

# A Simple Compound Control Method for Flexible-link Manipulator

Sanxiu Wang<sup>\*1,2</sup>, Li Yu<sup>1</sup>, Jianming Xu<sup>1</sup>, Kexing Xu<sup>1</sup>

<sup>1</sup>College of Information Engineering, Zhejiang University of Technology, Hangzhou 310032, China

<sup>2</sup>College of Physics and Electronic Engineering, Taizhou University, Taizhou 318000, China

\*Corresponding author, e-mail: wsx8188@163.com

## Abstract

*This paper presents a composite control scheme based on singular perturbation theory, which decouples the system of flexible-link manipulator into rigid motion (slow subsystem) and flexible vibration (fast subsystem) two different time scale subsystems. For the slow subsystem, a controller combined computer torque method and robust control is employed to track the desired trajectory, while, a state feedback control is used to stabilize the fast subsystem to suppress the vibration. The simulation results show that the proposed control strategy has good tracking performance and suppress the flexible-link vibration effectively.*

**Keywords:** flexible manipulator, singular perturbation, slow subsystem, fast subsystem

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## 1. Introduction

Flexible manipulators have lots of advantages, such as light weight, high speed and high payload to robot weight ratio. Compared to rigid robot, it require less material, have less overall cost, are more maneuverable and are safer to operate [1]. All of these make flexible manipulator have been widely applied in space, defense military, industry etc. and attracted more and more attention.

But the elastic vibration caused by joint flexibility of flexible robot makes the model of motion more complicate. For example, the order of the related dynamics become twice that of rigid robots and the number of degree of freedom is larger than the number of control input [2]. Therefore, the control of flexible manipulator is a complex and challenging problem.

Considerable approaches on the control problem of flexible joint robot have been introduced and various control algorithms have been proposed in the literatures. Feedback linearization [3], is good for tracking control of nonlinear systems, however, the method depends not only on the exact knowledge of the dynamic parameters but also on the joint acceleration [4]. To deal with the unavoidable uncertainties of flexible manipulator, adaptive control schemes[5-6] were developed, which can adjust its control law to the parameters change, but with the huge computation. Robust control [7] also has the ability to overcome parametric uncertainties and external disturbance, but the upper bound of uncertainty is needed. Sliding mode control, back-stepping approaches and PD controller etc. are all widely used in robot control.

Recently years, singular perturbation technique [8-9] has been shown to be a convenient strategy for the control of flexible manipulator. It is well known that the dynamics of singularly perturbed systems can be approximated by the dynamics of the corresponding reduced-order and boundary-layer subsystems for sufficiently small values of the singular perturbation parameter. the flexible manipulator can be decoupled into rigid motion and elastic vibrations by singular perturbation theory. In this way, the slow subsystem represents rigid motion is of the same order as the rigid manipulator and can also apply various control schemes developed for the rigid robot. While, the slow state variables being parameters in the fast subsystem, which represents flexible vibration.

This paper proposes a composite control scheme for flexible manipulators based on the singularly perturbation theory. A controller combined computer torque method and robust

control is employed to control the slow subsystem to track the desired trajectory while a state feedback control scheme is used to the fast subsystem to suppress the elastic vibration.

The paper is organized as follows: section 2 gives the dynamic description of flexible manipulator. Singularly perturbed model is depicted in section 3. Section 4 presents the controller of slow subsystem and fast subsystem respectively and forms the overall control input. Simulation results are addressed in section 5 to demonstrate the performance of the proposed method. Finally, a brief conclusion is given in section 6.

## 2. Dynamic Equation of Flexible-link Manipulator

From the Euler–Lagrange formulation and assumed modes approach, the dynamic equation of n-link flexible manipulator can be derived as:

$$\begin{bmatrix} D_{11}(\theta, q) & D_{12}(\theta, q) \\ D_{21}(\theta, q) & D_{22}(\theta, q) \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} C_1(\theta, \dot{\theta}) \\ C_2(\theta, \dot{\theta}) \end{bmatrix} + \begin{bmatrix} S_1(\theta, \dot{\theta}, q, \dot{q}) \\ S_2(\theta, \dot{\theta}, q, \dot{q}) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} \theta \\ q \end{bmatrix} = \begin{bmatrix} \tau \\ 0 \end{bmatrix} \quad (1)$$

Where,  $\theta = [\theta_1 \dots \theta_n]^T$  is the vector of joint variable.  $q = [q_1 \dots q_m]^T$  is the vector of modal variables. In which,  $q_i = [q_{i1} \dots q_{im}]^T$ .  $n, m$  represent the number of joint angle and modal respectively.

$D = \begin{bmatrix} D_{11}(\theta, q) & D_{12}(\theta, q) \\ D_{21}(\theta, q) & D_{22}(\theta, q) \end{bmatrix}$  is the positive definite symmetric inertia matrix.  $C_1, C_2$  is the vector corresponding to coriolis, centrifugal. In which, the effect of gravity is ignored, because the motion of each link is assumed to be in the horizontal plane.  $S_1, S_2$  is the vector containing terms of the interactions among joint angle, modal displacements and their time derivatives.  $K$  is the matrix corresponding to the structural stiffness of the flexible links.  $\tau$  is the control torque.

## 3. Singularly Perturbed Model

As previously mentioned, a successful solution to the control problem of flexible manipulator has been provided by the singular perturbation technique, which essentially uses a perturbation parameter to divide the complex dynamic systems into simpler subsystems at different time scales. The procedure to decouple the flexible manipulator model into a two-time-scale singular perturbation model is as follows.

Inertia matrix  $D$  is positive definite symmetric and non-singular, so there is an inverse matrix  $H$ , which can be represented as follow:

$$H = D^{-1} = \begin{bmatrix} H_{rr} & H_{rf} \\ H_{fr} & H_{ff} \end{bmatrix} \quad (2)$$

Where  $H_{rr} \in R^{n \times n}$ ,  $H_{rf} \in R^{n \times m}$ ,  $H_{ff} \in R^{m \times m}$ . The subscripts  $r$  and  $f$  refer to the sub matrices associated with the joint and modal variables, respectively.

Multiply (1) by (2) from the left, rearrange terms and then Equation (1) can be written as:

$$\ddot{\theta} = -H_{rr}C_1 - H_{rf}C_2 - H_{rr}S_1 - H_{rf}S_2 - H_{rf}Kq + H_{rr}\tau \quad (3)$$

$$\ddot{q} = -H_{fr}C_1 - H_{ff}C_2 - H_{fr}S_1 - H_{ff}S_2 - H_{ff}Kq + H_{fr}\tau \quad (4)$$

Now, let us define  $\mu = \frac{1}{k}$ , where,  $k = \min(k_{ij})$  is the smallest stiffness constant. Then

introduce new fast variable  $z = \frac{q}{\mu}$  and scaled stiffness matrix  $\tilde{K} = \mu K$

Using these new variables, Equation (3), (4) can be rewritten as:

$$\begin{aligned} \ddot{\theta} = & -H_{rr}(\theta, \mu z)C_1(\theta, \dot{\theta}) - H_{rf}(\theta, \mu z)C_2(\theta, \dot{\theta}) - H_{rr}(\theta, \mu z)S_1(\theta, \dot{\theta}, \mu z, \mu \dot{z}) \\ & - H_{rf}(\theta, \mu z)S_2(\theta, \dot{\theta}, \mu z, \mu \dot{z}) - H_{rf}(\theta, \mu z)\tilde{K}z + H_{rr}(\theta, \mu z)\tau \end{aligned} \quad (5)$$

$$\begin{aligned} \mu \dot{z} = & -H_{fr}(\theta, \mu z)C_1(\theta, \dot{\theta}) - H_{ff}(\theta, \mu z)C_2(\theta, \dot{\theta}) - H_{fr}(\theta, \mu z)S_1(\theta, \dot{\theta}, \mu z, \mu \dot{z}) \\ & - H_{ff}(\theta, \mu z)S_2(\theta, \dot{\theta}, \mu z, \mu \dot{z}) - H_{ff}(\theta, \mu z)\tilde{K}z + H_{fr}(\theta, \mu z)\tau \end{aligned} \quad (6)$$

On the right side of these two equations, the superscript indicates that the corresponding quantities have been multiplied by  $\tilde{K}$ .

It can be shown that the system described by Equation (5) have a boundary layer phenomenon in the fast variable  $z$  because of the presence of  $\mu$ . Formally, setting  $\mu = 0$  and substituting into Equation (5), (6) gives:

$$\begin{aligned} \ddot{\theta}_s = & -H_{rrs}(\theta, 0)C_{1s}(\theta, \dot{\theta}) - H_{rfs}(\theta, 0)C_{2s}(\theta, \dot{\theta}) - H_{rrs}(\theta, 0)S_{1s}(\theta, \dot{\theta}, 0, 0) \\ & - H_{rfs}(\theta, 0)S_{2s}(\theta, \dot{\theta}, 0, 0) - H_{rfs}(\theta, 0)\tilde{K}z_s + H_{rrs}(\theta, 0)\tau_s \end{aligned} \quad (7)$$

$$\begin{aligned} 0 = & -H_{frs}(\theta, 0)C_{1s}(\theta, \dot{\theta}) - H_{ffs}(\theta, 0)C_{2s}(\theta, \dot{\theta}) - H_{frs}(\theta, 0)S_{1s}(\theta, \dot{\theta}, 0, 0) \\ & - H_{ffs}(\theta, 0)S_{2s}(\theta, \dot{\theta}, 0, 0) - H_{ffs}(\theta, 0)\tilde{K}z_s + H_{frs}(\theta, 0)\tau_s \end{aligned} \quad (8)$$

Where, subscript  $s$  indicates that the system is considered in the slow time scale.  $\tau_s$  is the control torque in the slow time scale, under which, the actual output angle  $\theta$  will tend to  $\theta_d$ .

From Equation (8), we can obtain:

$$\begin{aligned} z_s = & \tilde{K}^{-1}H_{ffs}^{-1}(\theta, 0)(-H_{frs}(\theta, 0)C_{1s}(\theta, \dot{\theta}) - H_{ffs}(\theta, 0)C_{2s}(\theta, \dot{\theta}) \\ & - H_{frs}(\theta, 0)S_{1s}(\theta, \dot{\theta}, 0, 0) - H_{ffs}(\theta, 0)S_{2s}(\theta, \dot{\theta}, 0, 0) + H_{frs}(\theta, 0)\tau_s) \end{aligned} \quad (9)$$

And it is not difficult to find that there satisfies the following relation:

$$D_{11s}^{-1} = -H_{rfs}(\theta, 0)H_{ffs}^{-1}(\theta, 0)H_{frs}(\theta, 0) + H_{rrs}(\theta, 0) \quad (10)$$

Combining Equation (9), (10) and (7), yields the dynamic equation of slow subsystem.

$$D_{11s}(\theta, 0)\ddot{\theta}_s + C_{1s}(\theta, \dot{\theta}) = \tau_s \quad (11)$$

In order to deduce the dynamic equation of fast subsystem, It is essential to define a new fast time-scale and given as  $\sigma = t/\sqrt{\mu}$ . Introducing new fast variable  $z'_1 = z - z_s$ ,  $z'_2 = \sqrt{\mu}\dot{z}$ .

Then leads to the dynamic equation of fast subsystem as follows:

$$\frac{dz'_1}{d\sigma} = z'_2 \quad (12)$$

$$\frac{dz'_2}{d\sigma} = -H_{ffs}\tilde{K}z'_1 + H_{frs}\tau_f \quad (13)$$

Where  $\tau_f$  is the control torque of fast subsystem, which is used to suppress the elastic vibration.

#### 4. Control Law Design

From the above singular perturbation model of flexible manipulator, combining  $\tau_s$  and  $\tau_f$  forms a composite control law:

$$\tau = \tau_s + \tau_f \quad (14)$$

Where  $\tau_s$  is the tracking controller for the slow subsystem and  $\tau_f$  is the stabilizer for the fast subsystem,  $\tau$  is the overall control torque.

Figure 1 gives the structure for the composite controller.

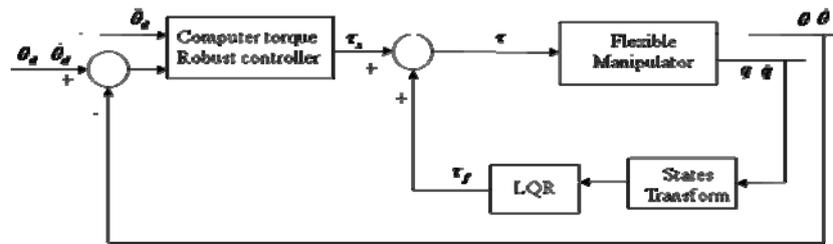


Figure 1. The Structure of Composite Controller

##### 4.1. Controller Design for the Trajectory Tracking Subsystem

When the slow moving subsystem (11) is exactly known, the following controller based on the computed torque method can guarantee the asymptotic convergence of the system output tracking error.

$$\tau_s = D_{11}(\ddot{\theta}_d + k_{ps} \times e + k_{vs} \times \dot{e}) \quad (15)$$

In which,  $\dot{\theta}_d$  is the desired joint,  $e = \theta_d - \theta$  is the tracking error.  $k_{ps}, k_{vs}$  are defined as the gain matrix of position and velocity, respectively.

Substituting (14) into (10) yields:

$$\ddot{e} + k_{vs} \dot{e} + k_{ps} e = 0 \quad (16)$$

Which indicates that the tracking error will converge to zero with proper choice of gains.

But in practice the robot system can not be accurately known. When the system exists unmodeled dynamic and external disturbance, the control law (15) is not able to guarantee a good dynamic performance and stability of system. In order to eliminate the impact of system uncertainty, we add the robust compensation item. Then the control law of slow subsystem can be rewritten as:

$$\tau_s = D_{11}(\ddot{\theta}_d + k_{ps} \times e + k_{vs} \times \dot{e}) + u_r \quad (17)$$

Where,  $u_r = -\frac{\rho^2 s}{\rho \|s\| + \varepsilon}$  is used to compensate the influence of uncertainty.  $s = \dot{e} + \lambda e$ ,  $\rho$  is the upper bounded of uncertainty,  $\varepsilon$  is a positive number.

##### 4.2. Controller Design for the Fast Subsystem

The state space representation of fast subsystem can be expressed as:

$$\dot{X} = AX + B\tau_f \quad (18)$$

Where:

$$X = \begin{bmatrix} z_f^1 \\ z_f^2 \end{bmatrix} \quad A = \begin{bmatrix} 0 & I \\ -H_{ffs} \tilde{K} & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ H_{frs} \end{bmatrix}$$

The system described by Equation (18) is a linear system, which can easily be shown to be completely controllable. A fast feedback controller can damp out the deflection at steady state as fast as possible, is represented as:

$$\tau_f = -k_f X \quad (19)$$

Where, the feedback gain  $k_f$  can be obtained through optimizing the cost function using LQR approach and is given by:

$$k_f = R^{-1} B^T P \quad (20)$$

Where  $P$  is obtained from the Riccati equation.

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (21)$$

In which, weighting matrix  $R$ ,  $Q$  are the positive definiteness matrix.

## 5. Experimental Results

In order to demonstrate the validation of the proposed method, in this section, a planar manipulator with one link is taken into consideration. The manipulator structure is shown in Figure 2.

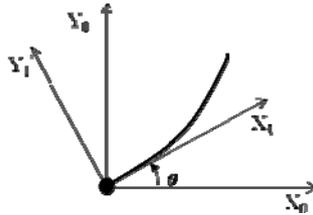


Figure 2. The Structural Model of One-link Flexible Manipulator

The model of one-link flexible manipulator has been description in section 2. The detailed matrix and its elements expression in the model can be seen in literature [10]. And parameters of the flexible manipulator for the experiment are shown in Table 1.

Table 1. Experiment Parameters

Beam length	Beam mass	Joint inertia	Flexural rigidity	Payload mass	Payload inertia
1m	0.2kg	1kg.m <sup>2</sup>	60N m <sup>2</sup>	0.1kg	0.001 kg.m <sup>2</sup>

A very simple desired trajectory is given by  $\theta_d = \sin(2\pi t)$ . The initial conditions of the modes and its time derivative have been consideration as  $q_1 = q_2 = 0$ ,  $\dot{q}_1 = \dot{q}_2 = 0$ .

The slow subsystem controller parameters in (17) are considered as  $k_{ps} = 60$ ,  $k_{vs} = 50$ ,  $\lambda = 2$ ,  $\rho = 1$  and  $\varepsilon = 0.2$ . For the fast subsystem, the gain in (19) is chosen as:

$$k_f = \begin{bmatrix} 0.1220 & 0.5616 \\ 0.5948 & 1.1316 \end{bmatrix}$$

The simulations are shown in Figure 3-5.

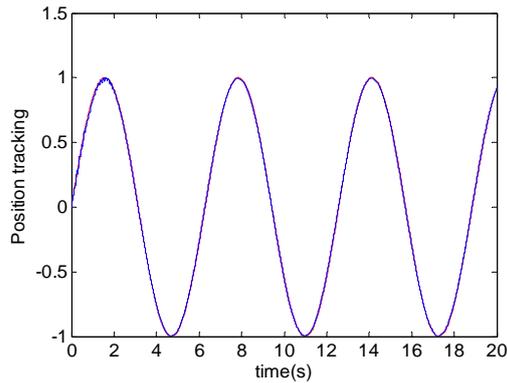


Figure 3. Tracking Response

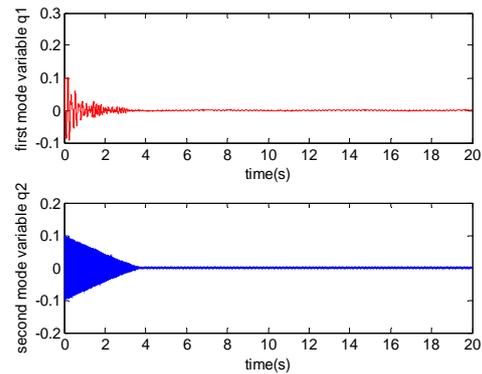


Figure 4. The First and Second Mode

Tracking response is depicted in Figure 3, which shows that position tracking control can be achieved using proposed control strategy. The actual output trajectory can tend to desired trajectory well. Figure 4 shows the first and second modes for the flexible-link, from which, we can find that the elastic vibration can be suppressed effectively by the method. Although there are tiny vibrations in the initial stage, the vibration can be restrained to almost zero in very short time. The overall control input is shown in Figure 5.

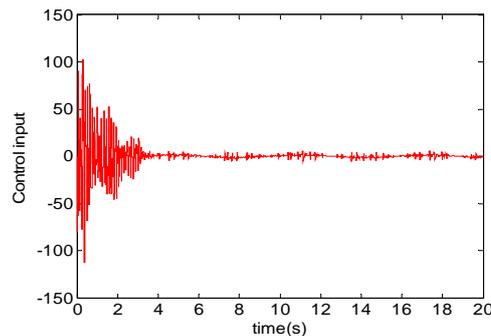


Figure 5. Control Torque

### 3. Conclusion

The paper proposes a composite control strategy based on singular perturbation model of flexible manipulator, which decouples the system of flexible manipulator into two different time scale subsystems. Representative rigid motion slow subsystem and flexible vibration fast subsystem respectively. The combination of computer torque method and robust compensation is employed to control the slow subsystem to track the desired trajectory. At the same time, state feedback based on LQR approach is used to stabilize the fast subsystem and control the elastic vibration. The simulation results on the one-link flexible manipulator show that the

proposed scheme is able to move the flexible link along the given trajectory while suppressing the vibration that are excited during the motion of the system.

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