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Synchronization of Hyperchaotic Systems under Active Adaptive Sliding Mode Control

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Abstract

Under the existence of system uncertainties and external disturbances, complete synchronization and anti-synchronization between two identical or different hyperchaotic systems are investigated in the paper. Firstly, an active control is used to eliminate the nonlinear part of the error. Then the sliding mode controller is designed based on suitable sliding surface. After that the adaptive updating law is designed to estate the bound of the uncertainties and external disturbances under the combination of active sliding mode control and adaptive control. The structure of the master and slave hyperchaotic systems has no restrictive assumption about the bound of the uncertainties and external. The active adaptive sliding mode controller (AASMC) is proposed to drive the state of slaver system trajectories into or opposites to the state of master system. The active adaptive sliding mode controller is proposed to realize synchronization and anti-synchronization by changing the parameter in the control function respectively. Moreover, a strict proof of the stability of the error dynamics is derived based on the Lyapunov stability theory. Finally, the corresponding numerical simulations are demonstrated the robustness and efficiency of the proposed controller.

Keywords: synchronization, hyperchaotic system, active adaptive sliding model, uncertainties, external disturbances

1. Introduction

Chaos is a very interesting nonlinear phenomenon due to its high sensitive dependence on initial conditions. Synchronization between two chaotic systems is one of the important processes in the control of complex phenomena for chemical, physical, and biological systems [1]. Since the pioneering work by Pecoraet al. in 1991 [2], chaos synchronization has received increasing attention and various researches have focused on the complete synchronization [3], lag synchronization [4], generalized synchronization [5] etc. At the same time, many effective technologies have been developed for instance nonlinear feedback control[6], impulsive method [7], adaptive method [8-9], sliding mode control [10], back-stepping control method [11] etc.

Among several control methods, sliding mode control has received a great deal of attentions due to its robustness to parameters uncertainty and invariance to unknown disturbance. Haeri and Tavazoe [12] have studied the synchronization of chaotic systems with uncertainty using active sliding mode control. Aghababa [13] have proposed synchronization of two different chaotic systems with unknown parameters via sliding mode technique. H.Zhang and X.K. Ma [14] have achieved synchronization of chaotic systems with parametric uncertainty using active sliding mode control.

Nevertheless, the previous methods have studied chaotic systems with known bounds of uncertainties and external disturbances. For instance, Cai et al. [15] have reported modified projective chaos synchronization between two different chaotic systems with external disturbances. Wafaa Jawaada has proposed a robust active sliding mode for anti-synchronization of hyperchaotic systems with uncertainties and external disturbances. However, in the practical and real applications, it is difficult to determine the bounds of the uncertainties and external disturbances. The adaptive control is considered for this problem. W. Guo et al have designed a simple adaptive-feedback controller to synchronize a chaotic system [16].

However, the most of the aforementioned works were involved mainly with low-dimensional chaos system, characterized by one positive Lyapunov exponent and the

aforementioned methods did not deal with the problem of hyperchaotic complete synchronization and anti-synchronization [17]. In this paper, the complete synchronization and anti-synchronization is accomplished via changed the parameter in the control function. The adaptive updating law is designed to estate the bound of the uncertainties and external disturbances under the combination of active sliding mode control and adaptive control. The stability of error dynamics are demonstrated based on the Lyapunov stability theory. Numerical simulation of hyperchaotic system illustrates the effectiveness of the proposed control method.

The rest of this paper is organized as follows. The synchronized problem is described in Section 2. Section 3 presents a brief description of designing active adaptive sliding mode controller. Simulation results presented in section 4 confirm the effectiveness and the applicability of the proposed method. Section 5 briefly concludes this paper.

2. Systems Description and Synchronized Problem Formulations

Consider the nonlinear hyperchaotic system as master system:

$$\dot{x} = (A_1 + \Delta A_1)x + f_1(x) + \delta_1(t) \tag{1}$$

And another hyperchaotic system as slave system:

$$\dot{y} = (A_2 + \Delta A_2)y + f_2(y) + \delta_2(t) + u(t) \tag{2}$$

Where $x, y \in R^n$ are the n-dimensional state vectors of the system. $A_1, A_2 \in R^{n \times n}$ denote the linear parts of the system dynamics and $f_1(x), f_2(y) : R^n \to R^n$ represent the nonlinear parts of the system. $\Delta A_1, \Delta A_2 \in R^{n \times n}$ are the matrixes of uncertainties and $\delta_1(t), \delta_2(t) \in R^{n \times n}$ are the vectors denoting external disturbances. In (2), $u(t) \in R^n$ is the n-dimensional vector that is the output of the controller.

If $A_1 = A_2$ and $f_1(x) = f_2(x)$, master system and slave system are two identical chaotic (hyperchaotic) system. Otherwise they represent two different chaotic (hyperchaotic) systems.

The synchronous error system between (1) and (2) can be written as follows:

$$e = y + \beta x \tag{3}$$

When $\beta = [\beta_1, \beta_2, \cdots \beta_{n-1}, \beta_n]^T = [-1, -1, \cdots -1, -1]^T$, the synchronization type of system is complete synchronization and if $\beta = [\beta_1, \beta_2, \cdots \beta_{n-1}, \beta_n]^T = [1, 1, \cdots 1, 1]^T$, the synchronization type is anti-synchronization.

The dynamics of the synchronization and anti-synchronization error can be expressed as:

$$\dot{e} = (A_2 + \Delta A_2)y + f_2(y) + \delta_2(t) + \beta((A_1 + \Delta A_1)x + f_1(x) + \delta_1(t)) + u(t)$$
$$= A_2 e + \beta(A_2 + A_1)x + f_2(y) + \beta f_1(x) + \delta_2(t) + \beta \delta_1(t) + \Delta A_2 y + \beta \Delta A_1 x + u(t) \tag{4}$$

For convenience, the following assumption is made:

$$F(x, y) = f_2(y) + \beta f_1(x) + \beta(A_1 - A_2)x \tag{5}$$

So (4) is expressed as:

$$\dot{e} = A_2 e + F(x, y) + \Delta A_2 y + \beta \Delta A_1 x + \delta_2(t) + \beta \delta_1(t) + u(t) \tag{6}$$

The master system and the slave system are to be synchronized by designing an appropriate control that is added into the slave system such that:

$$\lim_{t \to \infty} \|e(t)\| = \lim_{t \to \infty} \|y + \beta x\| = 0 \tag{7}$$

Where $\| \ \|$ is the Euclidean norm. Assumption. $\|\Delta A_2 + \beta \Delta A_1\| < \psi$ and $\|\delta_2 + \beta \delta_1\| < \eta$. Where ψ, η are unknown positive constants.

3. Active Sliding Mode Controller Design and Analysis

According to the active control design procedure, the output of controller is used to eliminate the nonlinear part of the error dynamics. Therefore the $u(t)$ is considered as

$$u(t) = G(t) - F(x, y) \tag{8}$$

Hence the error system can be rewritten as:

$$\dot{e} = A_2 e + \Delta A_2 y + \beta \Delta A_1 x + \delta_2(t) + \beta \delta_1(t) + G(t) \tag{9}$$

There are many possible methods for the control input G(t). Without loss of generality, we choose the sliding mode control law as follows:

$$G(t) = Kv(t) \tag{10}$$

Where $K = [k_1, k_2, \cdots, k_{n-1}, k_n]^{\mathrm{T}}$ is a constant gain vector and $v(t)$ satisfies:

$$v(t) = \begin{cases} v^+(t) & s \geq 0 \\ v^-(t) & s < 0 \end{cases} \tag{11}$$

Where $s = s(e)$ is a switching surface.

The sliding mode control method involves two major stages: (1) Choosing a suitable sliding surface; and (2) designing the sliding mode controller.

In general, the switching surface can be represented as follows:

$$s = Ce \tag{12}$$

Where $C = [C_1, C_2, \cdots, C_{n-1}, C_n]^{\mathrm{T}}$ is a constant vector.

When in sliding surface, the controlled system should satisfy the following conditions:

$$s = Ce = 0 \text{ and } \dot{s} = C\dot{e} = 0 \tag{13}$$

Generally the sliding mode control method applies the constant plus proportional rate reaching law. The reaching law is expressed as:

$$\dot{s} = -\varepsilon \operatorname{sgn}(s) - rs \tag{14}$$

Where ε, r are positive real numbers.

From (9) and (14), it is obvious that

$$
\begin{aligned}
\dot{s} &= -\varepsilon \cdot \operatorname{sgn}(s) - rs = C\dot{e} \\
&= C\left[A_2 e + \Delta A_2 y + \beta \Delta A_1 x + \delta_2(t) + \beta \delta_1(t) + Kv(t)\right]
\end{aligned} \tag{15}
$$

Hence, the $v(t)$ can be expressed as follow:

$$v(t) = -(CK)^{-1}\left[CA_2 e + C\Delta A_2 y + \beta C\Delta A_1 x + C\delta_2(t) + \beta C\delta_1(t) + rs + \varepsilon \cdot \operatorname{sgn}(s)\right] \tag{16}$$

Where the existence of $(CK)^{-1}$ is a necessary condition.

There exist system uncertainties and external disturbances in (16). In this regard, we propose the following control law:

$$v(t) = -(CK)^{-1}\left[CA_2e + \|C\|\hat{\psi}\,\mathrm{sgn}(s) + \|C\|\hat{\eta}\,\mathrm{sgn}(s) + rs + \varepsilon \cdot \mathrm{sgn}(s) \right] \tag{17}$$

Where $\hat{\psi}, \hat{\eta}$ are estimations for ψ, η of Assumption 2.

To tackle the bounds of the error system uncertainties and external disturbances, the suitable adaptive laws are defined as follow:

$$\dot{\hat{\psi}} = \|Cs\|, \dot{\hat{\eta}} = \|Cs\| \tag{18}$$

To prove that the error dynamics (6) is asymptotically stable, we choose the Lyapunov function defined by the equation

$$V = \frac{1}{2}\,s^2 + \frac{1}{2}\tilde{\psi}^2 + \frac{1}{2}\tilde{\eta}^2 \tag{19}$$

Where $\tilde{\psi} = \hat{\psi} - \psi, \tilde{\eta} = \hat{\eta} - \eta$.

Taking derivative of the Lyapunov function candidate with respect to time, one has

$$
\begin{aligned}
\dot{V} &= s\dot{s} + \tilde{\psi}\dot{\tilde{\psi}} + \tilde{\eta}\dot{\tilde{\eta}} \\
&= sC\{A_2e - K(CK)^{-1}(CA_2e + \|C\|\hat{\psi}\,\mathrm{sgn}(s) + \|C\|\hat{\eta}\,\mathrm{sgn}(s) + \varepsilon\,\mathrm{sgn}(s) + rs) \\
&\quad + \Delta A_2y + \beta\Delta A_1x + \delta_2(t) + \beta\delta_1(t)\} + \tilde{\psi}\dot{\tilde{\psi}} + \tilde{\eta}\dot{\tilde{\eta}} \\
&= -\varepsilon s\,\mathrm{sgn}(s) - rs^2 - \|C\|\hat{\psi}s\,\mathrm{sgn}(s) - \|C\|\hat{\eta}s\,\mathrm{sgn}(s) + sC(\Delta A_2y + \beta\Delta A_1x) \\
&\quad + sC(\delta_2(t) + \beta\delta_1(t)) + \tilde{\psi}\dot{\tilde{\psi}} + \tilde{\eta}\dot{\tilde{\eta}} \\
&\le -\varepsilon s\,\mathrm{sgn}(s) - rs^2 - \|C\|\hat{\psi}s\,\mathrm{sgn}(s) - \|C\|\hat{\eta}s\,\mathrm{sgn}(s) + \|Cs\|\|\Delta A_2y + \beta\Delta A_1x\| \\
&\quad + \|Cs\|\|\delta_2(t) + \beta\delta_1(t)\| + \tilde{\psi}\dot{\tilde{\psi}} + \tilde{\eta}\dot{\tilde{\eta}} \\
&\le -\varepsilon s\,\mathrm{sgn}(s) - rs^2 - \|C\|\hat{\psi}s\,\mathrm{sgn}(s) - \|C\|\hat{\eta}s\,\mathrm{sgn}(s) + \|Cs\|\psi + \|Cs\|\eta + \tilde{\psi}\dot{\tilde{\psi}} + \tilde{\eta}\dot{\tilde{\eta}} \\
&= -\varepsilon s\,\mathrm{sgn}(s) - rs^2 - \|Cs\|\hat{\psi} - \|Cs\|\hat{\eta} + \|Cs\|\psi + \|Cs\|\eta + (\hat{\psi} - \psi)\dot{\tilde{\psi}} + (\hat{\eta} - \eta)\dot{\tilde{\eta}} \\
&= -\varepsilon s\,\mathrm{sgn}(s) - rs^2 - \|Cs\|(\hat{\psi} - \psi) - \|Cs\|(\hat{\eta} - \eta) + (\hat{\psi} - \psi)\dot{\tilde{\psi}} + (\hat{\eta} - \eta)\dot{\tilde{\eta}} \\
&= -\varepsilon |s| - rs^2 \le 0
\end{aligned}
\tag{20}
$$

Therefore, the condition for Lyapunov stability is satisfied. States of the error system can reach the sliding surface asymptotically.

4. Numerical Simulations

In this section, the numerical simulation results of two methods with different control law are discussed and validate the effectiveness and superiority of sliding mode controller that we proposed in Section 3.

The hyperchaotic Chen system is described by:

$$
\begin{cases}
\dot{x}_1 = a(x_2 - x_1) + x_4 \\
\dot{x}_2 = bx_1 - x_1x_3 + cx_2 \\
\dot{x}_3 = x_1x_2 - dx_3 \\
\dot{x}_4 = x_2x_3 + rx_4
\end{cases}
\tag{21}
$$

Where $a = 35, b = 7, c = 12, d = 3, r = 0.5$.

The hyperchaotic Lorenz system is described by:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_4 \\ \dot{x}_2 = -x_1 x_3 + bx_{1} - x_2 \\ \dot{x}_3 = x_1 x_2 - cx_3 \\ \dot{x}_4 = -x_1 x_3 + dx_4 \end{cases} \tag{22}$$

Where $a = 10, b = 28, c = 8/3, d = 1.3$.

4.1. Synchronization between Two Identical Chen Systems

When $\beta = [\beta_1, \beta_2, \cdots \beta_{n-1}, \beta_n]^{\mathrm{T}} = [-1, -1, \cdots -1, -1]^{\mathrm{T}}$, the synchronization type is complete synchronization. We choose the Chen system for the master and slave system. Let us consider that:

$$\Delta A_1 = \begin{pmatrix} 3 & 1 & 0 & 1 \\ 0 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \delta_1(t) = \begin{pmatrix} -0.5\cos(50t) \\ 0.5\sin(50t) \\ \sin(50t) \\ -\sin(50t) \end{pmatrix} \tag{23}$$

$$\Delta A_2 = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \delta_2(t) = \begin{pmatrix} 0.1\cos(50t) \\ -0.3\sin(50t) \\ -0.2\sin(50t) \\ -0.2\sin(50t) \end{pmatrix} \tag{24}$$

Hence, the master system can be rewritten as the following:

$$\begin{cases} \dot{x}_1 = 35(x_2 - x_1) + x_4 + 3x_1 + x_2 + x_4 - 0.5\cos(50t) \\ \dot{x}_2 = 7x_1 - x_1 x_3 + 12x_2 - 2x_2 + 0.5\sin(50t) \\ \dot{x}_3 = x_1 x_2 - 3x_3 + x_1 + \sin(50t) \\ \dot{x}_4 = x_2 x_3 + 0.5 x_4 + x_3 - \sin(50t) \end{cases} \tag{25}$$

The slave system can be rewritten as the following:

$$\begin{cases} \dot{y}_1 = 35(y_2 - y_1) + y_4 + 2y_1 + y_2 + y_4 - 0.5\cos(50t) + u_1(t) \\ \dot{y}_2 = 7y_1 - x_2 y_3 + 12y_2 - 2y_2 + 0.3\sin(50t) + u_2(t) \\ \dot{y}_3 = y_1 y_2 - 3y_3 + 2y_1 - 0.2\sin(50t) + u_3(t) \\ \dot{y}_4 = y_2 y_3 + 0.5 y_4 + y_3 - 0.2\sin(50t) + u_4(t) \end{cases} \tag{26}$$

Make $e = y - x$, we can get the error system:

$$\begin{cases} \dot{e}_1 = 35(e_2 - e_1) + 2e_4 + 2e_1 + e_2 - x_1 + u_1(t) \\ \dot{e}_2 = 7e_1 + (12 - 2)e_2 - (x_2 y_3 - x_1 x_3) - 0.2\sin(50t) + u_2(t) \\ \dot{e}_3 = -3e_3 + e_1 + y_1 + y_1 y_2 - x_1 x_2 - 1.2\sin(50t) + u_3(t) \\ \dot{e}_4 = 0.5e_4 + e_3 + y_2 y_3 + x_2 x_3 + 0.8\sin(50t) + u_4(t) \end{cases} \tag{27}$$

The control parameters is chosen as $C = (0, 2, 1, -1)$, $K = (1, 1, 0, 1)^{\mathrm{T}}$ then the sliding surface $s = 2e_2 + e_3 - e_4$.

In simulation the initial values are assumed as $(x_1(0), x_2(0), x_3(0), x_4(0)) = (1, 2, -1, 2)$, $(y_1(0), y_2(0), y_3(0), y_4(0)) = (2, 5, -1, 2)$. The numerical simulation results are shown in Figure 1 and Figure 2. Figure 1 shows the states of the master system and the slave system. It can be

seen that the slave system can trace the master system successfully when output of the proposed controller is in action at 5 second.
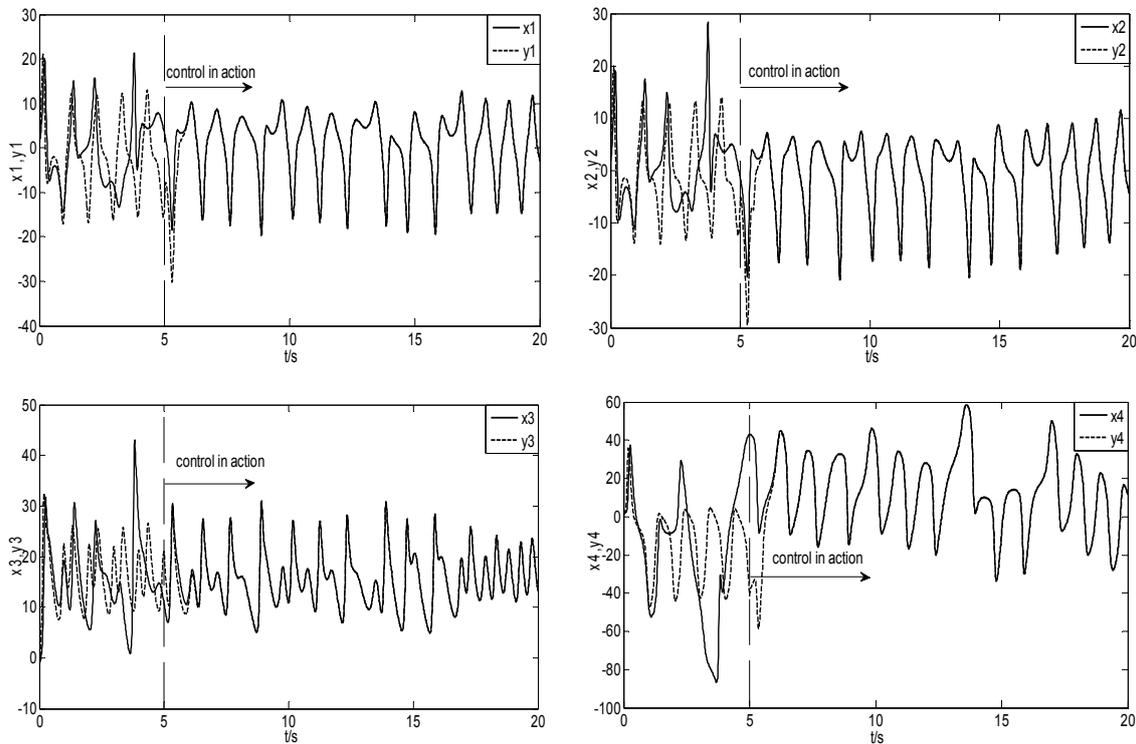
Figure 1. System States under the Proposed Method in the Paper
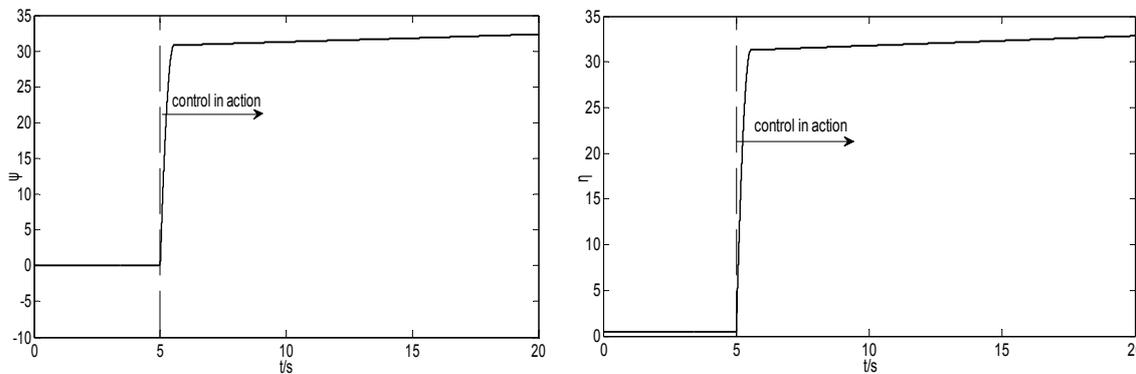
Figure 2. Time Response of the Update Parameter ψ,η

4.2 Anti-synchronization between Lorenz System and Chen System

The $\beta = \left[\beta_1, \beta_2, \cdots \beta_{n-1}, \beta_n\right]^{\mathrm{T}} = \left[-1, -1, \cdots -1, -1\right]^{\mathrm{T}}$, the synchronization type is anti-synchronization. We choose the Chen system and Lorenz system for the master and slave system. Let us consider that:

$$\Delta A_1 = \Delta A_2 = \begin{pmatrix} 3 & 1 & 0 & 1 \\ 0 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \delta_1(t) = \delta_2(t) = \begin{pmatrix} -0.5\cos(50t) \\ 0.5\sin(50t) \\ \sin(50t) \\ -\sin(50t) \end{pmatrix} \tag{28}$$

Hence, the master system can be rewritten as the following:

$$\begin{cases} \dot{x}_1 = 35(x_2 - x_1) + x_4 + 3x_1 + x_2 + x_4 - 0.5\cos(50t) \\ \dot{x}_2 = 7x_1 - x_1x_3 + 12x_2 - 2x_2 + 0.5\sin(50t) \\ \dot{x}_3 = x_1x_2 - 3x_3 + x_1 + \sin(50t) \\ \dot{x}_4 = x_2x_3 + 0.5x_4 + x_3 - \sin(50t) \end{cases} \tag{29}$$

The slave system can be rewritten as the following:

$$\begin{cases} \dot{x}_1 = 10(y_2 - y_1) + y_4 + 3y_1 + y_2 + y_4 - 0.5\cos(50t) + u_1(t) \\ \dot{x}_2 = -y_1y_3 + 28y_1 - y_2 - 2y_2 + 0.5\sin(50t) + u_2(t) \\ \dot{x}_3 = y_1y_2 - 28/3y_3 + y_1 + \sin(50t) + u_3(t) \\ \dot{x}_4 = -y_1y_3 + 1.3y_4 + y_3 - \sin(50t) + u_4(t) \end{cases} \tag{30}$$

Make $e = y + x$, the error system shown that:

$$\begin{cases} \dot{e}_1 = 10(e_2 - e_1) + 3e_1 + 2e_4 + e_2 + 25(x_2 - x_1) - \cos(50t) + u_1(t) \\ \dot{e}_2 = 7e_1 + 21y_1 + 2e_2 - (y_1y_3 + x_1x_3) + \sin(50t) + u_2(t) \\ \dot{e}_3 = -3e_3 + e_1 - 19/3y_3 + y_1y_2 + x_1x_2 + 2\sin(50t) + u_3(t) \\ \dot{e}_4 = 0.5e_4 + e_3 + 0.8y_4 - y_1y_3 + x_2x_3 - 2\sin(50t) + u_4(t) \end{cases} \tag{31}$$

The control parameters is chosen as $C = (0, 2, 1, -1)$, $K = (1, 1, 0, 1)^T$ then the sliding surface $s = 2e_2 + e_3 - e_4$.

In simulation the initial values are assumed as $(x_1(0), x_2(0), x_3(0), x_4(0)) = (1,2,-1,2)$, $(y_1(0), y_2(0), y_3(0), y_4(0)) = (2,5,-1,2)$. The numerical simulation results are shown in Figure 3 and Figure 4. Figure 3 shows the states of the master system and the slave system. Obviously, the anti-synchronization is realized when the control input are active at 5 second.
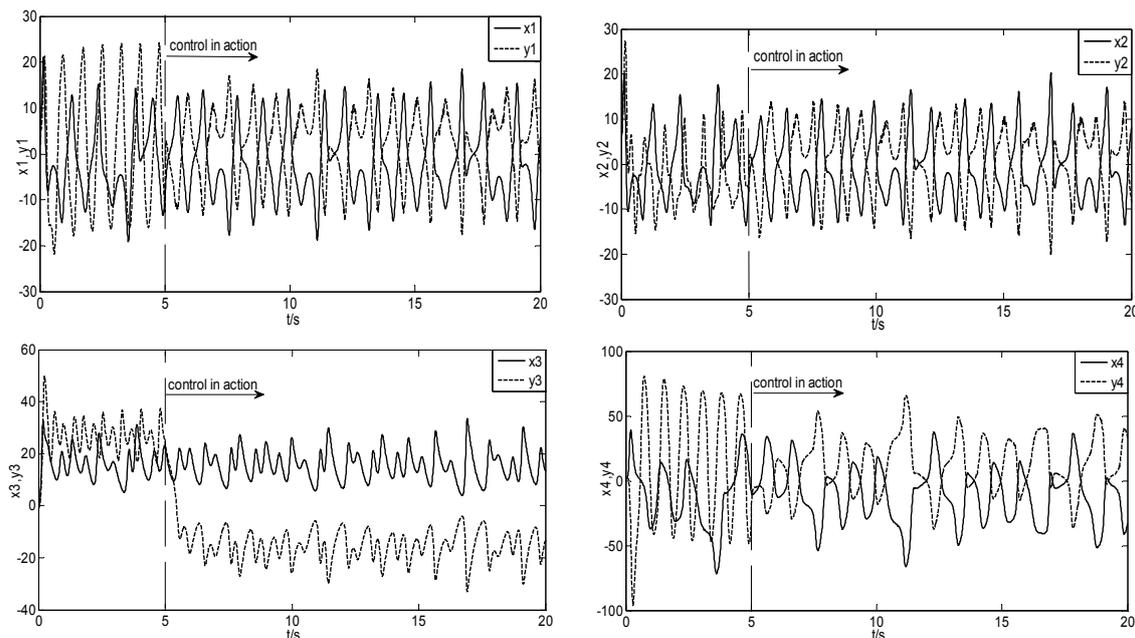
Figure 3. System States under the Proposed Method in the Paper
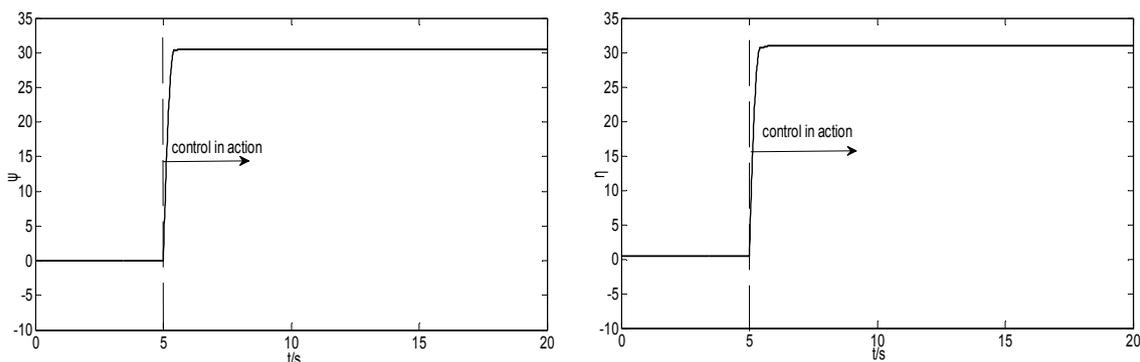
Figure 4. Time Response of the Update Parameter ψ,η

The time responses of the adaptive parameters ψ and η are shown in Figure 2 and Figure 4 in two examples. Obviously, the adaptive parameters gradually converge to some constant.

In comparison with the sliding mode method, our proposed scheme estimated the bound of the uncertainties and external disturbances, which in turn proves that our adaptive gain law is effective.

5. Conclusion

Chaotic systems which are required synchronization and anti-synchronization need good control systems. By choosing the appropriate parameters, the active adaptive sliding mode controller can have good synchronization result. The simulation results have validated the proposed controller.

Acknowledgment

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