z(10)

From Table 1, we can see that z(1), z(3), z(6) and z(10) are received on time, z(2), z(5) and z(9) are lost, z(4), z(7) and z(8) are delayed. It is known that the on-time arriving rate is $\mathrm{Prob}\{\xi(t)=1\} = \alpha$, one-step delay rate is $\mathrm{Prob}\{\xi(t)=0,\ \xi(t+1)=0\} = (1-\alpha)^2$ and packet dropout rate is $\mathrm{Prob}\{\xi(t)=0,\ \xi(t+1)=1\} = (1-\alpha)\alpha$ for the data at *t* instant. So, model (3) describes possible one-step transmission delay and inconsecutive packet dropouts.

In this paper, the expectation E operates on $\xi(t)$ and/or $w(t)$ and $v(t)$. *I* and 0 are an identity matrix and a zero matrix with suitable dimensions, respectively. Also, the following assumptions are used.

Assumption 1. $w(t)$ and $v(t)$ are uncorrelated white noises with zero means and variances $Q_w \ge 0$ and $Q_v > 0$.

Assumption 2. The initial state $x(0)$ is uncorrelated with $w(t)$ and $v(t)$, and

$$\mathrm{E}[x(0)] = \mu_0,\ \mathrm{E}[(x(0)-\mu_0)(x(0)-\mu_0)^{\mathrm{T}}] = P_0 \tag{4}$$

Our aim is to find the recursive linear unbiased optimal filter of the following Kalman-like form:

$$\hat{x}(t+1) = F(t)\hat{x}(t) + G(t)u(t) + K(t)y(t+1) \tag{5}$$

With the initial value is $\hat{x}(0) = \mu_0$. We will solve the gain matrices $F(t)$, $G(t)$ and $K(t)$ such that the linear filter (5) satisfies unbiasedness and least mean square criterion, i.e. $\mathrm{E}[\hat{x}(t)] = \mathrm{E}[x(t)]$ and $\min \mathrm{E}[(x(t)-\hat{x}(t))^{\mathrm{T}}(x(t)-\hat{x}(t))]$. Note that we will design the linear filter (5) dependent on the probability α, which can be computed offline.

Remark 1. From the distribution of $\xi(t)$, we have $\mathrm{E}[\xi(t)] = \alpha$, $\mathrm{Cov}[\xi(t)] = \alpha(1-\alpha)$, $\mathrm{E}[\xi^2(t)] = \alpha$, $\mathrm{E}[(1\text{-}\xi(t))^2] = 1-\alpha$, $\mathrm{E}[\xi(t)(1\text{-}\xi(t))] = 0$, $\mathrm{E}[\xi(k)(1\text{-}\xi(t))] = \alpha(1-\alpha)$, $k \neq t$.

3. Linear Unbiased Optimal Filter

In this section, a linear unbiased optimal filter as (5) will be designed for system (1)-(3). Theorem 1 and Theorem 2 give the results.

Theorem 1. For system (1)-(3) with Assumptions 1 and 2, the state second-order moment matrix $q(t) = \mathrm{E}[x(t)x^{\mathrm{T}}(t)]$ is computed by the following Lyapunov equation:

$$q(t+1) = \Phi q(t)\Phi^{\mathrm{T}} + Bu(t)u^{\mathrm{T}}(t)B^{\mathrm{T}} + \Gamma Q_w \Gamma^{\mathrm{T}} \quad (6)$$

With the initial value $q(0) = \mu_0\mu_0^{\mathrm{T}} + P_0$.
The mean $\bar{x}(t) = \mathrm{E}[x(t)]$ of the state $x(t)$ satisfies the following difference equation:

$$\bar{x}(t+1) = \Phi\bar{x}(t) + Bu(t) \quad (7)$$

With the initial value $\bar{x}(0) = \mu_0$.

Proof. Substitute (1) into the definition $q(t+1) = \mathrm{E}[x(t+1)x^{\mathrm{T}}(t+1)]$, and it yields (6). Equation (7) directly follows from taking expectation on (1).

Theorem 2. For system (1)-(3) with Assumptions 1 and 2, the gain matrices of the linear unbiased optimal filter (5) are computed by:

$$F(t) = \Phi - K(t)M \quad (8)$$

$$M = \alpha H\Phi + (1-\alpha)^2 H \quad (9)$$

$$G(t) = B - \alpha K(t)HB \quad (10)$$

$$K(t) = \Omega(t)\Lambda^{-1}(t) \quad (11)$$

Where:

$$\begin{aligned}
&\Omega(t) = \Phi P(t)M^{\mathrm{T}} + \\
&\alpha(1-\alpha)^2\Phi K(t-1)H[\Phi-(1-\alpha)I]\; q(t-1)\Phi^{\mathrm{T}}H^{\mathrm{T}} + \\
&\alpha[(1-\alpha)^2\Phi K(t-1)H + I]\Gamma Q_w\Gamma^{\mathrm{T}}H^{\mathrm{T}} \qquad (12)\\
&\Lambda(t) = (1-\alpha)H\{\alpha\Phi q(t)\Phi^{\mathrm{T}} + (1-\alpha)[1-(1-\alpha)^2]q(t) - \\
&\alpha(1-\alpha)\Phi q(t) - \alpha(1-\alpha)q(t)\Phi^{\mathrm{T}}\}H^{\mathrm{T}} + MP(t)M^{\mathrm{T}} + \alpha(1-\alpha)HBu(t)u^{\mathrm{T}}(t)B^{\mathrm{T}}H^{\mathrm{T}} + \\
&\alpha(1-\alpha)H\{\Phi\bar{x}(t)u^{\mathrm{T}}(t)B^{\mathrm{T}} + Bu(t)\bar{x}^{\mathrm{T}}(t)\Phi^{\mathrm{T}} - \\
&(1-\alpha)\bar{x}(t)u^{\mathrm{T}}(t)B^{\mathrm{T}} - (1-\alpha)Bu(t)\bar{x}^{\mathrm{T}}(t)\}H^{\mathrm{T}} + \\
&\alpha H\Gamma Q_w\Gamma^{\mathrm{T}}H^{\mathrm{T}} + \alpha Q_v + (1-\alpha)^2 Q_v + \\
&\alpha(1-\alpha)^2 H\Phi q(t-1)\Phi^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}(t-1)M^{\mathrm{T}} + \alpha(1-\alpha)^2 MK(t-1)H\Phi q(t-1)\Phi^{\mathrm{T}}H^{\mathrm{T}} - \\
&\alpha(1-\alpha)^3 H\Phi q(t-1)H^{\mathrm{T}}K^{\mathrm{T}}(t-1)M^{\mathrm{T}} - \alpha(1-\alpha)^3 MK(t-1)Hq(t-1)\Phi^{\mathrm{T}}H^{\mathrm{T}} + \\
&\alpha(1-\alpha)^2 H\Gamma Q_w\Gamma^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}(t-1)M^{\mathrm{T}} +
\end{aligned}$$

$$\alpha(1-\alpha)^2 MK(t-1)H\Gamma Q_w\Gamma^{\mathrm{T}}H^{\mathrm{T}} \quad (13)$$

The filtering error variance matrix is given by:

$$P(t+1)=\Phi P(t)\Phi^{\mathrm{T}}+\Gamma Q_w\Gamma^{\mathrm{T}}-K(t)\Lambda(t)K^{\mathrm{T}}(t) \quad (14)$$

Where $P(t)$ is the filtering error covariance matrix with the initial value $P(0)=P_0$.

Proof. Substituting (2) into (3) and using (1) and (5), we have the filtering error equation.

$$\begin{aligned}&\tilde{x}(t+1)=[\Phi-F(t)-\xi(t+1)K(t)H\Phi-(1-\xi(t+1))(1-\xi(t))K(t)H]x(t)+F(t)\tilde{x}(t)+\\&[B-G(t)-\xi(t+1)K(t)HB]u(t)+\\&\Gamma w(t)-\xi(t+1)K(t)H\Gamma w(t)-\xi(t+1)K(t)v(t+1)-\\&(1-\xi(t+1))(1-\xi(t))K(t)v(t)\end{aligned} \quad (15)$$

Where the filtering error $\tilde{x}(t)=x(t)-\hat{x}(t)$. From the unbiasedness, it requires that $\tilde{x}(0)=0$ and

$$\begin{aligned}&\mathrm{E}[\Phi-F(t)-\xi(t+1)K(t)H\Phi-\\&(1-\xi(t+1))(1-\xi(t))K(t)H]=0\end{aligned} \quad (16)$$

And,

$$\mathrm{E}[B-G(t)-\xi(t+1)K(t)HB]=0 \quad (17)$$

Then it follows from (16) and (17) that:

$$F(t)=\Phi-K(t)[\alpha H\Phi+(1-\alpha)^2 H] \quad (18)$$

$$G(t)=B-\alpha K(t)HB \quad (19)$$

Which give (8)-(10).

Substituting (18) and (19) into (15) yields:

$$\begin{aligned}&\tilde{x}(t+1)=K(t)H\{(\alpha-\xi(t+1))\Phi+[(1-\alpha)^2-(1-\xi(t+1))(1-\xi(t))]I\}x(t)+\\&F(t)\tilde{x}(t)+(\alpha-\xi(t+1))K(t)HBu(t)+\\&\Gamma w(t)-\xi(t+1)K(t)H\Gamma w(t)-\xi(t+1)K(t)v(t+1)\\&-(1-\xi(t+1))(1-\xi(t))K(t)v(t)\end{aligned} \quad (20)$$

From (20), we have the filtering error variance as:

$$\begin{aligned}&P(t+1)=(1-\alpha)K(t)H\{\alpha\Phi q(t)\Phi^{\mathrm{T}}+(1-\alpha)[1-(1-\alpha)^2]q(t)-\alpha(1-\alpha)\Phi q(t)-\\&\alpha(1-\alpha)q(t)\Phi^{\mathrm{T}}\}H^{\mathrm{T}}K^{\mathrm{T}}(t)+F(t)P(t)F^{\mathrm{T}}(t)+\\&\alpha(1-\alpha)K(t)HBu(t)u^{\mathrm{T}}(t)B^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}(t)+\\&\alpha(1-\alpha)K(t)H\{\Phi\overline{x}(t)u^{\mathrm{T}}(t)B^{\mathrm{T}}+Bu(t)\overline{x}^{\mathrm{T}}(t)\Phi^{\mathrm{T}}-\\&(1-\alpha)\overline{x}(t)u^{\mathrm{T}}(t)B^{\mathrm{T}}-(1-\alpha)Bu(t)\overline{x}^{\mathrm{T}}(t)\}H^{\mathrm{T}}K^{\mathrm{T}}(t)\\&+\Gamma Q_w\Gamma^{\mathrm{T}}+\alpha K(t)H\Gamma Q_w\Gamma^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}(t)-\alpha K(t)H\Gamma Q_w\Gamma^{\mathrm{T}}-\alpha\Gamma Q_w\Gamma^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}(t)+\\&\alpha K(t)Q_vK^{\mathrm{T}}(t)+(1-\alpha)^2K(t)Q_vK^{\mathrm{T}}(t)-\alpha(1-\alpha)^2K(t)H\Phi q(t-1)\Phi^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}(t-1)F^{\mathrm{T}}(t)+\\&\alpha(1-\alpha)^3K(t)H\Phi q(t-1)H^{\mathrm{T}}K^{\mathrm{T}}(t-1)F^{\mathrm{T}}(t)-\alpha(1-\alpha)^2K(t)H\Gamma Q_w\Gamma^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}(t-1)F^{\mathrm{T}}(t)-\\&\alpha(1-\alpha)^2F(t)K(t-1)H\Phi q(t-1)\Phi^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}(t)+\alpha(1-\alpha)^3F(t)K(t-1)Hq(t-1)\Phi^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}(t)-\\&\alpha(1-\alpha)^2F(t)K(t-1)H\Gamma Q_w\Gamma^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}(t)\end{aligned} \quad (21)$$

Where the uncorrelation of $x(t)$ and $w(t)$, $v(t+1)$, $v(t)$ and the uncorrelation of $\tilde{x}(t)$ and $w(t)$, $v(t+1)$ are used. Substituting (8) into (21) and completing the square, we can rewrite (21) as:

$$\begin{aligned}&P(t+1)=\Phi P(t)\Phi^{\mathrm{T}}+\Gamma Q_w\Gamma^{\mathrm{T}}+\\&[K(t)-\Omega(t)\Lambda^{-1}(t)]\Lambda(t)\,[K(t)-\Omega(t)\Lambda^{-1}(t)]^{\mathrm{T}}\\&-\Omega(t)\Lambda^{-1}(t)\Omega^{\mathrm{T}}(t)\end{aligned} \tag{22}$$

Where $\Omega(t)$ and $\Lambda(t)$ are defined by (12) and (13).

To minimize the right hand side of (22), the filtering gain *K*(*t*) only needs to satisfy (11) which further leads to (14).

Remark 2. It is worthwhile noting that the gain and variance matrices of the filter designed in Theorem 2 are affected by the input *u*(*t*), which is different from the standard Kalman filter [16]. The reason is that there are random delay and packet dropout. So the steady-state filter does not exist generally. In next section, we will study the steady-state property.

4. Steady-State Property

In the section 3, the linear unbiased optimal filter in the finite horizon has been designed. In this section, we will study the steady-state property in the infinite horizon for $0<\alpha<1$.

Theorem 3. For system (1)-(3), if the matrix Φ is stable and the input $u(t)$ is constant, the solutions $q(t)$ and $\bar{x}(t)$ of equations (6) and (7) with any initial conditions $q(0)$ and $\bar{x}(0)$ will converge to the unique positive semi-definite solutions *q* and $\bar{x}$ of the following algebraic Lyapunov equation and difference equation:

$$q=\Phi q\Phi^{\mathrm{T}}+Buu^{\mathrm{T}}B^{\mathrm{T}}+\Gamma Q_w\Gamma^{\mathrm{T}} \tag{23}$$

And,

$$\bar{x}=\Phi\bar{x}+Bu \tag{24}$$

Proof. Let the matrix $A=\Phi\otimes\Phi$ where $\otimes$ is the Kronecker product, from the stability of Φ, it can be easily known that $\rho(A)<1$, where $\rho(A)$ is the spectrum radius of the matrix *A*. Also the input $u(t)$ is constant, then $q=\lim_{t\to\infty}q(t)$ satisfies (23) [10]. From the stability of Φ and the constant input u, then $\bar{x}=\lim_{t\to\infty}\bar{x}(t)$ satisfies (24).

Theorem 4. For system (1)-(3), if the matrix Φ is stable and the input $u(t)$ is constant, the solution $P(t)$ of Equation (14) with any initial condition $P(0)\geq 0$ will converges to the unique positive semi-definite solution Σ of the following algebraic Riccati equation:

$$\begin{aligned}&\Sigma=(1-\alpha)KH\{\alpha\Phi q\Phi^{\mathrm{T}}+(1-\alpha)[1-(1-\alpha)^2]q-\alpha(1-\alpha)\Phi q-\alpha(1-\alpha)q\Phi^{\mathrm{T}}\}H^{\mathrm{T}}K^{\mathrm{T}}+\\&(\Phi-KM)\Sigma(\Phi-KM)^{\mathrm{T}}+\\&\alpha(1-\alpha)KHBuu^{\mathrm{T}}B^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}+\\&\alpha(1-\alpha)KH\{\Phi\bar{x}u^{\mathrm{T}}B^{\mathrm{T}}+Bu\bar{x}^{\mathrm{T}}\Phi^{\mathrm{T}}-\\&(1-\alpha)\bar{x}u^{\mathrm{T}}B^{\mathrm{T}}-(1-\alpha)Bu\bar{x}^{\mathrm{T}}\}H^{\mathrm{T}}K^{\mathrm{T}}+\\&\Gamma Q_w\Gamma^{\mathrm{T}}+\alpha KH\Gamma Q_w\Gamma^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}-\\&\alpha KH\Gamma Q_w\Gamma^{\mathrm{T}}-\alpha\Gamma Q_w\Gamma^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}+\alpha KQ_vK^{\mathrm{T}}+(1-\alpha)^2KQ_vK^{\mathrm{T}}-\\&\alpha(1-\alpha)^2KH\Phi q\Phi^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}(\Phi-KM)^{\mathrm{T}}+\alpha(1-\alpha)^3KH\Phi qH^{\mathrm{T}}K^{\mathrm{T}}(\Phi-KM)^{\mathrm{T}}-\\&\alpha(1-\alpha)^2KH\Gamma Q_w\Gamma^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}(\Phi-KM)^{\mathrm{T}}-\alpha(1-\alpha)^2(\Phi-KM)KH\Phi q\Phi^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}+\\&\alpha(1-\alpha)^3(\Phi-KM)KHq\Phi^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}-\\&\alpha(1-\alpha)^2(\Phi-KM)KH\Gamma Q_w\Gamma^{\mathrm{T}}H^{\mathrm{T}}K^{\mathrm{T}}\end{aligned} \tag{25}$$

Then we have $K = \lim_{t\to\infty} K(t)$ and $P = \lim_{t\to\infty} P(t)$. Moreover, the steady-state filter:

$$\hat{x}(t+1) = (\Phi - KM)\hat{x}(t) + (I - \alpha KH)Bu(t) + Ky(t+1) \quad (26)$$

Is asymptotically stable.

Proof. From Theorem 3, we have $q = \lim_{t\to\infty} q(t)$ and $\bar{x} = \lim_{t\to\infty} \bar{x}(t)$. Moreover, the stability of Φ means that the system is detectable and stabilizable. Then, from Kalman filtering theory [16], it is known that the solution $P(t)$ of equation (14) with any initial condition $P(0) \geq 0$ converges to the unique positive semi-definite solution Σ of (25), and $\Phi - KM$ is a stable matrix, which implies the stability of the steady-state filter (26).

5. Simulation Example

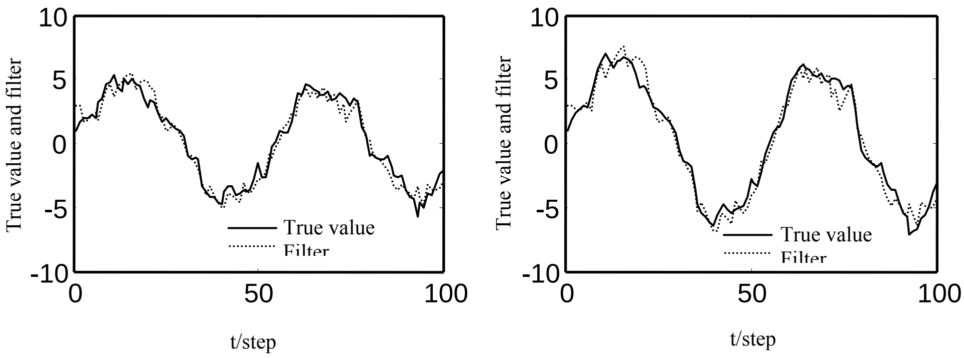
Consider a time-invariant example:

$$x(t+1) = \begin{bmatrix} 0.8 & 0 \\ 0.9 & 0.2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0.6 \\ 0.5 \end{bmatrix} w(t) \quad (27)$$

$$z(t) = [1 \quad 1]x(t) + v(t) \quad (28)$$

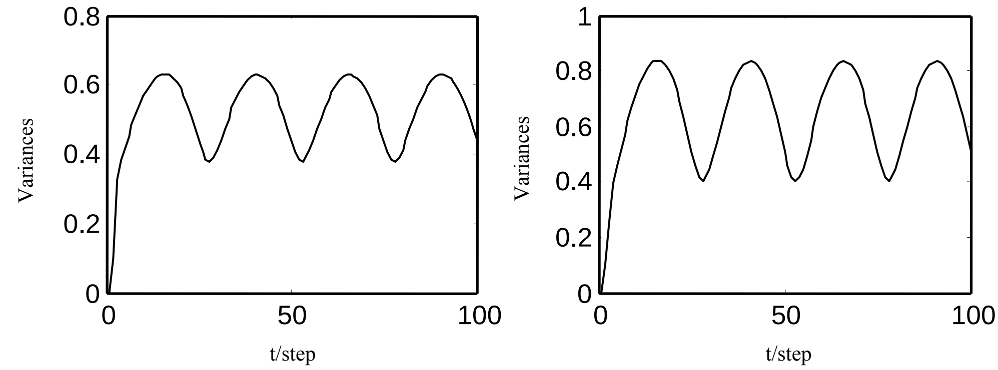
$$y(t) = \xi(t)z(t) + (1-\xi(t))(1-\xi(t-1))z(t-1) \quad (29)$$

In the simulation, we take $u(t) = \sin(4\pi t / N)$, $\alpha = 0.5$ and the initial values $\hat{x}(0) = [3,3]^{\mathrm{T}}$ and $P_0 = 0.1I_2$, where I_2 is the identity matrix. We take *N*=100 sampling data. Applying Theorems 1 and 2, we can obtain the linear unbiased optimal filter $\hat{x}(t)$. The filter is shown in Figure 1. Figure 2 shows the filtering error variance. It can be seen that the steady state values do not exist since the variance is affected by the time-varying input. To verify the steady-state property, we set the input $u(t) = 0.2$. The filter is shown in Figure 3. The corresponding filtering error variance is shown in Figure 4. It can be seen that the steady-state values exist, which is consistent to the theory analysis. The comparison of the steady-state filtering error variances in this paper, [6, 7] and [15] for $0.1 \leq \alpha \leq 1$ and *u*(*t*)=0 is shown in Figure 5. From Figure 5, we can see that our filter has the better accuracy than [6] since our filter has possible one-step delay but [6] is only noise when the present packet is lost. While our filter has the lower accuracy than that in [7] since [7] only has random delay. Compared with [15] where there is compensation of packet dropout, our filter has better accuracy at the lower arrival rate while worse accuracy at the higher arrival rate than [15].



(a) Filtering for the first state component (b) Filtering for the second state

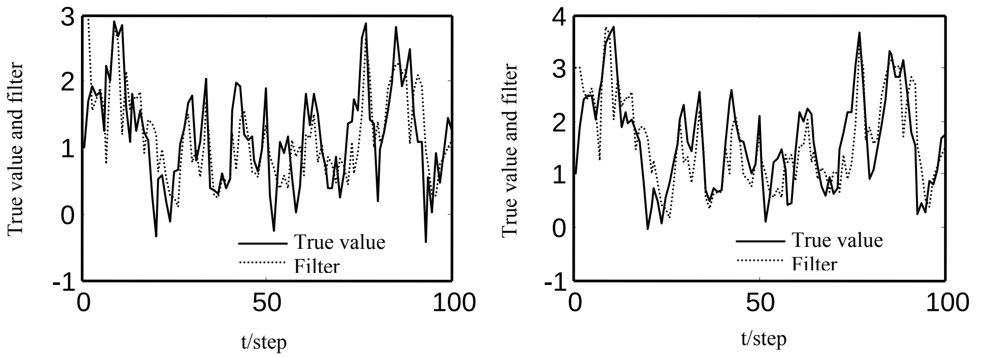
Figure 1. Linear Unbiased Optimal Filter with $\alpha = 0.5$ and $u(t) = \sin(4\pi t / N)$



(a) Filtering error variances for the first state component

(b) Filtering error variances for the second state component

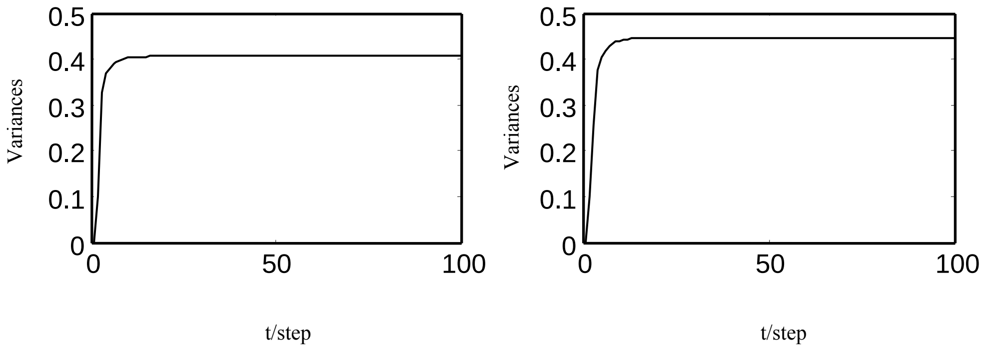
Figure 2. Filtering Error Variances with $\alpha = 0.5$ and $u(t) = \sin(4\pi t / N)$



(a) Filtering for the first state component

(b) Filtering for the second state component

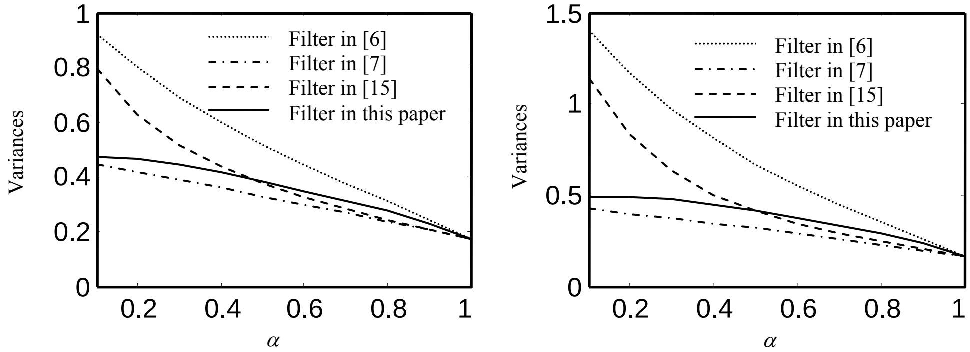
Figure 3. Linear Unbiased Optimal Filter with $\alpha = 0.5$ and u(t)=0.2



(a) Filtering error variances for the first state component

(b) Filtering error variances for the second state component

Figure 4. Filtering Error Variances with $\alpha = 0.5$ and $u(t) = 0.2$



(a) Filtering error variances for the first state component

(b) Filtering error variances for the second state component

Figure 5. Comparison for Filtering Error Variances in this Paper, [6], [7] and [15] with u(t)=0 and Different $0.1 \le \alpha \le 1$

6. Conclusion

For the discrete-time linear stochastic control systems with one-step random transmission delay and inconsecutive packet dropout, we have derived the recursive linear unbiased optimal filter in the linear minimum variance sense, which depends on the data arrival rate, state mean, state second-order moment and control input. The solution is given in terms of three equations including one Riccati, one Lyapunov and a simple difference equation. The asymptotic stability of the proposed filter has been analyzed. A sufficient condition for the existence of the steady-state filtering has been given.

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